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Diversity-Guided Dynamic Step Firefly Algorithm

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Abstract

Firefly Algorithm is a nature-inspired optimization method, which has been shown to implement well on numerous optimization problems. But it can easily fall into the local optima and low precision. Therefore, it is very important to overcome these defects. In this paper, we use a dynamic strategy for step setting, which takes into account the population diversity of fireflies. The experiments show that the proposed algorithm improves the performance of original firefly algorithm.

Keywords: *firefly algorithm; meta-heuristic algorithm diversity; optimization; step*

1. Introduction

Firefly algorithm (FA) is a novel swarm intelligence method for optimization introduced by Xin-She Yang [1]. It simulates the social behavior of fireflies and is a meta-heuristic algorithm that can be applied to solving many optimization problems. Since FA is relatively easy to implement without requiring complex evolutionary operations[2], it has been widely utilized as an optimization tool in various applications such as multi-objective economic emission dispatch solution[3], optimum design of tower[4], circular antenna array synthesis [5], multi-objective hybrid flowshop scheduling problems[6], dynamic multidimensional knapsack problems[7], image watermarking scheme[8], optimization of SVM classifier[9], continuous optimization[10], and so on.

Although FA has demonstrated good performance in solving lots of optimization problems, it has a weakness that is premature convergence which results in a low optimization precision. Meanwhile, for FA is population-based and has a tendency to converge to local optima. This shortcoming may has a variety of reasons, but a major cause is due to lose the diversity in the population[11]. When the algorithm executes, the diversity generally decreases since sample generating bias or selection pressure, it will increase the difficulty of escaping local optima. Therefore, maintaining a higher diversity in FA is a crucial task. To improve this default of FA, several variants of FA have been proposed. In [7], an effective FA, which introduces diversity with partial random restarts and with an adaptive movable procedure, is developed and proposed for solving dynamic multidimensional knapsack problems. Coelho and Bora[12] propose a novel multi-objective variant which uses the beta probability distribution in the tuning of control parameters and it is useful to maintain the diversity of solutions. In this study, we adopt a novel strategy to evaluate the population diversity in search stages. Therefore we use the evaluation strategy of diversity to dynamically adjust the step of FA, realizing a good balance between exploration and exploitation.

The rest of the paper is organized as follows. Review of FA is summarized in Section 2. Section 3 gives the proposed method. In Section 4, the simulation experiment of the proposed method on 16 test benchmark functions is executed and the results of simulation are compared. Finally in Section 5, we draw the conclusion based on the comparative analysis.

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2. Firefly Algorithm Schema

In nature, firefly uses the flashing as a semaphore to attract each other. FA mimics the social behavior of fireflies. There are three idealized assumptions as following: 1) Every firefly will be attracted to other fireflies no matter what their gender because they are unisexual; 2) Fireflies attract each other proportionally to their brightness. The less brightness firefly will be attracted by the brighter one, the more the distance the less attractiveness. If there is no brighter firefly nearby, it will move casually; and 3) The brightness of a firefly can be computed by the value of the objective function[1].

In basic FA, there are four important issues. For simplicity, we supposed that the attractiveness of a firefly can be computed by its brightness which determined by the encoded objective function[13].

Light intensity: In basic FA, based on Yang's[1] definition, the light intensity I(r) of a firefly is associated with fitness function value which can be defined as the following equation:

$$I(r) = I_0 e^{-\gamma r^2}, \quad (1)$$

where I_0 represents the initial light intensity, γ is a fixed light absorption factor, and r denotes the distance between two individuals.

Attractiveness: The attractiveness function $\beta(r)$ can be any monotonically decreasing function defined as the following form:

$$\beta(r) = \beta_0 e^{-\gamma r^2}, \quad (2)$$

where β_0 is the attractiveness at $r = 0$.

Distance: The distance between any two individuals i and j at Xi and Xj can be computed based on Cartesian distance:

$$r_{i,j} = \left\| X_i - X_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2}, \quad (3)$$

where $x_{i,k}$ is the k-th element of the i-th firefly and d is the dimension of the object function .

Movement: The motion of a firefly i is attracted to other more attractive one j, is defined by

$$X_i = X_i + \beta_0 e^{-\gamma r_{i,j}^2} (X_j - X_i) + \alpha(rand - \frac{1}{2}), \quad (4)$$

where the first and second expression is determined by the attraction, while the third expression is a randomization with α being the step parameter, and rand is a random number generator uniformly distributed range from 0 to 1. The basic steps of the FA are summarized as the pseudo code shown in Figure 1.


```
Firefly Algorithm
begin
  Objective function f(X), X=(x1,...,xd)^T
  Generate initial population of fireflies Xi(i=1,...,n)
  Brightness Ii at Xi is determined by f(Xi)
  Define light absorption coefficient γ
  while (t<MaxGeneration)
   for i=1:n all n fireflies
    for j=1:n all n fireflies
       if (Ij>Ii)
          Move firefly i towards j in d-dimension
       end if
       Attractiveness varies with distance via exp[- γr^2]
       Evaluate new solutions and update brightness
     end for j
   end for i
  Rank the fireflies and find the current best
  end while
  Post process results and visualization
end
```

Figure 1. Pseudocode of the Basic Firefly Algorithm

3. The Proposed FA

3.1. Population Diversity Definition

Diversity is important in population based algorithms. There are many studies on diversity of evolutionary algorithms, such as particle swarm optimization [14-16], memetic algorithm [17], genetic algorithm [18,19], differential evolution algorithm[20] and cultural algorithm[21]. From these literatures, we know that the ability of exploration and exploitation of an optimization algorithm is very important in search process. An optimization algorithm can explore more regions in the search space if it has good exploration ability. What's more, it also can help to discover the optimum eventually. Therefore, an optimization algorithm should have a good balance between exploration and exploitation. Accordingly, the population diversity of FA is also helpful to adjust its ability of exploration or exploitation dynamically.

There are two approaches to measure population diversity: one is average-distance-amongst-points and another is population-distribution-entropy. The former is the simplest way which obtains the distribution information of population at current position. By means of this way, the distribution information can be got by computing the mean distance from one individual to all the other individuals. The diversity measure of the swarm can be computed as[22]:

$$Diversity(t) = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum\nolimits_{j=1}^{D}(x_{ij}(t)-\overline{x_j(t)})^2}\,, \tag{5}$$

where N is the population size, D is the dimensionality of the given function, x_{ij} is the j-th value of the i-th firefly and $\overline{x_j(\mathrm{t})}$ is the mean distance of the j-th dimension over all individuals, *i.e.* $\overline{x_j(t)} = \frac{\sum_{i=1}^{N} x_{ij}(t)}{N}$.

Population-distribution-entropy, another method of diversity measure was initially stimulated by the rule of Shannon entropy. The conceptual framework of this way is that the problem space is divided into S intervals with same size. The amount of individuals in every area is expressed as F_k, from which can be defined the probability ($S_k = F_k / N$) that individuals are located in the k-th area. Then, the diversity of population is computed as following:

$$D(t) = -\sum_{k=1}^{S} S_k \log_e S_k \ . \tag{6}$$

From the above two diversity definitions, it can be easily find that the former *Diversity (t)* describes the discrete degree of various distribution statuses, while the latter *D(t)* represents the circumstance where the individuals are distributed among the various ranges in problem space. From (6), it can be seen that a higher value of *D(t)* means a larger population distribution, and vice versa.

3.2. Dynamic Control of Step

In basic FA, the method of setting step α is static or linear. It depends on maximum generation, which cannot really reflect the searching process. In general, it is useful for fireflies to explore new search space with a large step, but it is not helpful to the convergence of global optimum. If step has a small value, the result is contrary. Therefore, the step α has a great effect on the exploration and convergence of the algorithm. It would be good for balancing the ability of local exploitation and global exploration, and it should also be concerned with its current situation such as population distribution. For this reason, we design a dynamic adjusting scheme of step α which can be controlled according to the population diversity. In this paper, we use the population-distribution-entropy method and design a dynamic adjusting scheme of step. The step α can be calculated as following:

$$\alpha(D_t) = \frac{1}{0.4 + 1.6e^{-2D_t \ln 4}} \ . \tag{7}$$

where D_t is calculated by the equation (6). The step of FA can adapt to the population distribution changed by D_t during the search process. The procedure of diversity-guided dynamic step firefly algorithm (DDSFA) is described as follows:

Step 1: Create the original population of fireflies, $\{x_1, x_2, ..., x_n\}$;

Step 2: Calculate the intensity for every firefly individual, $\{I_1, I_2, ..., I_n\}$;

Step 3: Calculate the diversity of population by Eq. (6) and update the step α according to Eq. (7);

Step 4: Move each firefly towards other brighter fireflies; the position of each firefly is computed by Eq. (4);

Step 5: Renew the solution set;

Step 6: Stop if a termination criterion is achieved or else go to Step 2.

4. Experimental Verifications

The proposed DDSFA and the basic FA are executed on 16 benchmark functions which be given as Table 1. These functions used here are all minimization problems. The simulations are run with 2GB-RAM, WIN-XP OS and MATLAB 2010b software. To avoid stochastic discrepancy, we adopted 100 independent runs for two optimization algorithms involving 100 different original solutions. The number of fireflies was N=30, the dimensions of $f_{11} - f_{16}$ are 20 and the maximum iteration number was 1000. For convenience, the search space is divided into S=30 intervals with same size. According to the previously used and suggested values in[23], we used $\beta_0 = 1$and $\gamma = 1$.

Table 1. Benchmark Functions

Functions	Formulations	Range
f_1	$\min f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$	[-1,1]
f_2	$\min f(x) = (4 - 2.1x_1^2 + \frac{x_1^4}{3})x_1^2 + x_1x_2 + 4(x_2^2 - 1)x_2^2$	[-3,3]
f_3	$\min f(x) = (\lvert x_1 \rvert - 5)^2 + (\lvert x_2 \rvert + 5)^2$	[-10,10]
f_4	$\min f(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	[-5,5]
f_5	$\min f(x) = 100(x_2 - x_1)^2 + (6.4(x_2 - 0.5)^2 - x_1 - 0.6)^2$	[-5,5]
f_6	$\min f(x) = 100(x_2^2 - x_1^2) + (x_1 - 1)^2$	[-2.048,2.048]
f_7	$\min f(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	[-10,10]
f_8	$\min f(x) = (1 + (x_1 + x_2 + 1))^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)^2(30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))$	[-2,2]
f_9	$\min f(x) = 0.5 + \frac{(\sin\sqrt{x_1^2+x_2^2})^2 - 0.5}{(1+0.001(x_1^2+x_2^2))^2}$	[-10,10]
f_{10}	$\min f(x) = (x_2 + x_1^2 - 11)^2 + (x_1 + x_2^2 - 7)^2 + x_1$	[-5,5]
f_{11}	$\min f(x) = \sum_{i=1}^{D} x_i^2$	[-5.12,5.12]
f_{12}	$\min f(x) = \sum_{i=1}^{D} (x_i - 1)^2$	[-2,2]
f_{13}	$\min f(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$	[-10,10]
f_{14}	$\min f(x) = \sum_{i=1}^{D} \lvert x_i \rvert \exp(-\sum_{i=1}^{D} x_i^2)$	[-10,10]
f_{15}	$\min f(x) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$	[-30,30]
f_{16}	$\min f(x) = \sum_{i=1}^{D} (x_i + 0.5)^2$	[-100 100]

The result comparison of the best solutions and the worst ones, the average value and the standard deviations on the benchmarks are presented in Table 2, where the best results are highlighted in boldface. From the results, it illustrates the prominent potential of the DDSFA in gaining the optimal solution with high accuracy and the solutions of DDSFA is better than the solution found by basic FA. It should also be indicated that the stability of any algorithm relies on the setting of the arguments.

Table 2. Results of Objective Functions in 100 Runs

Function	Best Value		Worst Value		Average Value		Standard Deviation	
	FA	DDSFA	FA	DDSFA	FA	DDSFA	FA	DDSFA
f_1	-1.999994E+00	**-1.999999E+00**	-1.999399E+00	**-1.999417E+00**	-1.999858E+00	**-1.999889E+00**	1.161993E-04	1.201730E-04
f_2	-1.031628E+00	-1.031628E+00	-2.154370E-01	**-1.031624E+00**	-1.023382E+00	**-1.031628E+00**	8.161449E-02	**7.933336E-07**
f_3	2.500003E+01	**2.500002E+01**	2.500177E+01	**2.500138E+01**	2.500060E+01	**2.500038E+01**	3.354487E-04	**2.358388E-04**
f_4	3.978874E-01	3.978874E-01	3.979008E-01	**3.978995E-01**	3.978907E-01	**3.978892E-01**	2.931106E-06	**2.066431E-06**
f_5	3.040799E-07	**5.983732E-08**	3.567641E-04	**5.321874E-05**	5.364283E-05	**1.273179E-05**	5.664919E-05	**1.400776E-05**
f_6	1.263531E-07	**6.849244E-08**	5.934528E-05	**1.661544E-05**	1.157003E-05	**3.265676E-06**	1.159878E-05	**3.083703E-06**
f_7	1.204366E-08	**4.767502E-09**	1.493901E-05	**5.778882E-06**	3.116501E-06	**6.986347E-07**	3.072022E-06	**9.899130E-07**
f_8	3.000005E+00	**3.000002E+00**	3.001576E+01	**3.000088E+01**	3.810522E+00	6.780153E+00	4.629984E+00	9.415897E+00
f_9	1.119313E-07	**7.278939E-08**	3.722408E-02	3.722408E-02	8.619434E-03	**7.902872E-03**	5.467298E-03	**4.933285E-03**
f_{10}	-3.783961E+00	-3.783961E+00	-2.812558E+00	**-3.783070E+00**	-3.723564E+00	**-3.783848E+00**	2.270129E-01	**1.305018E-04**
f_{11}	2.403423E+00	**1.256736E-01**	5.319622E+00	**2.387431E-01**	4.045057E+00	**1.858756E-01**	5.481454E-01	**2.641672E-02**
f_{12}	3.058342E-01	**1.718415E-02**	7.031559E-01	**5.371085E-02**	5.634570E-01	**3.687674E-02**	8.280601E-02	**6.774503E-03**
f_{13}	1.126592E+02	**7.889010E+01**	1.556594E+02	**1.139600E+02**	1.374433E+02	**9.655319E+01**	7.990968E+00	**7.686018E+00**
f_{14}	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
f_{15}	8.366610E+00	**3.436237E+00**	1.062262E+01	**4.354501E+00**	9.866501E+00	**4.047120E+00**	4.437812E-01	**2.081583E-01**
f_{16}	6.579413E+02	**5.674727E+01**	1.938634E+03	**1.220516E+02**	1.510798E+03	**9.511529E+01**	2.371815E+02	**1.318064E+01**

Figure 2-9 shows the convergence curves of two algorithms. Due to space limitations, we give only eight functions' convergence graphs.

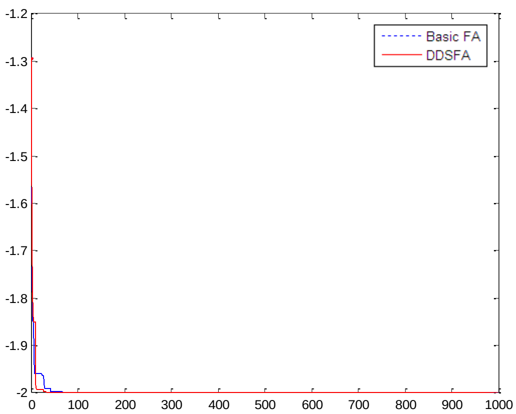

Figure 2. The Convergence Curves of f_1

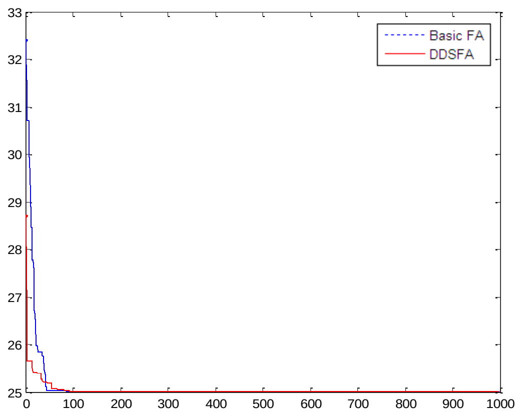

Figure 3. The Convergence Curves of f_3

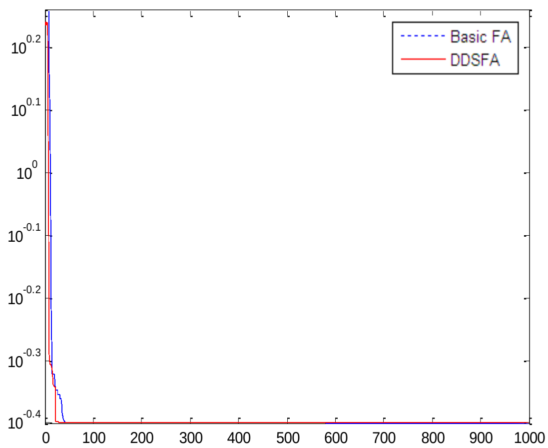

Figure 4. The Convergence Curves of

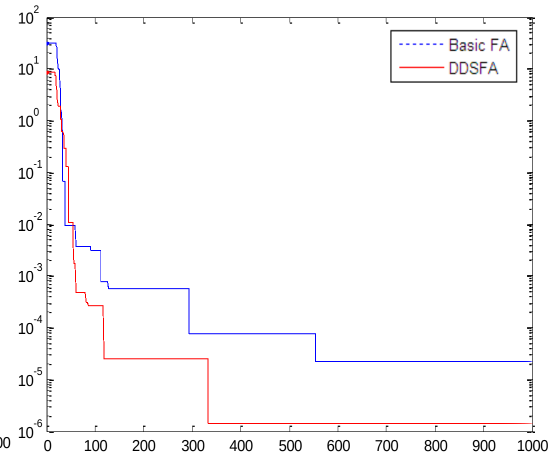

Figure 5. The Convergence Curves of f_5

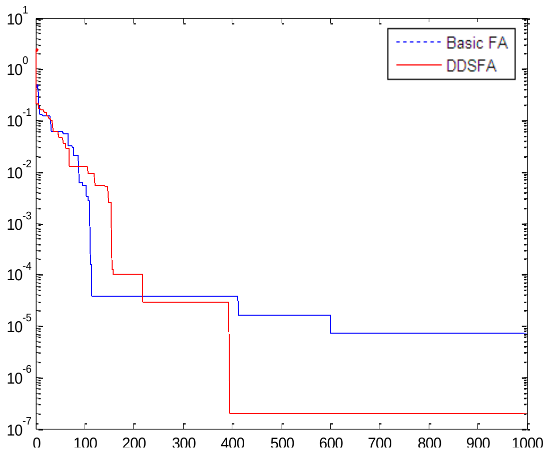

Figure 6. The Convergence Curves of f_6

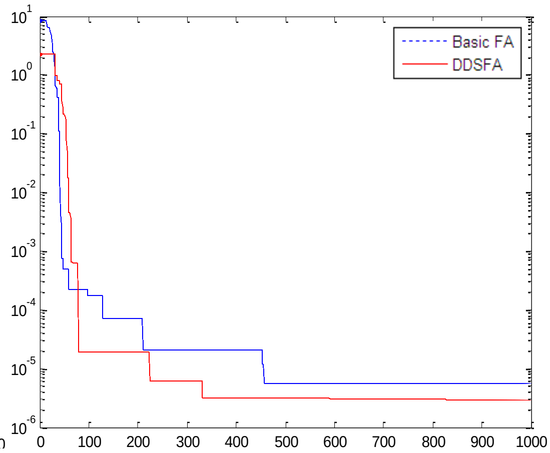

Figure 7. The Convergence Curves Of f_7

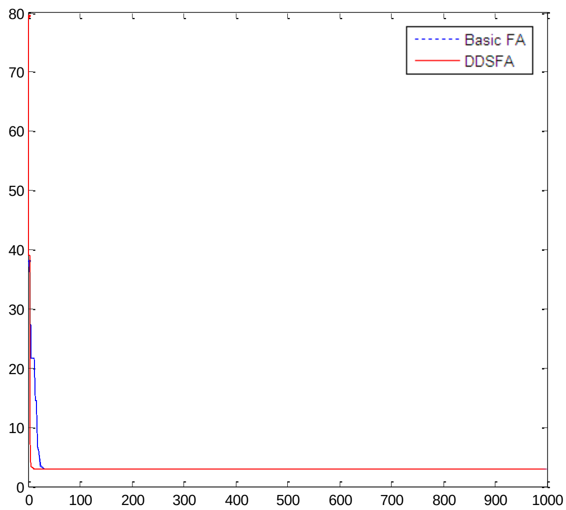

Figure 8. The Convergence Curves of f_8

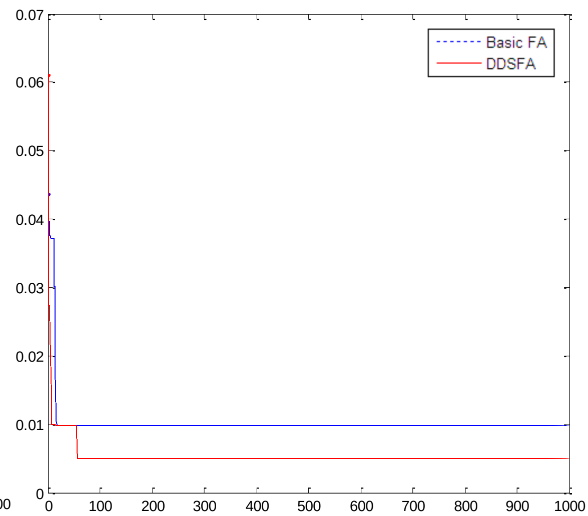

Figure 9. The Convergence Curves of f_9

It can be seen clearly from the graphs that DDSFA can behave well in given functions both convergence rate and quality of the solution. The main reason is that DDSFA adopts the diversity-guided adaptive step α during the course of evolution. This strategy can help DDSFA to avoid stagnating and premature converging. The main reason is that the strategy can keep the population diversity of FA and help fireflies to move to the global optimum.

In order to appraise DDSFA more adequately, a non-parametric statistical tool, Wilcoxon's test is also carried out. Table 3 shows the results of applying Wilcoxon's test for functions $f_1 - f_{16}$. The level of significance considered is α=0.05.

Table 3. Wilcoxon's Test Considering Functions $f_1 - f_{16}$

	f1	f2	f3	f4	f5	f6	f7	f8
Wilcoxon W	8958.000	6691.000	7910.000	8208.000	6891.000	7199.000	6940.000	8754.000
Z	-2.668	-8.207	-5.229	-4.501	-7.719	-6.966	-7.599	-3.167
p-value (2-tailed)	.008	.000	.000	.000	.000	.000	.000	.002
	f9	f10	f11	f12	f13	f14	f15	f16
Wilcoxon W	9655.500	9003.000	5050.000	5050.000	5051.00	10050.00	5050.000	5050.000
Z	-.964	-2.558	-12.217	-12.217	-12.215	.000	-12.217	-12.217
p-value (2-tailed)	.335	.011	.000	.000	.000	1.000	.000	.000

According to the suggestion given in literature[24], the smaller the p-value, the stronger the evidence against the null hypothesis. Therefore, the results in Table 3 are showing a significant difference in the performance of DDSFA and basic FA for the given functions. As for the functions of large p-values, the test shows no statistical significant difference in the performance of the algorithms. Only for f_9, Wilcoxon's test obtains p-value larger than the level of significance. The p-value of f_{14} is 1.000 because two algorithms obtain the same results, we can see in Table 2. On the whole, it is apparent from the results of the Wilcoxon's tests that DDSFA has significantly better performance than basic FA.

5. Conclusion

This paper presents a diversity-guided dynamic step firefly algorithm called DDSFA to solve optimization problems. The proposed algorithm employed a diversity-guided strategy to dynamically adjust the step. This strategy uses the dynamic changing step to adjust the FA's ability between exploration and exploitation. Experiments on 16 testing functions illustrate that the proposed algorithm has some significant improvements. How to resolve the number of intervals with same size in search space, and how to apply it in practice such as PID[25] is our future work.

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