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Research on Nonlinear Sensitive Medical Joint Automation

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Abstract

The main objective in this research is comparative study between three types of control methodology: proportional-integral-derivative (PID) controller, computed torque controller (CTC), and sliding mode controller (SMC) with application to active multi degrees of freedom actuators. PID controller is a linear model-free controller; to control of nonlinear system based on PID controller system linearization is the main challenge. Computed torque controller and sliding mode controller are model-base and nonlinear. In order to design computed torque controller, an accurate dynamic model of nonlinear system plays an important role. To modelling an accurate dynamic system, modelling of complex parameters is needed to form the structure of system's dynamic model. It may be very difficult to include all the complexities in the system dynamic model. Computed torque controller is work very good in certain condition but in uncertainty, PID and CTC have some challenges. Conventional switching sliding mode controller is an apparent nominates to design a controller using the bounds of the uncertainties and external disturbance. In partly uncertainties, sliding mode controller is more robust than CTC and PID. In this paper these three types controller are test in MATLAB/SIMULINK.

Keywords: *proportional-integral-derivative controller, sliding mode controller, computed torque controller, multi degrees of freedom actuator, system uncertainty*

1. Introduction

Robot manipulators are set of links which connected by joints, they are multi input and multi output (MIMO), nonlinear, time variant, uncertain dynamic systems and are developed either to replace human work in many fields such as in industrial or in the manufacturing. Complexities of the tasks caused to design mechanical architectures and robot manipulator with nonlinear behavior. These factors are:

- Time-varying parameters based on tear and ware.
- Simplifying suppositions in system modelling caused to have un-modelled dynamic parameters.
- External disturbance and noise measurement, which it is caused to generate uncertainties.
- To control of multi degrees of freedom actuator, three purposes are very important:
- **Stability**: Stability is due to the proper functioning of the system. A system is called stable if for any bounded input signal the system's output will stay bounded. Therefore limitation of output deviation is very important for any design.
- **Robust**: Robust method is caused to achieve robust and stable performance in the presence of uncertainty and external disturbance. A system is robust when it would also work well under different conditions and external disturbance.
- **Reliability:** to control of nonlinear and uncertain systems, reliability play important role and most of model-base controller are reliable.

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As a result, design a controller based on these three factors are the main challenge in this work. Based on control theory; controls of nonlinear systems are divided into two main collections:

- Conventional control theory: conventional control theories are work based on nonlinear dynamic parameters, these controllers are divided into two main categories: Linear control method and nonlinear control method.
- intelligent control theory: intelligent control theory is worked based on intelligent control theory and it is free of nonlinear dynamic parameters.

According to the dynamic formulation of multi degrees of freedom actuator, it is uncertain and there exist strong coupling effects between joints. The problem of coupling effects play important role to get best performance. In linear controller, this challenge has been reduced, with the following two methods:

- Limiting the performance of the system
- Design high gear ratio joint actuator

Conventional nonlinear control theories are highly sensitive to system's behavior and work based on cancelling decoupling and nonlinear terms of dynamic parameters. Computed Torque Control (CTC) and Sliding Mode Control (SMC) are two nonlinear conventional controller which introduced by many researchers [1-10].

Computed Torque Controller (CTC) is one of the effective nonlinear controllers [1-7]. Consequently, to have a good performance, linearization and decoupling without using many gears, feedback linearization (computed-torque) control methodologies is presented. To design computed torque controller, an accurate dynamic model of multi degrees of freedom actuator has important role. To modelling an accurate dynamic system, modelling of complex parameters is needed to form the structure of system's dynamic model. It may be very difficult to include all the complexities in the system dynamic model [11-16]. Dynamic parameters may not be constant over time and measure the acceleration term should be measured is very expensive, thus this problem is the main challenge to select the computed torque controller.

To eliminate the actual acceleration measurement and the computation burden as well improve the stability, efficiency and robust controller, sliding mode controller is recommended. This controller works very well in certain and partly uncertain condition [17-23]. This controller has two important subparts, switching part and equivalent part. Switching part of controller is used to design suitable tracking performance based on very fast switching. This part has essential role to have a good trajectory performance in all joints. However this part is very important in uncertain condition but it is caused to chattering phenomenon in system performance. Chattering phenomenon can cause some important mechanical problems [18]. The second subpart in sliding mode controller is equivalent part especially in uncertain condition. Sliding mod controller is a nonlinear model based controller and equivalent part is a dynamic formulation of nonlinear system, that is used in control formulation to eliminate the decoupling and nonlinear term of dynamic parameters [1-19]. However this part is very essential to reliability but in uncertain condition or highly nonlinear dynamic systems it can cause some problem.

This paper is organized as follows; section two, is served as an introduction to the dynamic formulation of multi degrees of freedom joints. Part three, introduces and describes the methodology. Section four presents the simulation results and discussion of this algorithm and the final section is describing the conclusion.

2. Theory

Dynamic of surgical joints: Dynamic modeling of multi degrees of freedom is used to describe the behavior of spherical motor such as linear or nonlinear dynamic behavior, design of model based controller such as pure sliding mode controller which design this controller is based on nonlinear dynamic equations, and for simulation. The dynamic modeling describes the relationship between motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (e.g., inertia, coriolios, centrifugal, and the other parameters) to behavior of system. Spherical motor has nonlinear and uncertain dynamic parameters 3 degrees of freedom (DOF) motor [10-15].

The equation of a spherical motor governed by the following equation:

$$\boldsymbol{H(q)}\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix}+\mathbf{B}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}\dot{\beta}\\ \dot{\alpha}\dot{\gamma}\\ \dot{\beta}\dot{\gamma}\end{bmatrix}+\boldsymbol{C(q)}\begin{bmatrix}\dot{\alpha}^2\\ \dot{\beta}^2\\ \dot{\gamma}^2\end{bmatrix}=\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix} \quad (1)$$

Where τ is actuation torque, H (q) is a symmetric and positive define inertia matrix, B(q) is the matrix of coriolios torques, C(q) is the matrix of centrifugal torques.

This is a decoupled system with simple second order linear differential dynamics. In other words, the component $\ddot{q}$ influences, with a double integrator relationship, only the variableq_i, independently of the motion of the other parts. Therefore, the angular acceleration is found as to be:

$$\ddot{\boldsymbol{q}}=\boldsymbol{H^{-1}(q)}.\{\boldsymbol{\tau}-\{\boldsymbol{B}+\boldsymbol{C}\}\} \quad (2)$$

This technique is very attractive from a control point of view. Study of spherical motor is classified into two main groups: kinematics and dynamics. Calculate the relationship between rigid bodies and final part without any forces is called Kinematics. Study of this part is pivotal to design with an acceptable performance controller, and in real situations and practical applications. As expected the study of kinematics is divided into two main parts: forward and inverse kinematics. Forward kinematics has been used to find the position and orientation of task frame when angles of joints are known. Inverse kinematics has been used to find possible joints variable (angles) when all position and orientation of task frame be active.

The main target in forward kinematics is calculating the following function:

$$\boldsymbol{\Psi(X,q)=0} \quad (3)$$

Where $\Psi(.)\in R^n$ is a nonlinear vector function, $X=[X_1,X_2,\dots\dots,X_l]^T$ is the vector of task space variables which generally task frame has three task space variables, three orientation, $q=[q_1,q_2,\dots.,q_n]^T$ is a vector of angles or displacement, and finally n is the number of actuated joints. The Denavit-Hartenberg (D-H) convention is a method of drawing spherical motor free body diagrams. Denvit-Hartenberg (D-H) convention study is necessary to calculate forward kinematics in this motor.

A systematic Forward Kinematics solution is the main target of this part. The first step to compute Forward Kinematics (F.K) is finding the standard D-H parameters. The following steps show the systematic derivation of the standard D-H parameters.

1. Locate the spherical motor
2. Label joints
3. Determine joint rotation (θ)
4. Setup base coordinate frames.
5. Setup joints coordinate frames.
6. Determineα_i, thatα_i, link twist, is the angle between Z_i and Z_{i+1} about an X_i.
7. Determine d_i and a_i , that a_i, link length, is the distance between Z_i and Z_{i+1} along X_i. d_i, offset, is the distance between X_{i-1} and X_i along Z_i axis.
8. Fill up the D-H parameters table. The second step to compute Forward kinematics is finding the rotation matrix (R_n^0). The rotation matrix from$\{F_i\}$ to $\{F_{i-1}\}$ is given by the following equation;

$$\boldsymbol{R_i^{i-1}=U_{i(\theta_i)}V_{i(\alpha_i)}} \quad (4)$$

Where $U_{i(\theta_i)}$ is given by the following equation;

$$\boldsymbol{U_{i(\theta_i)}}=\begin{bmatrix}\boldsymbol{cos(\theta_i)} & \boldsymbol{-sin(\theta_i)} & \boldsymbol{0}\\ \boldsymbol{sin(\theta_i)} & \boldsymbol{cos(\theta_i)} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}\end{bmatrix} \quad (5)$$

and $V_{i(\alpha_i)}$ is given by the following equation;

$$\boldsymbol{V_{i(\theta_i)}} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{cos(\alpha_i)} & \boldsymbol{-sin(\alpha_i)} \\ \mathbf{0} & \boldsymbol{sin(\alpha_i)} & \boldsymbol{cos(\alpha_i)} \end{bmatrix} \quad \textbf{(6)}$$

So (R_n^0) is given by

$$\boldsymbol{R_n^0 = (U_1V_1)(U_2V_2)\ldots\ldots\ldots(U_nV_n)} \quad \textbf{(7)}$$

The final step to compute the forward kinematics is calculate the transformation ${}_n^0T$ by the following formulation [3-10]

$$\boldsymbol{{}_n^0T = {}_1^0T.{}_2^1T.{}_3^2T\ldots\ldots.{}_n^{n-1}T} = \begin{bmatrix} \boldsymbol{R_n^0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \quad \textbf{(8)}$$

3. Methodology

Linear Control Technique: Linear control theory is used in linear and nonlinear systems. This type of theory is used in industries, because design of this type of controller is simple than nonlinear controller. However this type of controller used in many applications but it cannot guarantee performance in complex systems. Simple linear controllers are including proportional algorithm, Proportional-Derivative algorithm, Integral algorithm, Proportional-Integral algorithm and Proportional-Integral-Derivative algorithm.

Proportional Algorithm: It is used to responds immediately to difference of control input variables by immediately changing its influences variables, but this type of control is unable to eliminate the control input difference. Figure 1 shows the block diagram of proportional controller with application to nonlinear system.

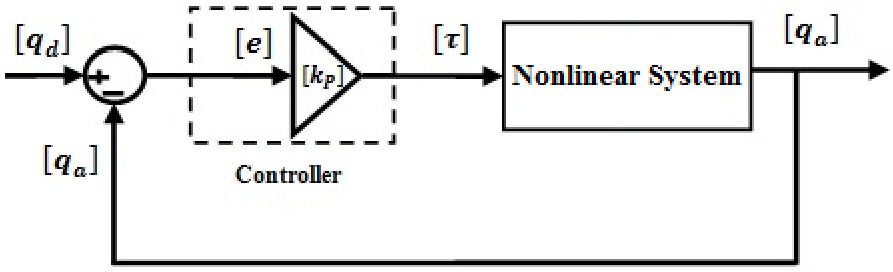

Figure 1. Block Diagram of Proportional Controller

Proportional plus Derivative (PD) control: This type of linear controller is widely used in control process where the results are sensitive to exceeded of set point. This controller, like Proportional controller, has permanent variation in presence of self-limitation control. In mathematically, the formulation of Proportional-Derivative part calculated as follows;

$$\boldsymbol{U_{PD} = K_p \times e + K_v(\frac{de}{dt}) = K_p \times e + K_v\dot{e}} \quad \textbf{(9)}$$

The Derivative component in this type of methodology is used to cancel outs the change process variables change in presence of quick change in controllers input. Figure 2 shows the block diagram of Proportional-Derivative (PD) control of nonlinear system.

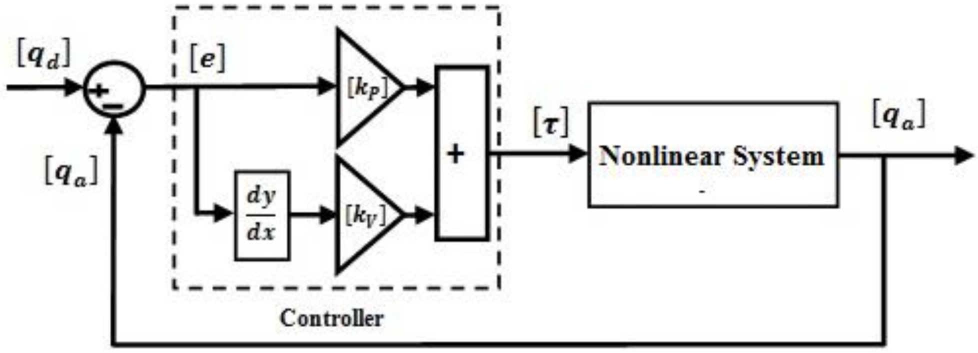

Figure 2. Block Diagram of PD Control

Integral (I) control: This category, integrate the input signal deviation over a period of time. This part of controller is used to system stability after a long period of time. Figure 3 shows the block diagram of Integral (I) controller with application to nonlinear system. In contrast of Proportional type of controller, this type of controller used to eliminate the deviation.

In mathematically, the formulation of integral part calculated as follows;

$$I = \frac{1}{T}\int e.dt = \sum e \quad (10)$$

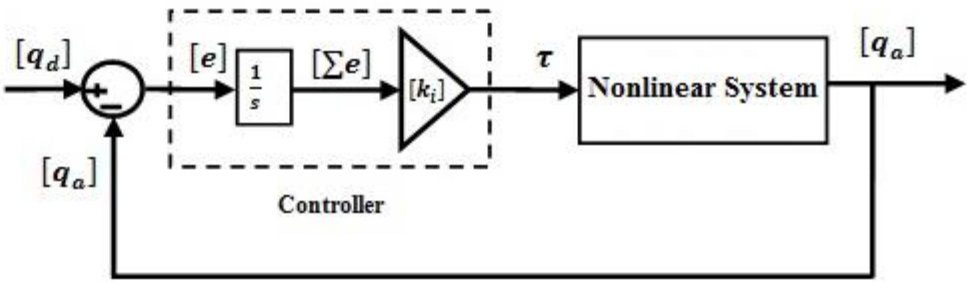

Figure 3. Block Diagram of Integral Control

Proportional plus Integral (PI) control: According to integral type of controller, it takes relatively long time. The proportional type controller used to immediately response to the input variations. The proportional-integral (PI) controller has the advantages of both proportional and integral controller; it is rapid response to the input deviation as well as the exact control at the desired input. Figure 4 shows the block diagram of PI control.

$$U_{PI} = K_p \times e + K_i(\frac{1}{T}\int e.dt) = K_p \times e + K_i \sum e \quad (11)$$

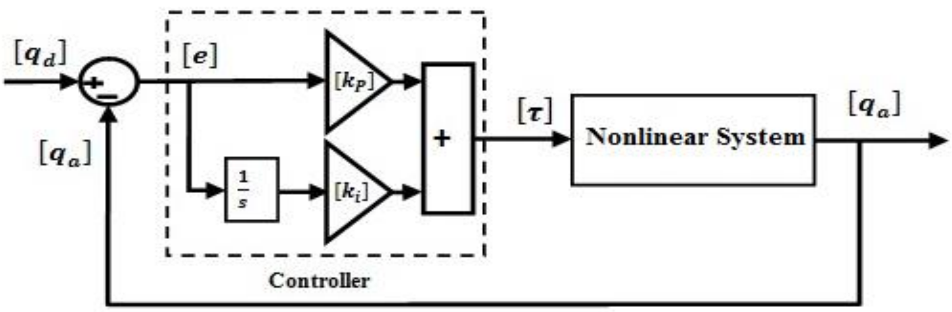

Figure 4. Block Diagram of PI Control

Proportional plus Integral plus Derivative (PID) control: The combination of proportional (P) component, integral (I) component with a derivative (D) controller offered advantages in each case. This type of controller has rapid response to the input deviation, the exact control at the desired input as well as fast response to the disturbances. The PID controller takes the error between the desired joint variables and the actual joint variables to control. A proportional-derivative integral control system can easily be implemented. This method does not provide sufficient control for systems with time-varying parameters or highly nonlinear systems. Figure 5 shows the block diagram of PID control. The formulation of PID controller calculated as follows;

$$U_{PID} = K_p \times e + K_i(\frac{1}{T}\int e.dt) + K_v(\frac{de}{dt}) = K_p \times e + K_i \sum e + K_v \dot{e} \quad (12)$$

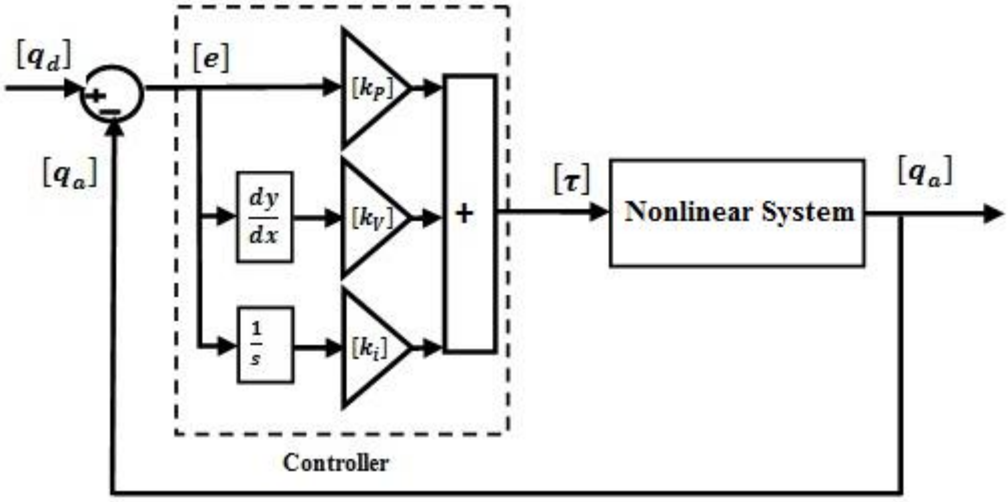

Figure 5. Block Diagram of PID Control of Robot Manipulator

Proportional-Integral-Derivative (PID) controller has rapid response to the input deviation, the exact control at the desired input as well as fast response to the disturbances. The PID controller takes the error between the desired joint variables and the actual joint variables to control the three dimension of joint. The equation of PID controller for control of 3 degrees of freedom joint is;

$$\begin{bmatrix} \widehat{\tau_1} \\ \widehat{\tau_2} \\ \widehat{\tau_3} \end{bmatrix} = \begin{bmatrix} K_{i1} \sum e_1 + K_{v1}\dot{e}_1 + K_{p1}e_1 \\ K_{i2} \sum e_2 + K_{v2}\dot{e}_2 + K_{p2}e_2 \\ K_{i3} \sum e_3 + K_{v3}\dot{e}_3 + K_{p3}e_3 \end{bmatrix} \quad (13)$$

Where $e = q_d - q_a$, q_d is desired joint variable and q_a is actual joint variable.

In PID controller the control law is given by the following equation;

$$\tau = K_p e + K_v \dot{e} + K_i \sum e \quad (14)$$

Where $e = q_{i_d} - q_{i_a}$

In this theory K_p, K_i **and** K_v are positive constant. To show this controller is stable and achieves zero steady state error, the Lyapunov function is introduced;

$$V = \frac{1}{2}\left[\dot{q}^T A(q)\dot{q} + e^T K_p e\right] = \quad (15)$$

$$\frac{1}{2}\frac{d}{dt}\left[\dot{q}^T A\dot{q}\right] = \dot{q}\,\tau$$

If the conversation energy is written by the following form:

$$\frac{1}{2}\frac{d}{dt}[\dot{q}^T A\dot{q}] = \dot{q}\,\tau$$

Where $(\dot{q}\,\tau)$ shows the power inputs from actuator and $\frac{1}{2}\frac{d}{dt}[\dot{q}^T A\dot{q}]$ is the derivative of the robot kinematic energy.

$$\dot{V} = \dot{q}^T[\tau + K_p e] \quad (16)$$

Based on $\tau = -K_{p_i}e - K_{v_i}\dot{e} - K_i\sum e$, we can write:

$$\dot{V} = \dot{q}^T K_p \dot{q} \le 0 \quad (17)$$

If $\dot{V} = 0$, we have

$$\dot{q} = 0 \rightarrow \ddot{q} = 0 \rightarrow \ddot{q} = A^{-1}K_p e \rightarrow e = 0 \quad (18)$$

In this state, the actual trajectories converge to the desired state.

Computed torque controller (CTC): Computed torque controller (CTC) is one of the effective nonlinear control methodologies for second order nonlinear system (e.g., robot manipulator). Computed torque controller works based on behavior (dynamic formulation) of system, which caused to works very well when all dynamic and physical parameters are known. In uncertain dynamic parameters when the system has variation computed torque controller has challenges. In this type of controller, if the desired position trajectory for the manipulator defined as $q_d(t)$, and the actual position trajectory defined as $q_a(t)$, the tracking error calculated by;

$$e(t) = q_d(t) - q_a(t) \quad (19)$$

If state space equation is defined by;

$$\dot{x} = Ax + BU \quad (20)$$

According to the Brunousky canonical form U is the nonlinearity term and defined by;

$$U = \ddot{q}_d + A^{-1}(q).\{N(q.\dot{q}) - \tau\} \quad (21)$$

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} U \quad (22)$$

According to $x = [e^T\ \dot{e}^T]^T$

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} U \quad (23)$$

The requirement torque calculated by;

$$\tau = A(q)(\ddot{q}_d + U) + N(\dot{q}, q) \quad (24)$$

The formulation of PD computed torque controller is;

$$\tau = A(q)\left(\ddot{q}_d + K_v\dot{e} + K_p e\right) + N(q, \dot{q}) \quad (25)$$

The PI computed torque controller formulation is;

$$\tau = A(q)\left(\ddot{q}_d + K_p e + K_i\sum e\right) + N(q, \dot{q}) \quad (26)$$

The PID computed torque controller formulation is;

$$\tau = A(q)\left(\ddot{q}_d + K_p e + K_v\dot{e} + K_i\sum e\right) + N(q, \dot{q}) \quad (27)$$

Where K_p , K_v and K_i are the controller gain coefficients. This type of controller has two main important parts; partly linear part and nonlinear term of dynamic equivalent part. Figure 6 shows the block diagram of computed torque controller.

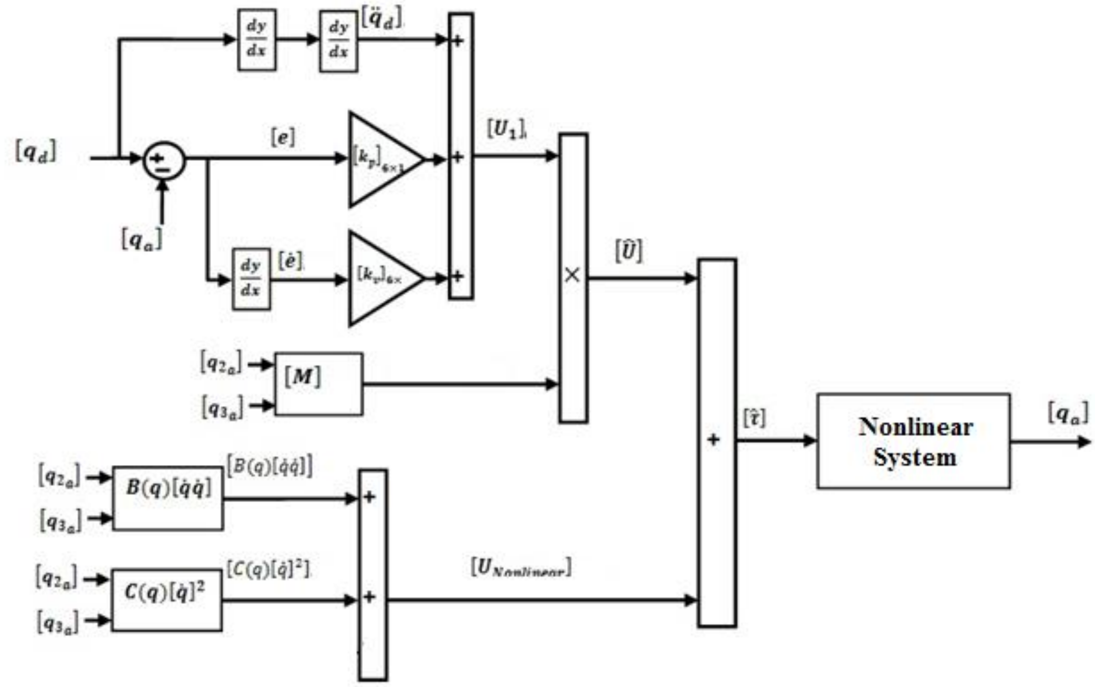

Figure 6. Block Diagram of Computed Torque Controller

Sliding Mode Controller: Sliding mode controller (SMC) is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. This theory was first proposed in the early 1950 by Emelyanov and several co-workers and has been extensively developed since then with the invention of high speed control devices The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness. Sliding mode control theory for control of nonlinear system has been proposed in 1978 by Young to solve the set point problem ($\dot{q}_d = 0$) by discontinuous method in the following form;

$$U_{(q,t)} = \begin{cases} U_i^+(q,t) & if\ S_i > 0 \\ U_i^-(q,t) & if\ S_i < 0 \end{cases} \quad \textbf{(28)}$$

where S_i is sliding surface (switching surface), $i = 1, 2, \dots\dots, n$ for *n*-DOF dental joint, $U_i(q,t)$ is the i^{th} torque of joint. Sliding mode controller is divided into two main sub controllers: discontinues controller(U_{dis}) and equivalent controller(U_{eq}).

Discontinues controller causes an acceptable tracking performance at the expense of very fast switching. Conversely in this theory good trajectory following is based on fast switching, fast switching is caused to have system instability and chattering phenomenon. Fine tuning the sliding surface slope is based on nonlinear equivalent part. However, this controller is used in many applications but, pure sliding mode controller has chattering phenomenon challenge. Chattering phenomenon (Figure 7) can causes some problems such as saturation and heats the mechanical parts of drivers.

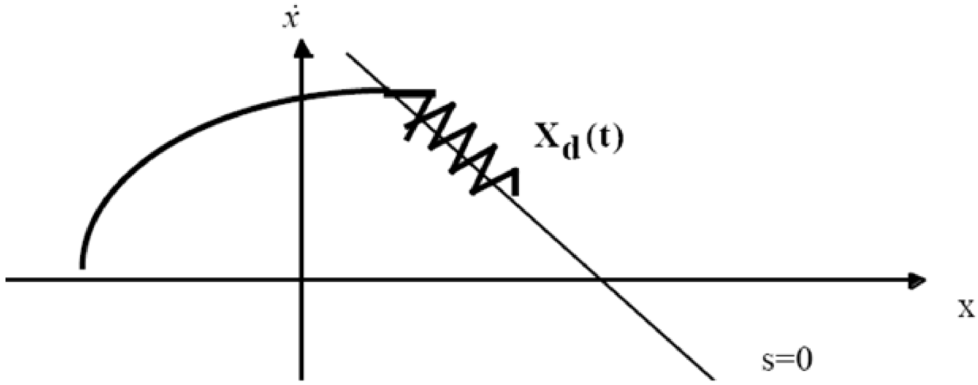

Figure 7. Chattering as a Result of Imperfect Control Switching

In boundary layer saturation method, the basic idea is the discontinuous method replacement by saturation (linear) method with small neighborhood of the switching surface. This replacement caused to increase the error performance against with the considerable chattering reduction. Design a robust controller for dental joint is essential because this type of joint has highly nonlinear dynamic parameters. Consider a nonlinear single input dynamic system is defined by:

$$\boldsymbol{x^{(n)} = f(\vec{x}) + b(\vec{x})u} \qquad \textbf{(29)}$$

$$\boldsymbol{\widetilde{x} = x - x_d = [\widetilde{x}, \dots, \widetilde{x}^{(n-1)}]^T} \qquad \textbf{(30)}$$

A time-varying sliding surface $\boldsymbol{s(x,t)}$ in the state space $\boldsymbol{R^n}$ is given by:

$$\boldsymbol{s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\, \widetilde{x} = 0} \qquad \textbf{(31)}$$

where λ is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows:

$$\boldsymbol{s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\left(\int_0^t \widetilde{x}\, dt\right) = 0} \qquad \textbf{(32)}$$

The main target in this methodology is kept the sliding surface slope $\boldsymbol{s(x,t)}$ near to the zero. Therefore, one of the common strategies is to find input $\boldsymbol{U}$ outside of $\boldsymbol{s(x,t)}$.

$$\boldsymbol{\frac{1}{2}\frac{d}{dt}s^2(x,t) \le -\zeta|s(x,t)|} \qquad \textbf{(33)}$$

where ζ is positive constant.

If $\mathbf{S(0)>0} \rightarrow \frac{d}{dt}\mathbf{S(t)} \le -\boldsymbol{\zeta}$ **(34)**

To eliminate the derivative term, it is used an integral term from t=0 to t=$\boldsymbol{t_{reach}}$

$$\boldsymbol{\int_{t=0}^{t=t_{reach}} \frac{d}{dt}S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S\,(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0)} \qquad \textbf{(35)}$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S(t_{reach} = 0) defined as

$$\boldsymbol{0 - S(0) \le -\eta(t_{reach}) \rightarrow t_{reach} \le \frac{S(0)}{\zeta}} \qquad \textbf{(36)}$$

and

$$\boldsymbol{if\; S(0)} < 0 \rightarrow 0 - S(\boldsymbol{0}) \le \boldsymbol{-\eta(t_{reach}) \rightarrow S(0) \le -\zeta(t_{reach}) \rightarrow t_{reach} \le \frac{|S(0)|}{\eta}} \qquad \textbf{(37)}$$

$$\boldsymbol{if\; S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0} \qquad \textbf{(38)}$$

suppose S is defined as

$$\boldsymbol{s(x,t)} = (\frac{\boldsymbol{d}}{\boldsymbol{dt}} + \boldsymbol{\lambda}) \quad \widetilde{\boldsymbol{x}} = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{39}$$

The derivation of S, namely, $\dot{S}$ can be calculated as the following;

$$\dot{\boldsymbol{S}} = (\ddot{\mathbf{x}} - \ddot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{40}$$

suppose the second order system is defined as;

$$\ddot{\boldsymbol{x}} = \boldsymbol{f} + \boldsymbol{u} \ \rightarrow \dot{\boldsymbol{S}} = \boldsymbol{f} + \boldsymbol{U} - \ddot{\boldsymbol{x}}_{\boldsymbol{d}} + \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{41}$$

Where $\boldsymbol{f}$ is the dynamic uncertain, and also since $S = 0 \; and \; \dot{S} = 0$, to have the best approximation ,$\widehat{\boldsymbol{U}}$ is defined as

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{42}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\boldsymbol{U_{dis}} = \widehat{\boldsymbol{U}} - \boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t}) \cdot \mathbf{sgn}(\boldsymbol{s}) \tag{43}$$

where the switching function $\mathbf{sgn}(\mathbf{S})$ is defined as

$$\boldsymbol{sgn}(\boldsymbol{s}) = \begin{cases} \mathbf{1} & \boldsymbol{s} > 0 \\ -\mathbf{1} & \boldsymbol{s} < 0 \\ \mathbf{0} & \boldsymbol{s} = \mathbf{0} \end{cases} \tag{44}$$

and the $\boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t})$ is the positive constant.

$$\frac{\mathbf{1}}{\mathbf{2}}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{s}^{2}(\boldsymbol{x},\boldsymbol{t}) = \dot{\mathbf{S}} \cdot \mathbf{S} = \left[\boldsymbol{f} - \widehat{\boldsymbol{f}} - \boldsymbol{K}\mathbf{sgn}(\boldsymbol{s})\right] \cdot \boldsymbol{S} = \left(\boldsymbol{f} - \widehat{\boldsymbol{f}}\right) \cdot \boldsymbol{S} - \boldsymbol{K}|\boldsymbol{S}| \tag{45}$$

The sliding surface can be calculated as

$$\boldsymbol{s}(\boldsymbol{x},\boldsymbol{t}) = (\frac{\boldsymbol{d}}{\boldsymbol{dt}} + \boldsymbol{\lambda})^{2} \left(\int_{\mathbf{0}}^{\boldsymbol{t}} \widetilde{\boldsymbol{x}} \, \boldsymbol{dt}\right) = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) - \boldsymbol{\lambda}^{\mathbf{2}}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{46}$$

in this method the approximation of $\boldsymbol{U}$ is computed as

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}^{\mathbf{2}}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{47}$$

$$\boldsymbol{U} = \boldsymbol{U_{eq}} + \boldsymbol{U_{dis}} \tag{48}$$

$$\boldsymbol{U_{eq}} = \left[\boldsymbol{H}^{-\mathbf{1}}(\boldsymbol{B} + \boldsymbol{C}) + \dot{\boldsymbol{S}}\right]\boldsymbol{H} \tag{49}$$

$$\boldsymbol{U_{dis}} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{50}$$

Figure 8 shows conventional sliding mode controller.

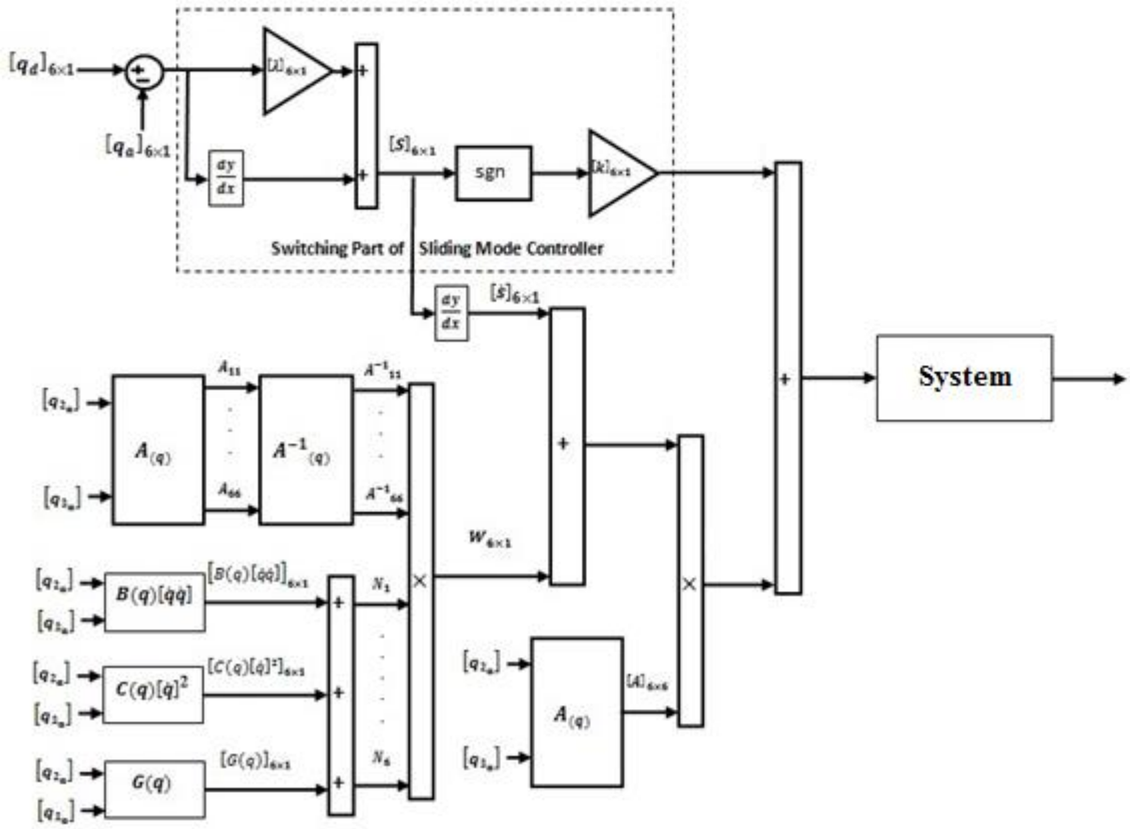

Figure 8. Block Diagram of PD Sliding Mode Controller

4. Results

In this research, three types of controller are compared, namely; conventional sliding mode controller, computed torque controller and PID controller. These three types of controller are tested in certain and uncertain situation.

Comparison of the Tracking Data and Information: the trajectory following for conventional sliding mode controller, computed torque controller and PID controller are compared in this section. According to Figure 9, traditional sliding mode controller has high frequency oscillation chattering phenomenon but this controller is a robust. However, computed torque controller does not have chattering or oscillation but this method has two challenges: robust and quality of performance. In rise time point of view, in some joints conventional sliding mode controller is faster than computed torque controller and PID controller. In error point of view, computed torque controller is better than conventional sliding mode controller and PID controller.

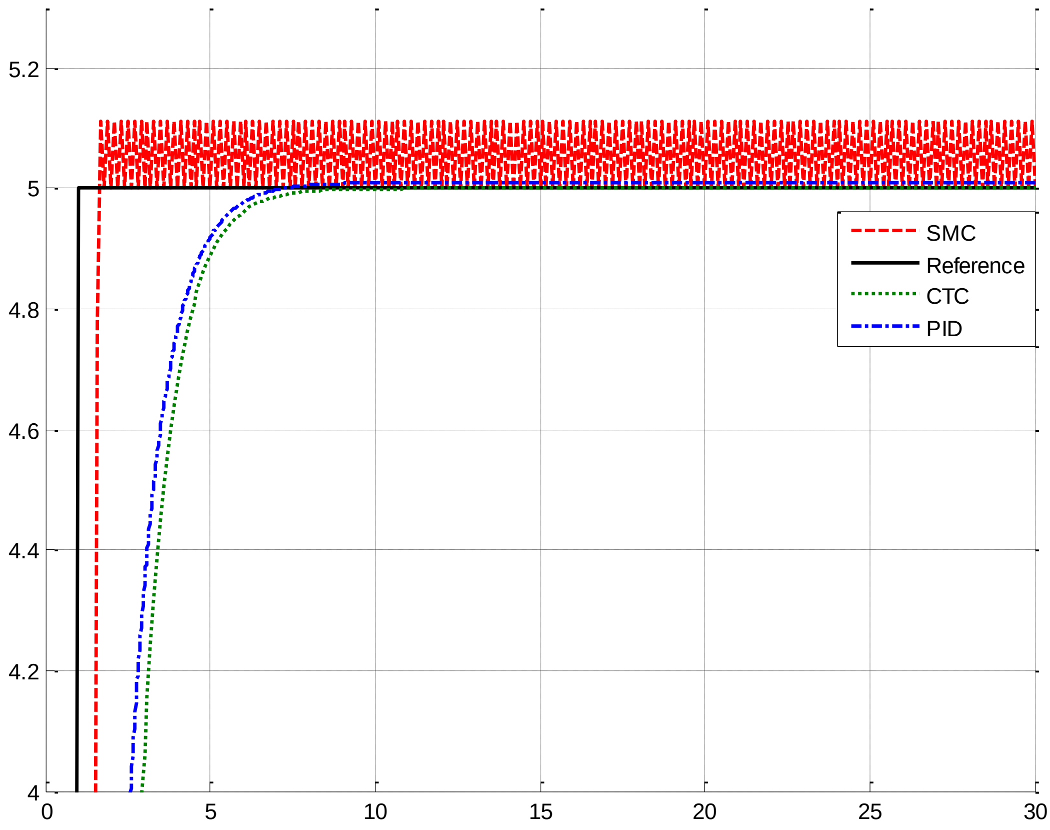

Figure 9.Tracking Data: Conventional SMC, CTC and PID Controller

Comparison the actuation torque(τ_i): the control input, forces the actuator to track the desired trajectories. Figure 10 shows the torque performance in conventional sliding mode controller, computed torque controller and PID controller. According to the following graph, computed torque controller and PID controller have steady stable torque performance.

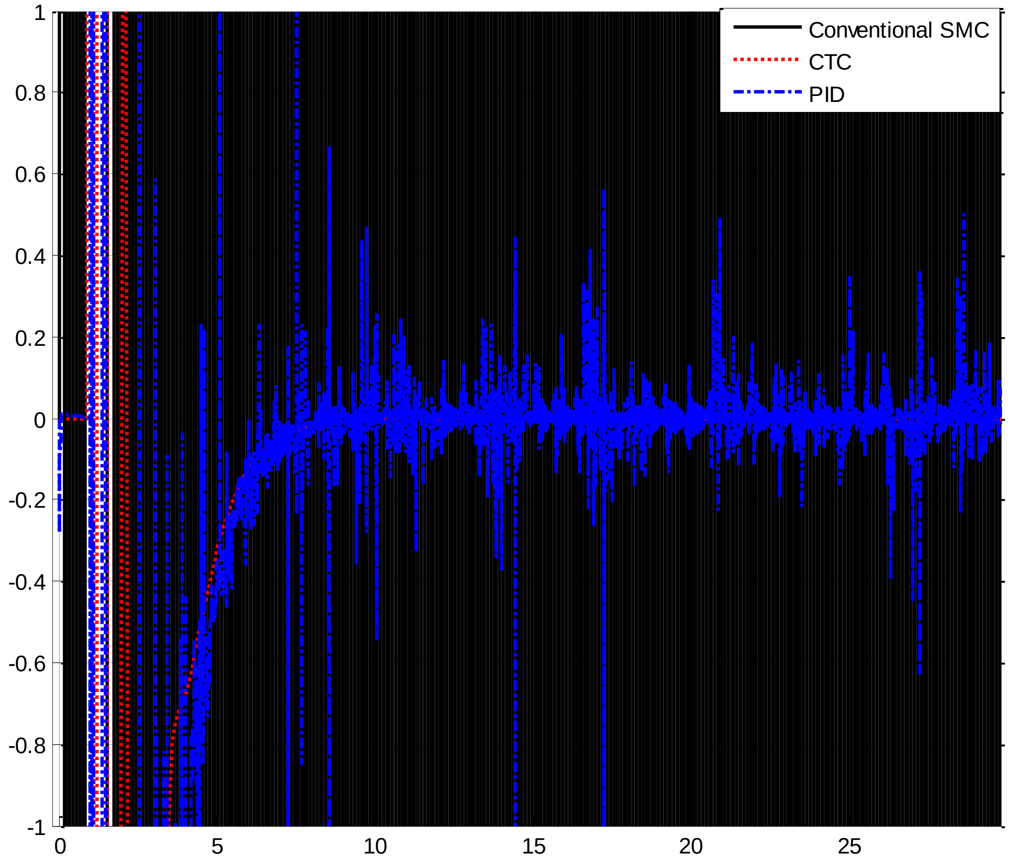

Figure 10. Torque Performance: Conventional SMC, CTC and PID Controller

In the control forces, smaller amplitude means less energy. According to Figure 10, the amplitude of the control forces in SMC controller is much larger than PID and CTC. Therefore, PID and CTC require less energy than the SMC controller.

Comparison the disturbance rejection: the power of disturbance rejection is very important to robust checking in any controllers. In this section trajectory accuracy, and torque performances are test under uncertainty condition. To test the disturbance rejection band limited white noise with 30% amplitude is applied to conventional sliding mode controller, computed torque controller and PID controller. In Figures 11 and 12, trajectory accuracy and torque performance are shown.

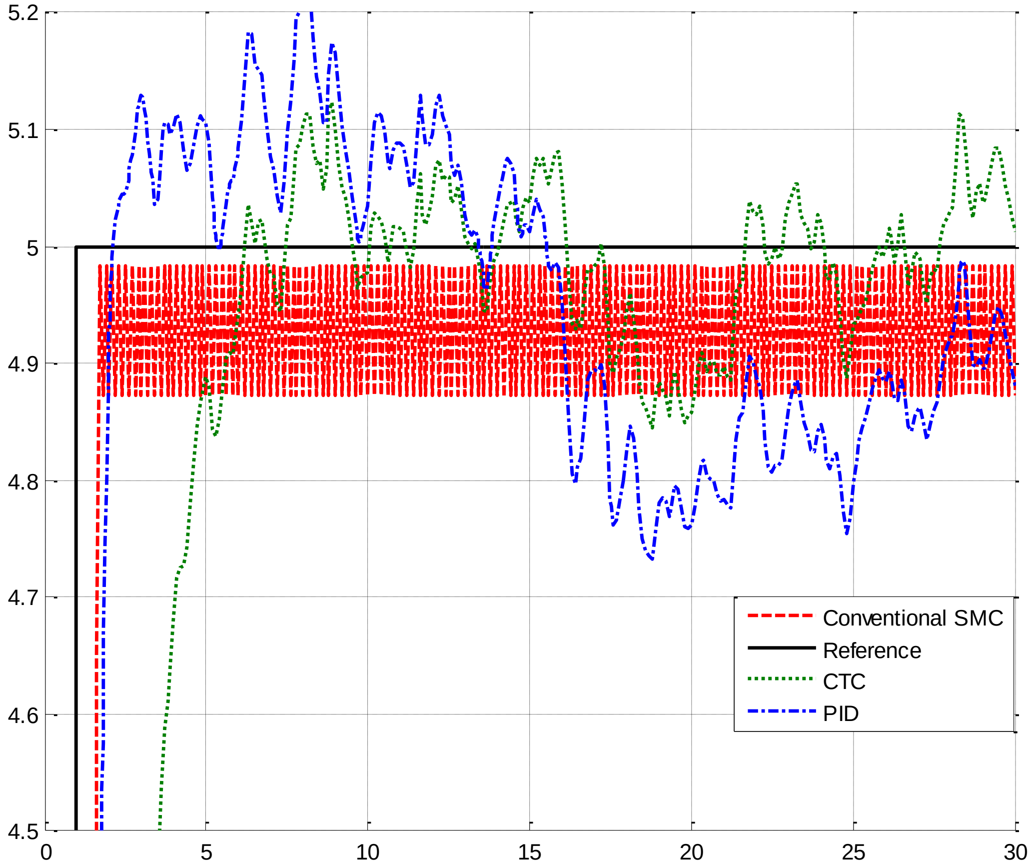

Figure 11. Tracking Data: Conventional SMC, CTC and PID Controller In Presence Of Uncertainty

According to above graph, however conventional sliding mode controller has suitable oscillation in presence of uncertainty but it is more robust than computed torque controller and PID controller. Computed torque controller and PID controller have very much fluctuations in presence of external disturbance. Figure 12 shows the torque performance in presence of uncertainty.

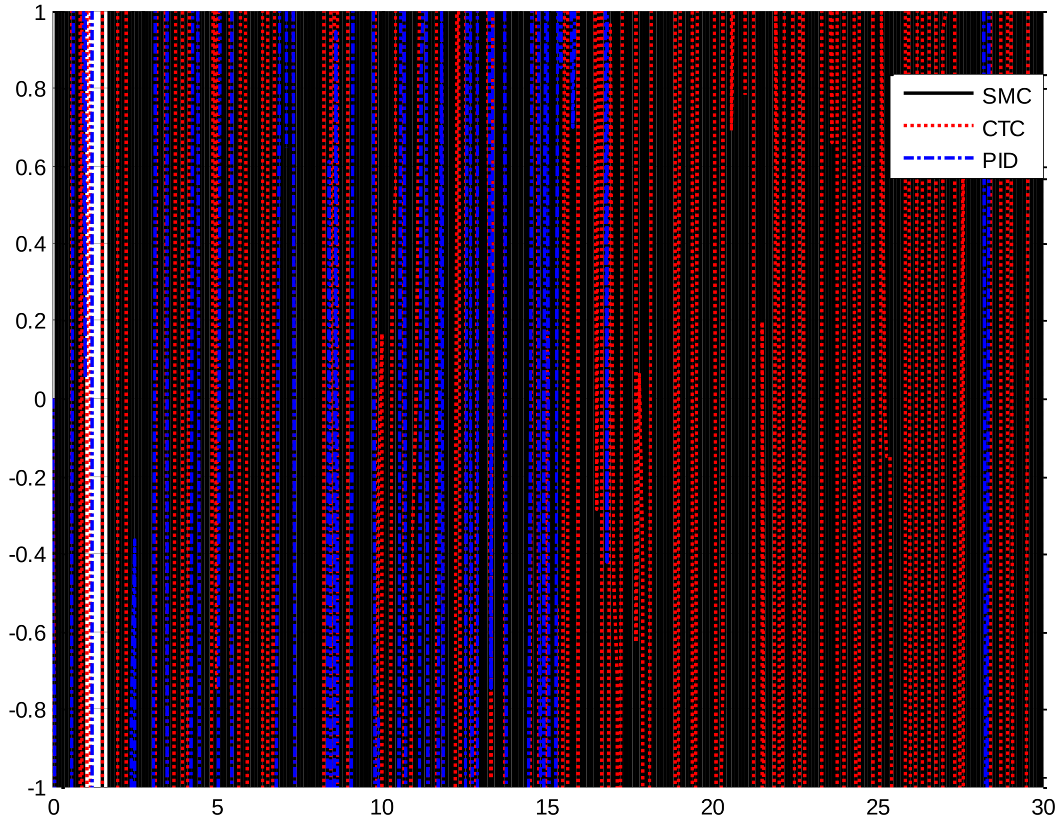

Figure 12. Torque Performance: Conventional SMC, CTC and PID Controller in Presence of Uncertainty

According to above graph, however sliding mode controller has chattering but it is more stable than CTC and PID controller in presence of uncertainties. After applied uncertainties the force amplitude in CTC and PID are increased which will lead to high energy consumption.

Tracking Error Comparison: in this part, tracking steady state error is compared. Figure 13 shows the steady state error in presence of uncertainties. According to this Figure, however conventional sliding mode controller has chattering but it is more stable than CTC and PID in presence of uncertainty. In the following graph CTC and PID have irregular fluctuations.

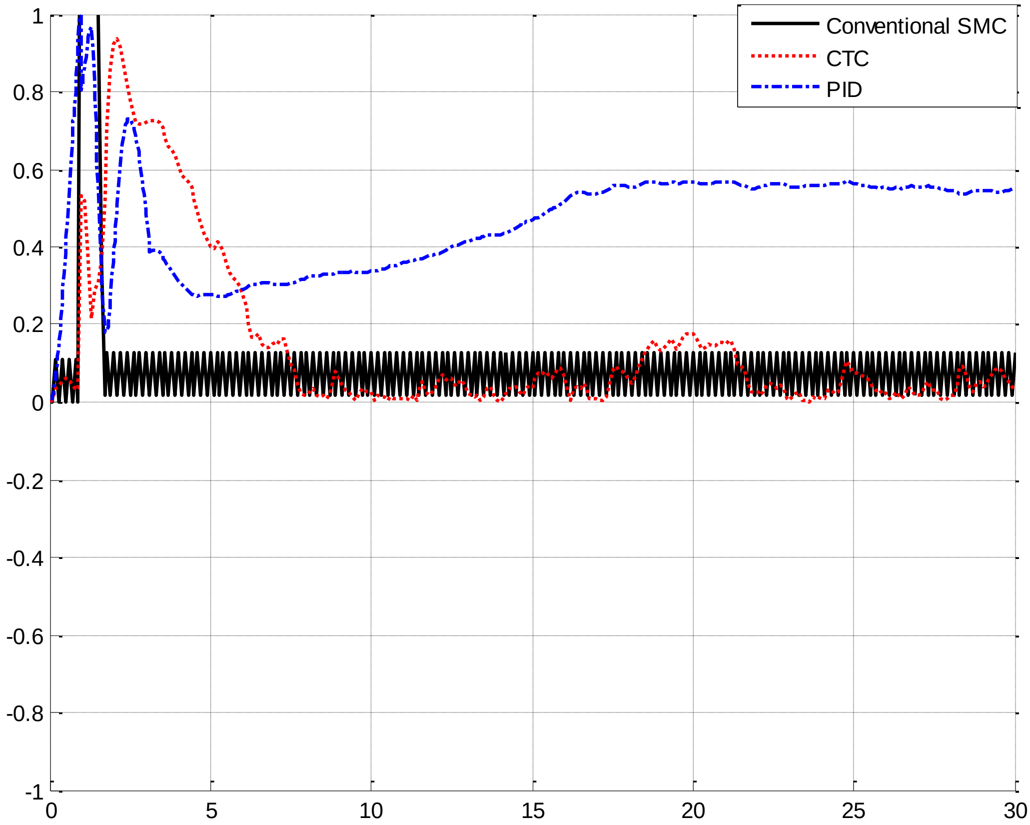

Figure 13. Steady State Error: Conventional SMC, CTC and PID Controller in Presence of Uncertainty

In CTC and PID fluctuations cause instability in presence of uncertainties. Figure 14 shows root means square (RMS) error in presence of uncertainty for conventional SMC, CTC and PID.

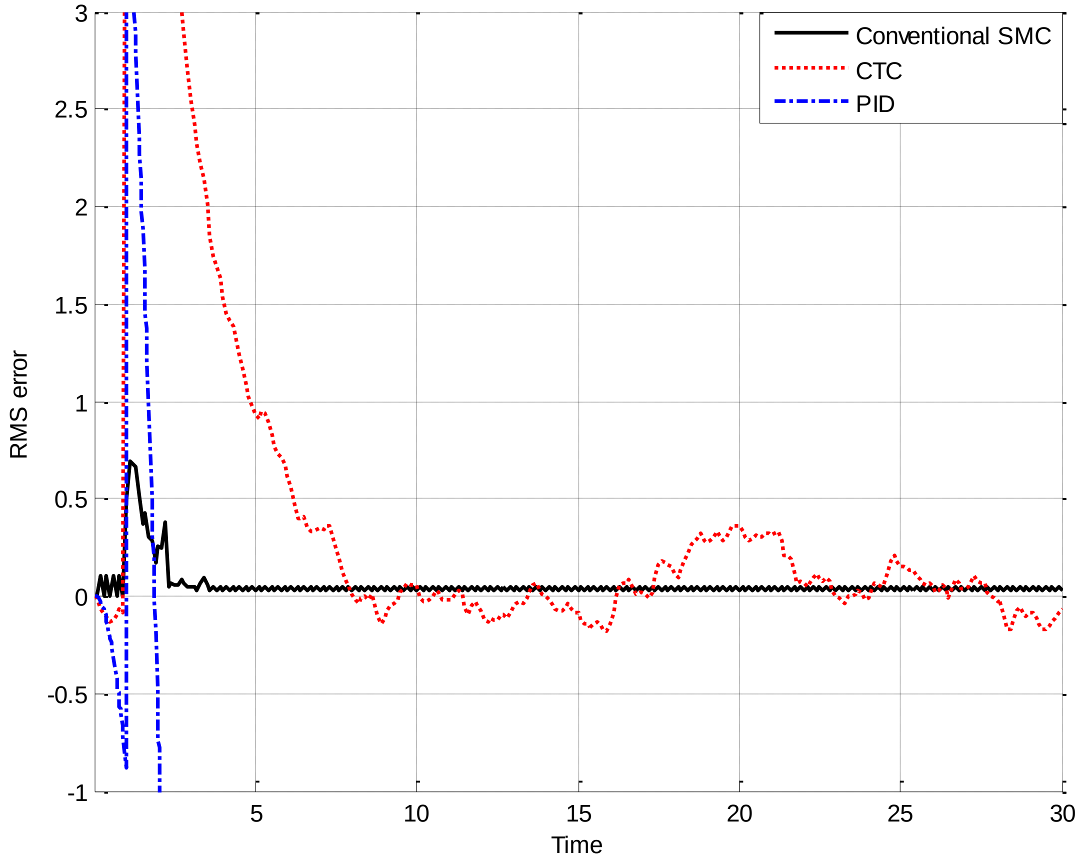

Figure 14. RMS Error: Conventional SMC, CTC and PID Controller in Presence of Uncertainty

Based on Figure 14, CTC and PID have more position deviations than conventional sliding mode controller. According to above graphs in presence of uncertainties conventional sliding mode controller has better performance than CTC and PID thus this type of controller is recommended. However SMC has better performance but this controller has chattering in certain and uncertain situations.

5. Conclusion

An Important question which comes to mind is that which type of controller is better for multi degrees of freedom system?

The dynamics of multi degrees of freedom actuator is highly nonlinear, time variant, MIMO, uncertain and there exist strong coupling effects between joints. The problem of coupling effects can be reduced, with the following two methods:

- Limiting the performance of the system according to the required velocities and accelerations, but now the applications demand for faster and lighter robot manipulators.
- Using a high gear ratio (e.g., 250 to 1) at the mechanical design step, in this method the price paid is increased due to the gears.

Therefore linear type of controller, such as PD or PID cannot be having a good performance. Consequently, to have a good performance, linearization and decoupling without using many gears, feedback linearization (computed-torque) control methodologies is presented. In order to design computed torque controller, an accurate dynamic model of system plays an important role. To modelling an accurate dynamic system, modelling of complex parameters is needed to form the structure of system's dynamic model. It may be very difficult to include all the complexities in the system dynamic model. Dynamic parameters may not be constant over time; subsequently adaptation methodology plays a vital role. System's dynamic parameter estimation in computed torque-based adaptive control methodology can be realized if acceleration term should be measured, but this work is very expensive. Furthermore, system's dynamic models through a large number of highly nonlinear parameters generate the problem of computation; as a result, it is caused to many challenges for real time applications. To eliminate the actual acceleration measurement and the computation burden as well as have stabile, efficiency and robust controller, sliding mode controller is introduced. Assuming unstructured uncertainties and structure uncertainties can be defined into one term and considered as an uncertainty and external disturbance, the problem of computation burden and large number of parameters can be solved to some extent. Now the most important target in this part is reducing the uncertainties limitation and assures the asymptotic stability for large area possible circumstances. The above discussion gives rational for selecting the comparison of these three controllers in this research.

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Project Title: "Design High Precision and Fast Dynamic Controller For Multi-Degrees Of Freedom Actuator"

Iranian center of Advance Science and Technology (IRAN SSP) is one of the independent research centers specializing in research and training across of Control and Automation, Electrical and Electronic Engineering, and Mechatronics & Robotics in Iran. At IRAN SSP research center, we are united and energized by one mission to discover and develop innovative engineering methodology that solve the most important challenges in field of advance science and technology. The IRAN SSP Center is instead to fill a long-standing void in applied engineering by linking the training a development function one side and policy research on the other. This center divided into two main units:

- Education unit
- Research and Development unit

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