

Improvement of the Method to Determine Weight Based on the Intuitionistic Fuzzy Entropy

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Abstract

The determining method of the weights of multiple attributes decision making needs to be improved. Firstly, three entropy measures are deduced through a similarity measure of intuitionistic fuzzy sets. Most of the entropy measures are not able to fully describe the uncertainty of things. So the weight determining results which based on the entropy, becomes somewhat inaccurate. The method of improving the effective information is proposed to determine the attribute weights in order to improve the accuracy of weights determining. Ultimately the ideal multiple attributes decision making analysis results are determined by adjustment weights with the parameter.

Keywords: *Weight, Effective Information, Multi-Attribute Decision Making, Entropy, Similarity*

1. Introduction

In the early stage of birth of fuzzy set theory, the fuzzy entropy was introduced as an important fuzzy information measure to be widely used in the fields of pattern recognition, image processing, neural networks. As the concept of fuzzy sets developing, entropy theory is applied in more and more areas.

Axiomatic definition of fuzzy entropy was given by De Luca and Termini in 1972 [1]. Bulgarian scholars Atanassov proposed the concept of intuitionistic fuzzy sets [2], which expanded the traditional fuzzy sets. Subsequently, Gau *et al.* proposed the concept of Vague sets [3]. Bustince and Burillo proved that these two concepts were essentially the same [4], and the first to propose the axiomatic definition of fuzzy entropy based on intuitionistic fuzzy sets [5]. The expanded fuzzy entropy and its associated theory have aroused widespread research interest. A large number of scholars carried out an in-depth study on intuitionistic fuzzy set and the fuzzy entropy of Vague sets. However, there was not compatibility between the definition of intuitionistic fuzzy entropy given by Burillo and Bustince and fuzzy entropy. Szmidt and Kacprzyk [6] proposed another axiomatic definition of intuitionistic fuzzy entropy, and successively had a deep systematic study on intuitionistic fuzzy entropy measure and similarity measure in the literature [7-12], and then a series of entropy measures are given. Zhang and Jiang [13] proposed a nonprobabilistic entropy of a vague set by means of the intersection and union of the membership degree and non-membership degree of the vague set. Xia and Xu [14] deduced a new entropy measure with the cross entropy measure of intuitionistic fuzzy sets. And Ye [15] proposed two intuitionistic fuzzy entropy measures based on trigonometric. Chen and Li [16] did classified research of intuitionistic fuzzy entropy. And in accordance with different meanings of intuitionistic fuzzy entropy measure, they summarized from four different aspects, including hesitation degree, geometry, probability, and non-probability frameworks. And by the means of experiment, they proposed a method of the objective weight determining method based on intuitionistic fuzzy entropy.

In multiple attributes decision making (MADM) analysis, a decision maker must select the reasonable attribute weights. The proper assessment of attribute weights plays an essential role in the MADM process because of the variation of weight values may result in different final rankings of alternatives [17]. In general, the weights in MADM can be classified as subjective weights and objective weights according to the methods of information acquisition [18]. Subjective weights are obtained from preference information given by the decision maker, who provides subjective intuition or judgments on specific attributes. Objective weights are derived from the information of a decision matrix through mathematical models. The well-known approaches for generating subjective weights include AHP [19] and the Delphi method [20]. In terms of determining objective weights, one of the most-representative approaches is the entropy method, which expresses the relative intensities of attribute importance to signify the average intrinsic information transmitted to the DM [21]. Determining the weights by entropy is one of the objective weights methods. The entropy measure, which determined by Burillo and Bustince's axiomatic definition, can only reflect the impact of hesitation degree on the uncertainty. And most of the entropies, which determined by Szmidt and Kacprzyk's axiomatic definition, can only reflect the impact of the degree of difference between membership and non-membership on the uncertainty. So it is not accurate enough to determine attribute weights with the above entropy. With the consideration of the effect of entropy and hesitation on uncertainty, the methods of determining the attribute weights by effective information are proposed. The ideal multiple attributes decision making analysis results are determined by adjustment weights with the parameter.

The rest of this paper is organized as follows. The intuitionistic fuzzy sets and some arithmetic properties are introduced in section 2. The intuitionistic fuzzy entropy and similarity measure are discussed and three entropy measures are induced through a similarity measure in section 3. The method to determine the weight by the effective information is presented and specific operational steps are given in section 4. Some real examples are analyzed with the above methods in section 5, and points out that the method of determining weight proposed in this paper is reasonable and effective. Finally, some conclusions are presented.

2. Intuitionistic Fuzzy Sets

Definition 1. Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set. An intuitionistic fuzzy set (IFS) A over X is an object having the form [4]:

$$A = \left\{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \mid x_i \in X \right\} \quad (1)$$

Where

$$\mu_A : X \rightarrow [0,1], \nu_A : X \rightarrow [0,1] \quad (2)$$

With the condition $0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1$ for all $x_i \in X$.

The numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ denote respectively the degree of membership and the degree of non membership of the element x_i to set A . For each IFS A in X , if

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) \quad (3)$$

Then $\pi_A(x_i)$ is called the intuitionistic index of the element x_i in the set A . It is a hesitancy degree of x_i to A . It is obvious that $0 \leq \pi_A(x_i) \leq 1$, $x_i \in X$.

The following expressions are defined in [1,2,22] for all A, B belonging to $\text{IFS}(X)$:

- 1) $A \subseteq B$ if and only if $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for all $x_i \in X$;
- 2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

- 3) $A \circ B$ if and only if $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for all $x_i \in X$;
- 4) $AI B = \left\{ \langle x_i, \min(\mu_A(x_i), \nu_A(x_i)), \max(\mu_A(x_i), \nu_A(x_i)) \rangle \mid x_i \in X \right\}$;
- 5) $AUB = \left\{ \langle x_i, \max(\mu_A(x_i), \nu_A(x_i)), \min(\mu_A(x_i), \nu_A(x_i)) \rangle \mid x_i \in X \right\}$;
- 6) $A^c = \left\{ \langle x_i, \nu_A(x_i), \mu_A(x_i) \rangle \mid x_i \in X \right\}$.

Definition 2 [23]. Let $a = (\mu, \nu)$ be an intuitionistic fuzzy value, the score of a is defined by $s(a) = \mu - \nu$; s is called the score function. The degree of accuracy of a is defined by $h(a) = \mu + \nu$; h is called the accuracy function.

Let $a_1 = (\mu_1, \nu_1)$, $a_2 = (\mu_2, \nu_2)$ be two intuitionistic fuzzy values, we say

If $s(a_1) < s(a_2)$, then $a_1 < a_2$;

If $s(a_1) = s(a_2)$, then

(i) If $h(a_1) < h(a_2)$, then $a_1 < a_2$;

(ii) If $h(a_1) = h(a_2)$, then $a_1 = a_2$.

3. Entropy of the Intuitionistic Fuzzy Sets and Similarity Measure

Definition 3 [6]. A mapping $E_{SK} : IFS(X) \rightarrow [0,1]$ is said to be an entropy if it satisfies the following axioms.

(E1) $E_{SK}(A) = 0$ if and only if A is crisp set;

(E2) $E_{SK}(A) = 1$ if and only if $\mu_A(x_i) = \nu_A(x_i)$ for every $x_i \in X$;

(E3) $E_{SK}(A) \leq E_{SK}(B)$ if $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$ or if $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$ for every $x_i \in X$;

(E4) $E_{SK}(A) = E_{SK}(A^c)$.

Definition 4. A real function $N : IFS \times IFS \rightarrow [0,1]$ is called similarity measure of intuitionistic fuzzy sets, if N satisfies the following properties:

(N1) $N(A, A^c) = 0$ if A is a crisp set;

(N2) $N(A, B) = 1$ if and only if $A = B$;

(N3) For all $A, B, C \in IFS$, if $A \subseteq B \subseteq C$, then $N(A, C) \leq N(A, B)$, $N(A, C) \leq N(B, C)$;

(N4) $N(A, B) = N(B, A)$.

It is easy to verify that the following formulas, $N_1(A, B)$, $N_2(A, B)$ and $N_3(A, B)$ are to satisfy the three similarity measures in definition 4.

$$N_1(A, B) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n \left((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 \right)} \quad (4)$$

$$N_2(A, B) = \frac{\sum_{i=1}^n (\mu_A(x_i) \wedge \mu_B(x_i) + \nu_A(x_i) \wedge \nu_B(x_i))}{\sum_{i=1}^n (\mu_A(x_i) \vee \mu_B(x_i) + \nu_A(x_i) \vee \nu_B(x_i))} \quad (5)$$

$$N_3(A, B) = 1 - \frac{1}{n} \sum_{i=1}^n (|\mu_A(x_i) - \mu_B(x_i)| \vee |\nu_A(x_i) - \nu_B(x_i)|) \quad (6)$$

Another transform method of setting up entropy of intuitionistic fuzzy set is proposed based on similarity measure of intuitionistic fuzzy sets.

For intuitionistic fuzzy set A , we define $f(A), g(A) \in \text{IFS}$, for every $x \in X$,

$$\mu_{f(A)}(x) = \frac{1 + (\mu_A(x) - v_A(x))^2}{2}, \quad v_{f(A)}(x) = \frac{1 - (\mu_A(x) - v_A(x))^4}{2},$$

$$\mu_{g(A)}(x) = \frac{1 - (\mu_A(x) - v_A(x))^2}{2}, \quad v_{g(A)}(x) = \frac{1 + (\mu_A(x) - v_A(x))^4}{2},$$

then we have the following theorem.

Theorem 1. Suppose N be similarity measure of intuitionistic fuzzy sets, $A \in \text{IFS}$, then $N(f(A), g(A))$ is entropy of intuitionistic fuzzy set A .

Proof.

(E1) If A is a crisp set, then for every $x \in X$, we have $\mu_A(x) = 1, v_A(x) = 0$ or $\mu_A(x) = 0, v_A(x) = 1$, we can get $|\mu_A(x) - v_A(x)| = 1$. Thus, for every $x \in X$, then $\mu_{f(A)}(x) = 1, v_{f(A)}(x) = 0, \mu_{g(A)}(x) = 0, v_{g(A)}(x) = 1$. It shows that $g(A) = (f(A))^c$, therefore, $N(f(A), g(A)) = N(f(A), (f(A))^c) = 0$.

(E2) Known by the definitions of $f(A)$ and $g(A)$, $f(A)$ and $g(A)$ are intuitionistic fuzzy sets, thus, $N(f(A), g(A)) = 1$ if and only if $f(A) = g(A)$, $f(A) = g(A)$ if and only if $\mu_A(x) = v_A(x)$.

(E3) Let $\mu_A(x) \leq \mu_B(x) \leq v_B(x) \leq v_A(x)$. Thus, we can get $|\mu_A(x) - v_A(x)| \geq |\mu_B(x) - v_B(x)|$.

It means that $g(A) \subseteq g(B) \subseteq f(B) \subseteq f(A)$, so we have $N(f(A), g(A)) \leq N(f(B), g(A)) \leq N(f(B), g(B))$.

With the same reason, when $\mu_A(x) \geq \mu_B(x) \geq v_B(x) \geq v_A(x)$, we also have $N(f(A), g(A)) \leq N(f(B), g(B))$.

(E4) Let $A = \{ \langle x, \mu_A(x), v_A(x) \rangle | x \in X \}$, then $A^c = \{ \langle x, v_A(x), \mu_A(x) \rangle | x \in X \}$. Known by the definitions of $f(A)$ and $g(A)$, we have $f(A) = f(A^c), g(A) = g(A^c)$, therefore

$$N(f(A), g(A)) = N(f(A^c), g(A^c)).$$

Hence, we complete the proof of Theorem 1.

According to Theorem 1 and $N_1(A, B), N_2(A, B), N_3(A, B)$, we can get the corresponding entropy measure.

$$E_1(A) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n \left((\mu_{f(A)}(x_i) - \mu_{g(A)}(x_i))^2 + (v_{f(A)}(x_i) - v_{g(A)}(x_i))^2 \right)} \quad (7)$$

$$E_2(A) = \frac{\sum_{i=1}^n (\mu_{f(A)}(x_i) \wedge \mu_{g(A)}(x_i) + v_{f(A)}(x_i) \wedge v_{g(A)}(x_i))}{\sum_{i=1}^n (\mu_{f(A)}(x_i) \vee \mu_{g(A)}(x_i) + v_{f(A)}(x_i) \vee v_{g(A)}(x_i))} \quad (8)$$

$$E_3(A) = 1 - \frac{1}{n} \sum_{i=1}^n \left(|\mu_{f(A)}(x_i) - \mu_{g(A)}(x_i)| \vee |v_{f(A)}(x_i) - v_{g(A)}(x_i)| \right) \quad (9)$$

4. Method of Determining Weight

4.1. Determine the Weight by Traditional Intuitionistic Fuzzy Entropy

Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of m alternatives and decision maker will choose the best one from A , according to a criterion set $C = \{C_1, C_2, \dots, C_n\}$ which include n criteria.

According to the actual case, firstly to determine the decision matrix J .

$$J = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \dots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \dots & (\mu_{2n}, \nu_{2n}) \\ \dots & \dots & \dots & \dots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \dots & (\mu_{mn}, \nu_{mn}) \end{bmatrix} \quad (10)$$

Step 1 : Calculate entropy values of each intuitionistic fuzzy number in the decision matrix J by using entropy measure which established on the basis of entropy axiomatic definition by Szmidt and Kacprzyk's [6].

$$E = \begin{bmatrix} E_{11} & E_{12} & \dots & E_{1n} \\ E_{21} & E_{22} & \dots & E_{2n} \\ \dots & \dots & \dots & \dots \\ E_{m1} & E_{m2} & \dots & E_{mn} \end{bmatrix} \quad (11)$$

Step 2 : Normalize the intuitionistic fuzzy entropy values in the decision matrix using the following equation:

$$t_{ij} = \frac{E_{ij}}{\max_i(E_{ij})}, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n.$$

We use t_{ij} to label the normalized value. The normalized decision matrix is thus shown as follows:

$$T = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ \dots & \dots & \dots & \dots \\ t_{m1} & t_{m2} & \dots & t_{mn} \end{bmatrix} \quad (12)$$

Step 3 : Calculate the objective attribute weights by applying the following transformer:

$$w_j = \frac{1 - \frac{1}{m} \sum_{i=1}^m t_{ij}}{\sum_{j=1}^n \left(1 - \frac{1}{m} \sum_{i=1}^m t_{ij} \right)}, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (13)$$

The core idea of the above method of determining the weight by entropy is that, the greater the uncertainty of information of the property, the smaller the weight of the attributes. However, the key issue we have to analysis is whether it includes all of the uncertainty of the information or not. The answer is no. Let us see the following analysis process.

4.2. Adopt Effective Information to Determine the Weight

In practical problems, the intuitionistic fuzzy number $\langle 0.5, 0.5, 0 \rangle$ and $\langle 0, 0, 1 \rangle$ are obviously not of the same role. But their entropies which calculated by intuitionistic fuzzy measures E_1, E_2 and E_3 are the same. And they have the equal weights according to the weight determining method in 4.1. Actually, as long as the absolute values of the difference between membership and non-membership of two intuitionistic fuzzy numbers are equal, the attribute weights they represent are equal. This result is clearly not reasonable. In order to accurately measure the attribute weights, we definite effective information as $k^\lambda(x)$, and $k^\lambda(x) = 1 - (\lambda E(x) + (1 - \lambda)\pi(x))$, Where λ is a parameter, and $\lambda \in [0, 1]$. The greater the effective information $k^\lambda(x)$, the more important the indicator. On the contrary, the smaller the effective information $k^\lambda(x)$, the less important the indicator. When $\lambda = 0$, $k^0(x) = 1 - \pi(x)$, which shows that the uncertainty information is completely determined by hesitation $\pi(x)$. When $\lambda = 1$, $k^1(x) = 1 - E(x)$, which shows that the uncertainty information is completely determined by entropy $E(x)$. Therefore, it can help to define the weight with $k^\lambda(x)$. Firstly, determine the effective information matrix

$$K(\lambda) = \begin{bmatrix} k_{11}^\lambda & k_{12}^\lambda & \dots & k_{1n}^\lambda \\ k_{21}^\lambda & k_{22}^\lambda & \dots & k_{2n}^\lambda \\ \dots & \dots & \dots & \dots \\ k_{m1}^\lambda & k_{m2}^\lambda & \dots & k_{mn}^\lambda \end{bmatrix} \quad (14)$$

Where $\lambda \in [0, 1]$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

We have weight vector $w(\lambda) = (w_1(\lambda), w_2(\lambda), \dots, w_n(\lambda))$, where

$$w_j(\lambda) = \frac{\sum_{i=1}^m k_{ij}^\lambda}{\sum_{j=1}^n \sum_{i=1}^m k_{ij}^\lambda} = \frac{\sum_{i=1}^m (1 - (\lambda E_{ij} + (1 - \lambda)\pi_{ij}))}{\sum_{j=1}^n \sum_{i=1}^m (1 - (\lambda E_{ij} + (1 - \lambda)\pi_{ij}))}, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (15)$$

Now, as the method of determining attribute weight is given, the decision steps are proposed as follows to multi-attribute decision problems:

Step1: Build entropy matrix E_1 and hesitation matrix π based on the intuitionistic fuzzy decision matrix and formula (7).

Step2: Build the effective information matrix $K(\lambda)$ according to formula (14) and build the weight vector $w(\lambda)$ by formula (15).

Step3: Get the Sub-function matrix $S = (s_{ij})_{mn}$ and the Precision matrix $H = (h_{ij})_{mn}$ based on Definition 2.

Step4: Get the composite score values with parameter λ_j in program i using $s_{ij}^* = \sum_{t=1}^n w_t(\lambda_j) s_{it}$. Finally, we will get composite score matrix $S^* = (s_{ij}^*)_{mr}$ with all of the parameters in every program. Similarly, we can determine the integrated precision matrix $H^* = (h_{ij}^*)_{mr}$, where $h_{ij}^* = \sum_{t=1}^n w_t(\lambda_j) h_{it}$.

Step5: Sort the decision according to Definition 2.

5. Example Analysis

Nowadays, with the expansion of banking business, the financial electrification has become the mainstream of the development of commercial banking business. Network Technology, as the basis of business innovation, has developed widely, which greatly improved the quality and efficiency of commercial banking service. While, as the degree of dependence of commercial banking business upon the internet increasing, many network security problems exposed accordingly. The harm of these problems, especially the financial computer crimes, has drawn the attention of experts and scholars from all areas. Network security assessment may be an effective method to solve those problems. While, the assessment involves many aspects, and there are many uncertainties in the process of it. Strict quantization or objective assessment cannot be achieved easily. So it is ideal to study it by using intuitionistic fuzzy sets.

There are 5 commercial banks as $Y = \{Y_1, Y_2, Y_3, Y_4, Y_5\}$, and they will be evaluated according to the following 4 indicators(or attributes): x_1 (Threat), x_2 (Fragility), x_3 (Asset), x_4 (Management).

The evaluating matrix given by experts is

$$J = \begin{pmatrix} (0.45, 0.17) & (0.72, 0.18) & (0.52, 0.33) & (0.13, 0.82) \\ (0.46, 0.16) & (0.14, 0.83) & (0.51, 0.33) & (0.73, 0.19) \\ (0.73, 0.16) & (0.84, 0.12) & (0.54, 0.27) & (0.11, 0.82) \\ (0.73, 0.16) & (0.75, 0.21) & (0.19, 0.68) & (0.85, 0.14) \\ (0.15, 0.64) & (0.88, 0.10) & (0.71, 0.19) & (0.76, 0.21) \end{pmatrix}.$$

Step1: Get the related entropy matrix E_1 and hesitation matrix π according to the above evaluating matrix.

$$E_1 = \begin{pmatrix} 0.9444 & 0.7852 & 0.9745 & 0.6271 \\ 0.9361 & 0.6271 & 0.9771 & 0.7852 \\ 0.7584 & 0.5871 & 0.9483 & 0.6008 \\ 0.7584 & 0.7852 & 0.8254 & 0.6008 \\ 0.8254 & 0.4964 & 0.8019 & 0.7765 \end{pmatrix}, \quad \pi = \begin{pmatrix} 0.38 & 0.10 & 0.15 & 0.05 \\ 0.38 & 0.03 & 0.16 & 0.08 \\ 0.11 & 0.04 & 0.19 & 0.07 \\ 0.11 & 0.04 & 0.13 & 0.01 \\ 0.21 & 0.02 & 0.10 & 0.03 \end{pmatrix}.$$

Step2: Get effective information matrix $K(\lambda_j)$ and weight $w(\lambda_j)$ based on different values of the parameter λ_j .

If $\lambda_3 = 0.5$, then $k^{0.5}(x) = 1 - (0.5E_1(x) + 0.5\pi(x))$,

$$K(0.5) = \begin{pmatrix} 0.3378 & 0.5574 & 0.4377 & 0.6614 \\ 0.3419 & 0.6714 & 0.4315 & 0.5674 \\ 0.5658 & 0.6865 & 0.4308 & 0.6646 \\ 0.5658 & 0.5874 & 0.5223 & 0.6946 \\ 0.4823 & 0.7418 & 0.5491 & 0.5968 \end{pmatrix},$$

and the weight is $w(0.5) = (0.2067, 0.2924, 0.2137, 0.2871)$.

If $\lambda_1 = 0$, then $k^0(x) = 1 - \pi(x)$, the weight is $w(0) = (0.2164, 0.2709, 0.2425, 0.2703)$.

If $\lambda_2 = 0.3$, then $k^{0.3}(x) = 1 - (0.3E_1(x) + 0.7\pi(x))$, the weight is $w(0.3) = (0.2117, 0.2814, 0.2285, 0.2784)$.

If $\lambda_4 = 0.7$, then $k^{0.7}(x) = 1 - (0.7E_1(x) + 0.3\pi(x))$, the weight is

$$w(0.7) = (0.1988, 0.3104, 0.1899, 0.3010).$$

If $\lambda_5 = 1$, then $k^1(x) = 1 - E_1(x)$, the weight is $w(1) = (0.1698, 0.3754, 0.1033, 0.3515)$.

The contrast of weights with each parameter are shown in Figure 1.

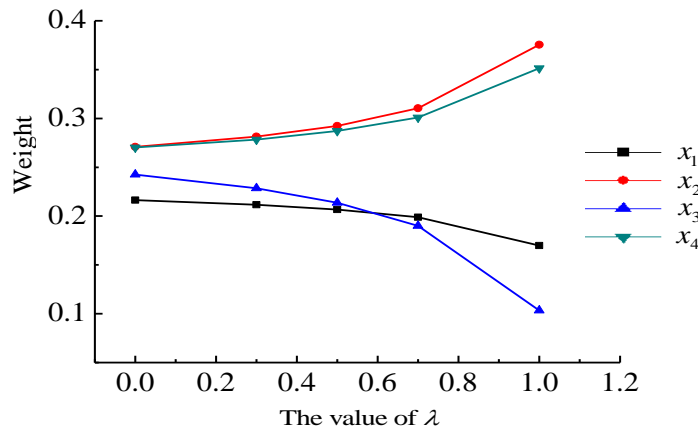


Figure 1. Weights of Different Parameters

Step3: The calculate results of score matrix S and precision matrix H are as follows:

$$S = \begin{pmatrix} 0.28 & 0.54 & 0.19 & -0.69 \\ 0.30 & -0.69 & 0.18 & 0.54 \\ 0.57 & 0.72 & 0.27 & -0.71 \\ 0.57 & 0.54 & -0.49 & 0.71 \\ -0.49 & 0.78 & 0.52 & 0.55 \end{pmatrix}, H = \begin{pmatrix} 0.62 & 0.90 & 0.85 & 0.95 \\ 0.62 & 0.97 & 0.84 & 0.92 \\ 0.89 & 0.96 & 0.81 & 0.93 \\ 0.89 & 0.96 & 0.87 & 0.99 \\ 0.79 & 0.98 & 0.90 & 0.97 \end{pmatrix}.$$

Step4: The calculate results of matrix S^* and integrated precision matrix H^* of composite score with all the parameters in every program are as follows.

$$S^* = \begin{pmatrix} 0.0664 & 0.0626 & 0.0583 & 0.0517 & 0.0274 \\ 0.0676 & 0.0608 & 0.0538 & 0.0422 & 0.0003 \\ 0.1920 & 0.1873 & 0.1822 & 0.1744 & 0.1454 \\ 0.3427 & 0.3583 & 0.3748 & 0.4016 & 0.4984 \\ 0.3800 & 0.3877 & 0.3958 & 0.4090 & 0.4567 \end{pmatrix},$$

$$H^* = \begin{pmatrix} 0.8409 & 0.8432 & 0.8457 & 0.8500 & 0.8649 \\ 0.8493 & 0.8523 & 0.8554 & 0.8608 & 0.8796 \\ 0.9005 & 0.9026 & 0.9048 & 0.9087 & 0.9221 \\ 0.9312 & 0.9330 & 0.9348 & 0.9381 & 0.9494 \\ 0.9169 & 0.9187 & 0.9207 & 0.9241 & 0.9360 \end{pmatrix}.$$

Step5: Decision results and analysis.

The composite score values of every bank with different parameters are shown in Figure 2. It can be seen that, if $\lambda_1 = 0$, the sort of network safety of the 5 commercial banks is Y_5 f Y_4 f Y_3 f Y_2 f Y_1 ; if $\lambda_5 = 1$, it will be Y_4 f Y_5 f Y_3 f Y_1 f Y_2 ; when $\lambda_2 = 0.3$, $\lambda_3 = 0.5$, $\lambda_4 = 0.7$, it is Y_5 f Y_4 f Y_3 f Y_1 f Y_2 . When $\lambda_1 = 0$, $\lambda_5 = 1$, the effective information is not accurate enough, and the decision results will be unreliable.

While, when $\lambda_2 = 0.3$, $\lambda_3 = 0.5$, $\lambda_4 = 0.7$, the effective information is accurate, and the decision results are reliable.

In summary, the sort of network safety of the 5 commercial banks is $Y_5 \succ Y_4 \succ Y_3 \succ Y_1 \succ Y_2$.

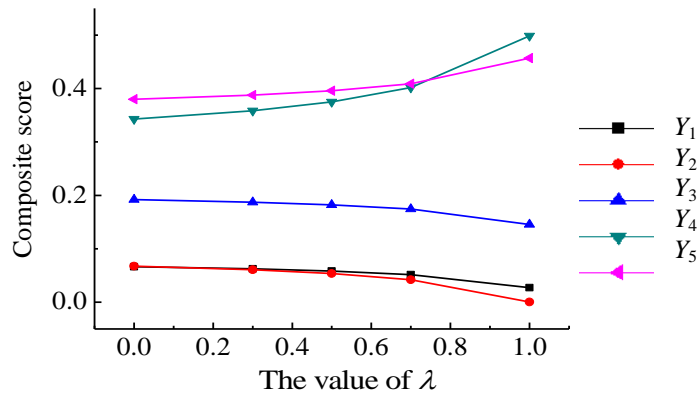


Figure 2. The Composite Score Values of Every Bank with Different Parameters

6. Conclusions

In this paper, the entropy measure and similarity measure are analyzed at first, and three entropy measures are induced with similarity measure. And we pointed out that most of the intuitionistic fuzzy entropy measure will not be able to fully describe the uncertainty of things. So the weight determine, which based on the entropy, becomes somewhat inaccurate. The method of determining weight by effective information, which is proposed in this paper, is flexible. It can determine the weight according to different parameters, and can get different decision results with various weights. Finally, ideal decision results were obtained based on the comparative analysis.

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