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A New Approach to Proposition Set Reduction Based on the Minimal Transversal of Waned Values Hypergraph

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Abstract

It is an important foundation for the in-depth application of the propositional logic to compute all reductions, and this is a hot subject at present. This paper defines the identity element concept and the zero element concept in concept lattice. It proves that the minimal transversal of waned values hypergraph of zero element concept is a reduction in inducible context, and it also proves that the minimal transversal set is a full reduction set. Furthermore, the paper presents a new method based on intent waned values and hypergraph minimal transversal of concept lattice. By this method, all reductions can be quickly computed all at once. It develops the reducing algorithm. Compared with the original algorithm, which compute a reduction every time in first and gradually aggregate all reductions, the operating number of the improved algorithm is remarkably reduced. The approach has the remarkable advantages and good results.

Keywords: *Concept lattice; Waned Values Hypergraph; Minimal Transversal; Proposition Set Reduction*

1. Introduction

It is important for propositional logic to compute the conclusion set $\mathrm{D}(\Gamma)$ of the proposition set Γ. With the increasing of the size of Γ, the time complexity of computing $\mathrm{D}(\Gamma)$ is too big to be afforded [1]. If a proper subset Γ_0 of Γ has the same conclusion set as Γ, using Γ_0 to compute the conclusion set will be easier, and the time complexity will be lower. Obviously, calculating the reduction is the primary problem. Liberatore P[2,3,4] researched on CNF, 2CNF, Horn proposition and non-monotonic proposition in 2005 and 2008. Their method checks whether a proposition set is a non -redundancy equivalent subset of another proposition every times. In 2006, Ren Yan, et al. have researched the reduction of computing the proposition set $(\xi)^*$ [5]. In 2009, Li LiFeng, et al. have researched the reduction of computing the n-Valued Propositional Logic [6,7]. In 2010 and 2011, Yu Peng, et al. have researched the proposition reduction based on the truth degree of proposition[8,9]. Literature [10] successfully solved the proposition reduction of $(\omega, A) \in I$ computing binary logic proposition set based on the concept lattice. The specific method is as follows: For a given atom proposition set $S=\{p_1,\cdots,p_m\}$ and its proposition set $\Gamma=\{A_1,\cdots,A_n\}$, they define an inducible context (Ω_Γ,Γ,I) , and give an algorithm of computing Γ_0 by inducible context, where $\Omega_\Gamma=\{\omega\in\Omega \mid \exists A\in\Gamma:\omega(A)=T\}$, Ω is the set of all assignments, and , if and only if $\omega(A)=T$. Firstly, order $A_1,\cdots,A_n$, for example , A_1 is the first one, A_2 is the second one, …, A_n is the n-th one. Secondly, let $\Gamma_1=\Gamma$, for A_1 in Γ_1, if each assignment ω which

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satisfies $\omega(A_1)=F$ exists $A'\in\Gamma_1-\{A_1\}$, and $\omega(A')=F$, then let $\Gamma_2=\Gamma_1-\{A_1\}$, otherwise let $\Gamma_2=\Gamma_1$; for A_2 in Γ_2, if each assignment ω which satisfies $\omega(A_2)=F$ exists $A'\in\Gamma_2-\{A_2\}$, and $\omega(A')=F$, then let $\Gamma_3=\Gamma_2-\{A_2\}$, otherwise let $\Gamma_3=\Gamma_2$, and so on, until A_n in Γ_n, if each assignment ω which satisfies $\omega(A_n)=F$ exists $A'\in\Gamma_n-\{A_n\}$, and $\omega(A')=F$, then let $\Gamma_0=\Gamma_n-\{A_n\}$, otherwise let $\Gamma_0=\Gamma_n$. Γ_0 is one of the reductions of Γ. But the reduction may be more than one, for computing all other reductions, we need change the order of $A_1,\cdots,A_n$, and repeat above method. Since the algorithm of literature [10] do not know whether the results are the same reduction, and do not know how many reduction it can get, algorithm must compute $n!$ possibilities of $A_1,\cdots,A_n$ to get all reductions. For instance, in the example 1 of [10], Γ has 6 propositions, so the number of all sorts is $6!$, and the loop number is $6!=720$. Since the reduction number of Γ is 4, there are 716 repeated reductions. Therefore, how to get all reductions at a time become an important theoretical problem. This paper presents a new proposition set reduction method based on the minimal transversal of waned values hypergraph, and the method can compute all reductions at a time.

The rest of the paper is structured as follows: The second chapter is the basic definition and principles of propositional logic; The third chapter is the relationship of concept lattice and propositional logic; The fourth chapter is the waned value hypergraph and minimal transversal definitions; The fifth chapter is the algorithm of proposition reduction; The sixth chapter is conclusion.

2. The Basic Definition and Principles of Propositional Logic

Definition 1. Let $S=\{p_1,\cdots,p_m\}$ be a set of atom Propositions, the recursive definition of proposition set $\mathrm{F}(S)$ generated by S is as follows:

(1) $p_1,\cdots,p_m\in\mathrm{F}(S)$.

(2) If $A,B\in\mathrm{F}(S)$, then $\neg A, A\to B\in\mathrm{F}(S)$.

If (1)、(2)are satisfied, then generated elements belong to $\mathrm{F}(S)$.

By Definition 1, $\mathrm{F}(S)$ is algebraic system $(S,\neg,\to)$, which defines 2 operations: “ $\neg$ ” and “ $\to$ ” on the set S. If $A\vee B=\neg A\to B$, $A\wedge B=\neg(A\to\neg B)$, $A\leftrightarrow B=\neg((A\to B)\to\neg(B\to A))$, then $A\vee B, A\wedge B, A\leftrightarrow B\in\mathrm{F}(S)$, where $A,B\in\mathrm{F}(S)$.

Definition 2. Let ω be a mapping from $\mathrm{F}(S)$ to $\{T,F\}$, and satisfies $\omega(\neg A)=\neg\omega(A)$, $\omega(A\to B)=\omega(A)\to\omega(B)$. Then ω is called as an assignment of $\mathrm{F}(S)$, and the set of all assignments is denoted as Ω. For some ω and $\Gamma\subset\mathrm{F}(S)$, if every proposition A is in Γ, and $\omega(A)=T$, then it is denoted as $\omega(\Gamma)=T$.

Definition 3. Suppose $\Gamma\subset\mathrm{F}(S)$, deduction of Γ is a proposition order $A_1,\cdots,A_p$. A_p is called as the conclusion of Γ and denoted as $\Gamma\vdash A_p$, where A_k $(1\le k\le p)$ is or a proposition in Γ, or an axiom (In other words, $A\to(B\to A)$, $(A\to(B\to C))\to((A\to B)\to(A\to C))$, and $(\neg A\to\neg B)\to(B\to A)$), or a proposition that is deduced by A_i ,A_j $(i,j<k)$ using principle MP (In other words, A and $A\to B$ deduce B). The set of all propositions $\{A\in\mathrm{F}(S)\mid\Gamma\vdash A\}$ that deduced by Γ is denoted as $\mathrm{D}(\Gamma)$. For $\Gamma_0\subset\Gamma$, if $\mathrm{D}(\Gamma_0)=\mathrm{D}(\Gamma)$ and $\forall\Gamma_1\subset\Gamma_0$, $\mathrm{D}(\Gamma_1)\ne\mathrm{D}(\Gamma)$, then Γ_0 is a reduction of Γ.

Theorem 1. [10] Suppose that Γ is a proposition set, and ω satisfies $\omega(\Gamma)=T$. Then $B\in \mathrm{D}(\Gamma)$, if and only if $\omega(B)=T$.

Theorem 2. Suppose $\Gamma_0 \subset \Gamma$. Γ_0 is a reduction of Γ, if and only if it satisfies conditions (1), (2).

(1) $\forall\omega\in\Omega, \omega(\Gamma_0)=T$ if and only if $\omega(\Gamma)=T$;

(2) $\forall\Gamma'\subset\Gamma_0$, there are more than one ω which satisfies $\omega(\Gamma')=T$ and $\omega(\Gamma)\neq T$;

Proof. Suppose $\forall\omega\in\Omega, \omega(\Gamma_0)=T$. Then $\omega(\Gamma)=T$ must be true. $\because \Gamma_0\subset\Gamma, \therefore$ if $\omega(\Gamma)=T$, then $\omega(\Gamma_0)=T$.In other words, $\omega(\Gamma_0)=T$, if and only if $\omega(\Gamma)=T$. $\because$ By the Theorem 1, $\omega(B)=T$ if and only if $\omega(\Gamma)=T$. $\therefore B\in\mathrm{D}(\Gamma)$, if and only if $\omega(\Gamma_0)=T$. $\because$ By Theorem 1, $B\in\mathrm{D}(\Gamma_0)$. $\therefore$ $\mathrm{D}(\Gamma_0)\supseteq\mathrm{D}(\Gamma)$. And $\because \Gamma_0\subset\Gamma . \therefore \mathrm{D}(\Gamma_0)\subseteq\mathrm{D}(\Gamma)$. $\therefore \mathrm{D}(\Gamma_0)=\mathrm{D}(\Gamma)$. $\because \forall\Gamma'\subset\Gamma_0$, there are more than one ω which satisfy $\omega(\Gamma')=T$ and $\omega(\Gamma)\neq T$ (That is, $\forall\Gamma'\subset\Gamma_0$, there are more than one $A\in\Gamma$ and ω, which satisfy $\omega(\Gamma)\neq T$).$\therefore A\notin\mathrm{D}(\Gamma')$. $\therefore \forall\Gamma'\subset\Gamma_0, \mathrm{D}(\Gamma')\neq\mathrm{D}(\Gamma)$. $\therefore \Gamma_0$ is a reduction of Γ.

Let Γ_0 be a reduction of Γ . $\because$ $\mathrm{D}(\Gamma_0)=\mathrm{D}(\Gamma)$, $\Gamma\subseteq\mathrm{D}(\Gamma)$, $\therefore \Gamma\subseteq\mathrm{D}(\Gamma_0)$ .By Theorem 1, $\forall\omega\in\Omega$, if $\omega(\Gamma_0)=T$, then $\omega(\mathrm{D}(\Gamma_0))=T$ and $\omega(\Gamma)=T$. $\because \forall\Gamma'\subset\Gamma_0$, $\mathrm{D}(\Gamma')\neq\mathrm{D}(\Gamma)$, $\therefore A\in\mathrm{D}(\Gamma)$, $A\notin\mathrm{D}(\Gamma')$. By Theorem 1, there are more than one ω which satisfy $\omega(\Gamma')=T$, $\omega(\mathrm{D}(\Gamma'))=T$. thus, $\omega(A)=F$, and $A\in\mathrm{D}(\Gamma)$, $\omega(\Gamma)=T$ (That is, $\forall\Gamma'\subset\Gamma_0$, there are more than one ω which satisfy $\omega(\Gamma')=T$ and $\omega(\Gamma)\neq T$).

3. The Relationship of Concept Lattice and Propositional Logic

Definition 4[11]. The tuple $\mathrm{K}=(U,M,I)$ is called formal context, where U is a set objects, M is a set of attributes, and $I\subseteq U\times M$ is the relation of U and M.

Definition 5[10]. $\mathrm{K}=(U,M,I)=(\Omega_\Gamma,\Gamma,I)$ is called inducible context, and $\overline{\mathrm{K}}=(\Omega_\Gamma,\Gamma,\Omega_\Gamma\times\Gamma-I)$ is called anti-inducible context, where Γ is a set of propositions, Ω is a set of assignments, $\Omega_\Gamma=\{\omega\in\Omega\,|\,\exists A\in\Gamma:\omega(A)=T\}$, and $\mathrm{K}=(U,M,I)$ is a formal context, $U=\Omega_\Gamma$, $M=\Gamma$, $(\omega,A)\in I$,if and only if $\omega(A)=T$.

Example 1. Suppose that $S=\{p,q,r\}$, $\Gamma=\{A,B,C,D,E,F\}$, where $A=p$, $B=q\to r$, $C=(p\to q)\to r$, $D=\neg q$, $E=(\neg q\to r)\wedge(r\to\neg q)$, $F=q\to p\wedge\neg r$. Then inducible context (Ω_Γ,Γ,I) is Table 1, and anti-inducible context $(\Omega_\Gamma,\Gamma,\Omega_\Gamma\times\Gamma-I)$ is Table 2.

Definition 6[11]. $\mathrm{K}=(U,M,I)$ is a formal context, $X\subseteq U, Y\subseteq M$. There are 2 functions: $f(X)\subseteq\{m\in M\,|\,\forall u\in X:(u,m)\in I\}$, $g(Y)\subseteq\{u\in U\,|\,\forall m\in Y:(u,m)\in I\}$.

If $\mathrm{f(X)=Y, g(Y)=X}$, then (X,Y) is a formal concept, known as concept for short. X is called the object of concept, and Y is called the attribute of concept. All concept of K is denoted as $\mathrm{B}(U,M,I)$.

Table 1. The Inducible Context of the Proposition Set

	p	q	r	A	B	C	D	E	F
ω_1	0	0	0		×		×		×
ω_2	1	0	0	×	×	×	×		×
ω_3	0	1	0					×	
ω_4	0	0	1		×	×	×	×	×
ω_5	1	1	0	×				×	×
ω_6	1	0	1	×	×	×	×	×	×
ω_7	0	1	1		×	×			
ω_8	1	1	1	×	×	×			

Table 2. The Anti-Inducible Context of the Proposition Set

	p	q	r	A	B	C	D	E	F
ω_1	0	0	0	×		×		×	
ω_2	1	0	0					×	
ω_3	0	1	0	×	×	×	×		×
ω_4	0	0	1	×					
ω_5	1	1	0		×	×	×		
ω_6	1	0	1						
ω_7	0	1	1	×			×	×	×
ω_8	1	1	1				×	×	

Definition 7 [11]. Let $\mathrm{K}=(U,M,I)$ be a formal context. $(X_1,Y_1),(X_2,Y_2)\in \mathrm{B}(U,M,I)$, If $X_1 \subseteq X_2$, (X_1,Y_1) is called the subconcept of (X_2,Y_2) , and (X_2,Y_2) is the superconcept of (X_1,Y_1), the relation between two concepts is denoted as $(X_1,Y_1)\le(X_2,Y_2)$. If $X_1 \subset X_2$, the relation is denoted as $(X_1,Y_1)<(X_2,Y_2)$. If $(X_1,Y_1)<(X_2,Y_2)$ and there is no (X_3,Y_3) with $(X_1,Y_1)<(X_3,Y_3)<(X_2,Y_2)$, then (X_1,Y_1) is the direct subconcept of (X_2,Y_2) , (X_2,Y_2) is the direct superconcept of (X_1,Y_1), and the relation of two concepts is denoted as $(X_1,Y_1)\prec(X_2,Y_2)$.

Property 1 [11]. Let $\mathrm{K}=(U,M,I)$ be a formal context, $X_1, X_2 \subseteq U$, $Y_1,Y_2 \subseteq M$, then

(1) if $X_1 \subseteq X_2$, $f(X_1) \supseteq f(X_2)$ (2) if $Y_1 \subseteq Y_2$, $g(Y_1) \supseteq g(Y_2)$

(3) $f(X_1 \cup X_2) = f(X_1) \cap f(X_2)$ (4) $g(Y_1 \cup Y_2) = g(Y_1) \cap g(Y_2)$

(5) $X_1 \subseteq g(f(X_1))$ (6) $Y_1 \subseteq f(g(Y_1))$

(7) $f(X_1) = f(g(f(X_1)))$ (8) $g(Y_1) = g(f(g(Y_1)))$

By Property 1(7), $\forall X \subseteq U$, $(g(f(X)), f(X))$ must be a concept. By the property 1(8), $\forall Y \subseteq M$, $(g(Y), f(g(Y)))$ must be a concept. Specially, if X has only one object u , $(g(f(u)), f(u))$ is called the object concept of u . If Y has only one attribute m , $(g(m), f(g(m)))$ is called the attribute concept of m . By Property 1(5), $U \subseteq g(f(U))$, and obviously, $g(f(U)) \subseteq U$, so $g(f(U))=U$, and $(U, f(U))=(g(f(U)), f(U))$ must be a concept. Because the object number of extensions don't more than U , All concepts are subconcept of U . Because if $X \subseteq U$, then $X \cap U = X$, $(U, f(U))$ is called unit meta-concept. By Property 1(6),

$M \subseteq f(g(M))$, and obviously, $f(g(M)) \subseteq M$, so $f(g(M)) = M$, and $(g(M), M) = (g(M), f(g(M)))$ must be a concept. Because the object number of intensions don't more than M , All concepts are superconcept of M . Because if $X \supseteq g(M)$, then $X \cup g(M) = X$, $(g(M), M)$ is called zero meta-concept.

Theorem 3. Let Γ be a proposition set, $\mathrm{K} = (\Omega_\Gamma, \Gamma, I)$ be the inducible context of Γ, and $\Gamma_0 \subset \Gamma$. Γ_0 is a reduction of Γ, if and only if it satisfies conditions (1), (2).

(1) $g(\Gamma_0) = g(\Gamma)$.

(2) $\forall \Gamma' \subset \Gamma_0, g(\Gamma') \neq g(\Gamma)$.

Proof. By Definition 6, $g(\Gamma) = \{\omega \in \Omega_\Gamma \mid \forall A \in \Gamma : (\omega, A) \in I\}$, and by Definition 6, $(\omega, A) \in I$, that is, $\omega(A) = T$, so $g(\Gamma) = \{\omega \in \Omega_\Gamma \mid \forall A \in \Gamma : \omega(A) = T\}$. By Definition 2, $\omega(\Gamma) = T$, so $\mathrm{g}(\Gamma) = \{\omega \in \Omega_\Gamma \mid \omega(\Gamma) = T\}$. Similarly, $g(\Gamma_0) = \{\omega \in \Omega_\Gamma \mid \omega(\Gamma_0) = T\}$. $\because$ if $\Gamma_0 \subset \Gamma$, $g(\Gamma) = g(\Gamma_0)$. $\therefore$ $\forall \omega \in \Omega_\Gamma$, if $\omega(\Gamma_0) = T$, $\omega(\Gamma) = T$. Similarly, by definition 5, $\forall \omega \in \Omega$, if $\omega(\Gamma_0) = T$, $\omega(\Gamma) = T$. By Property1(2), $\forall \Gamma' \subset \Gamma_0, g(\Gamma') \supseteq g(\Gamma)$, and $\because$ if $g(\Gamma') \neq g(\Gamma)$, $g(\Gamma') \supset g(\Gamma_0)$, $\therefore \forall \Gamma' \subset \Gamma_0$, there are more than one ω which satisfy $\omega \in g(\Gamma')$ and $\omega \notin g(\Gamma)$. In other words, $\forall \Gamma' \subset \Gamma_0$, there are more than one ω which satisfy $\omega(\Gamma') = T$, and make $\omega(\Gamma) \neq T$ satisfise Theorem 2. So Theorem 3 is true.

4. Waned Values Hypergraph and the Minimal Transversal

Definition 8. Let $\mathrm{K} = (U, M, I)$ be a formal context. (X, Y) is a concept of K , and $(\mathrm{X}_1, \mathrm{Y}_1)$ is a direct superconcept of (X, Y). Then $\mathrm{Y} - \mathrm{Y}_1$ is called an attribute waned value of (X, Y) or waned value of (X, Y) for short (unit meta-concept $(\mathrm{U}, f(\mathrm{U}))$ does not have waned value). The waned value set of (X, Y) is denoted as $\mathrm{V}(\mathrm{X}, \mathrm{Y})$ ($\mathrm{V}(U, f(U)) = \varnothing$), and the union set of all waned values in $\mathrm{V}(\mathrm{X}, \mathrm{Y})$ is denoted as $\mathrm{P}(\mathrm{X}, \mathrm{Y})$.

Example 2. The concept lattice of Table 1 is shown in Figure 1. The zero meta-concept $(6, ABCDEF)$ has 3 direct superconcept : $(26, ABCDF)$, $(46, BCDEF)$ 与 $(56, AEF)$, so waned values are E, A, BCD , $\mathrm{V}((6, ABCDE))= \{E, A, BCD\}$, and $\mathrm{P}((6, ABCDE)) = \cup \mathrm{V}(X, Y) = \{E\} \cup \{A\} \cup \{B, C, D\} = \{A, B, C, D, E\}$.

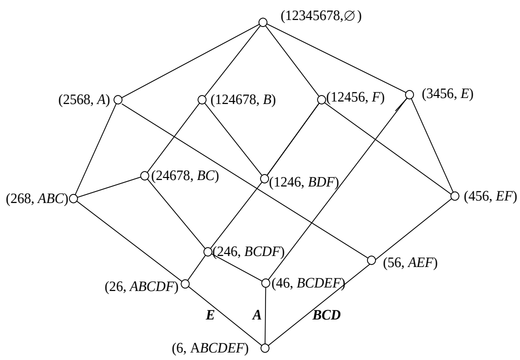

Figure 1. The Hasse Graph of Table 1

Definition 9[12]**.** Let H=(P,V) be a hypergraph, where P is a set of points, $V \subseteq 2^P$ is a set of hyper-edges. If $E \subseteq P$, $\forall v \in V$, $v \cap E \neq \varnothing$, then E is a transversal of H **.** If E is a transversal of H , and $\forall E' \subset E$, E' is not a transversal of H , then E is a minimal transversal of H **.**

Example 3. Suppose that $P=\{A,B,C,D,E\}$, $V=\{AE,ACD,BC\}$. Then the hypergraph H=(P,V) is shown as Figure 2. ABC is a transversal, but not a minimal transversal. AB is a minimal transversal. In transversal this example, there are more than one minimal transversals. For instance, BDE is a minimal transversal too.

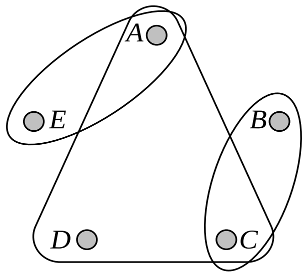

Figure 2. The Hypergraph of Example 3

Definition 10. Let $\mathrm{K}=(U,M,I)$ be a formal context. (X,Y) is a concept of K , and $P=\mathrm{P}(\mathrm{X},\mathrm{Y})$, $V=\mathrm{V}(\mathrm{X},\mathrm{Y})$. Then $\mathrm{H}(\mathrm{X},\mathrm{Y})=$ (P,V) is a waned value hypergraph of (X,Y).

Example 4. The waned value $\mathrm{H}(6,\mathrm{ABCDEF})$ hypergraph of zero meta-concept is shown as Figture 3.(The according concept lattice is shown as Figure 1).

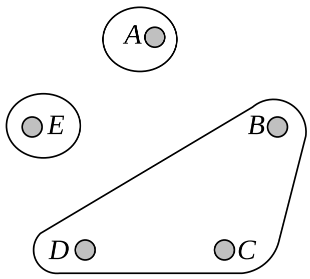

Figure 3. The Waned Value Hypergraph of Zero Meta-Concept

Lemma 1. Let K be formal context. If Y is a intension of K , and A is a transversal of the waned value hypergraph $\mathrm{H}(g(Y),Y)$, then $A \subseteq Y$.

Proof. $\because$ A is a transversal of the hypergraph $\mathrm{H}(g(Y),Y)$. $\therefore$ A is the subset of point set $\mathrm{P}(\mathrm{g(Y)},\mathrm{Y})$, that is, A is the subset of the waned value union set $\cup\mathrm{V}(\mathrm{g(Y)},\mathrm{Y})$. And $\because$ all waned values are the subset of Y . $\therefore$ $A \subseteq Y$.

Lemma 2. Let K be a formal context. Y is the intension of K , $A \subseteq Y$. $g(A)=g(Y)$ if and only if A is a transversal of the hypergraph $\mathrm{H}(g(Y),Y)$.

Proof. Suppose that $\mathrm{V}(g(Y),Y)=\{S_1,\cdots,S_m\}$, where the waned value S_i is generated by the direct superconcept (X_i,Y_i), that is, $S_i=Y-Y_i$, $1 \le i \le m$. Suppose that A is a transversal of the hypergraph $\mathrm{H}(g(Y),Y)$. By Lemma 2, $A \subseteq Y$. By Property 1(2), $g(A) \supseteq g(Y)$. Use proof by contradiction. If $g(A) \supset g(Y)$ is true, by definition 7, $(g(A), f(g(A)))$ is a superconcept of $(g(Y),Y)$. So there must be a direct superconcept $(g(Y_j),Y_j)$ of $(g(Y),Y)$ that satisfies $f(g(A))\subseteq Y_j$. $\therefore$ By the property 1(6), $A\subseteq f(g(A))\subseteq Y_j$. $\because$ A is a transversal of $\mathrm{H}(g(Y),Y)$. $\therefore$ $A\cap(Y-Y_i)\neq\varnothing$, $1\leq i\leq m$. $\therefore$ $A\not\subseteq Y_i$. The conclusion contradicts with $A\subseteq Y_j$. $\therefore$ $g(A)\supset g(Y)$ is not true. $\therefore$ $g(A)=g(Y)$.

Prove that if $g(A)=g(Y)$, then A is a transversal of $\mathrm{H}(g(Y),Y)$. Use proof by contradiction. Suppose A is not a transversal of $\mathrm{H}(g(Y),Y)$. Then there must be a S_i that satisfies $A\cap(Y-Y_i)=\varnothing$. $\because$ $g(A)=g(Y)$, $\therefore$ $f(g(A))=f(g(Y))$. $\because$ By property 1(6), $A\subseteq f(g(A))$. $\therefore$ $A\subseteq f(g(Y))$. $\because$ Y is a intension of K. $\therefore$ By Definition 6, $Y=f(g(Y))$. $\therefore$ $A\subseteq Y$. $\therefore$ By $A\cap(Y-Y_i)=\varnothing$, $A\subseteq Y_i$. $\therefore$ By Property 1(2), $g(A)\supseteq g(Y_i)$. $\because$ $(g(Y_i),Y_i)$ is the direct superconcept of $(g(Y),Y)$. $\therefore$ $g(Y_i)\supset g(Y)=g(A)$. $\therefore$ $g(A)\supseteq g(Y_i)\supset g(Y)=g(A)$, that is, $g(A)\supset g(A)$. The conclusion is not true.

Theorem 4. Let Γ be a proposition set, let $\mathrm{K}=(\Omega_\Gamma,\Gamma,I)$ be the inducible context of Γ, and $\Gamma_0\subset\Gamma$. Γ_0 is a reduction of Γ if and only if Γ_0 is the minimal transversal of the hypergraph of zero meta-concept $(g(\Gamma),\Gamma)$.

Proof. By Theorem 3 and Lemma 2, Γ_0 is a transversal of waned value hypergraph $(g(\Gamma),\Gamma)$, while Γ' is not a transversal of waned value hypergraph $(g(\Gamma),\Gamma)$. Therefore, Γ_0 is a minimal transversal of hypergraph $(g(\Gamma),\Gamma)$.

5. The Algorithm of Proposition Reduction

Definition 11. Let N be a set, and let $\mathrm{N}\subseteq 2^N$ be the set of all subsets of N. Then

$$\min(\mathrm{N})=\{P\in\mathrm{N}\mid\nexists P'\in\mathrm{N}:P'\subset P\}:$$

$$\max(\mathrm{N})=\{P\in\mathrm{N}\mid\nexists P'\in\mathrm{N}:P'\supset P\}:$$

Definition 12. Let $N_1,N_2,\cdots,N_k$ be k sets. Then their set product is

$$N_1\otimes N_2\otimes\cdots\otimes N_k=\{\{a_1,a_2,\cdots,a_k\}\mid a_1\in N_1,a_2\in N_2,\cdots,a_k\in N_k\}.$$

their set product is denoted as

$$\otimes(\{N_1,N_2,\cdots,N_k\}).$$

Theorem 5. Let Γ be a proposition set. The function that get anti-inducible context $\overline{\mathrm{K}}=(\Omega_\Gamma,\Gamma,\Omega_\Gamma\times\Gamma-I)$ is denoted as $\overline{f}$. Then the set of all reductions of Γ is

$$\min(\otimes(\min(\{\overline{f}(\omega)\mid\omega\in\Omega_\Gamma\}-\{\varnothing\}))).$$

Proof. By Theorem 4, the reduction of Γ is the minimal transversal generated by hypergraph of zero meta-concept $(g(\Gamma),\Gamma)$ of inducible context. if $Y\in\max(\{f(\omega)\mid\omega\in\Omega_\Gamma\}-\{\Gamma\})$, then $\exists\ \omega'\in\Omega_\Gamma$, $Y=f(\omega')$, $Y\neq\Gamma$. So (X,Y) must be a superconcept of zero meta-concept $(g(\Gamma),\Gamma)$, and (X,Y) must be a object concept. $\because$ There is the waned value $\Gamma-Y$. $\therefore$the set of all waned value of $(g(\Gamma),\Gamma)$ is $\left\{\Gamma-Y\middle|Y\in\max(\{f(\omega)\mid\omega\in\Omega_\Gamma\}-\{\Gamma\})\right\}$. $\because$ Y is a element in $\{f(\omega)\mid\omega\in\Omega_\Gamma\}-\{\Gamma\}$, and there are no element that is its proper superset. $\therefore$ $\Gamma-Y$ must be the element in $\{\overline{f}(\omega)\mid\omega\in\Omega_\Gamma\}-\{\varnothing\}$, and there are no other elements that is its proper subset in $\{\overline{f}(\omega)\mid\omega\in\Omega_\Gamma\}-\{\varnothing\}$. $\therefore$ the set of all waned values of $(g(\Gamma),\Gamma)$ is

$\min(\{\overline{f}(\omega) \mid \omega \in \Omega_\Gamma\} - \{\varnothing\})$, the transversal set of waned value hypergraph is $\otimes(\min(\{\overline{f}(\omega) \mid \omega \in \Omega_\Gamma\} - \{\varnothing\}))$, and the minimal transversal set is $\min(\otimes(\min(\{\overline{f}(\omega) \mid \omega \in \Omega_\Gamma\} - \{\varnothing\})))$.

Algorithm. Calculate all reductions of proposition set

Input: Atom proposition set $S = \{p_1, \cdots, p_m\}$ and proposition set $\Gamma = \{A_1, \cdots, A_n\}$.

Output: All reductions of Γ.

Step:

(1) Generate anti-inducible context $\overline{K} = (\Omega_\Gamma, \Gamma, \Omega_\Gamma \times \Gamma - I)$ by S and Γ.

(2) Get the set $S_1 = \{\overline{f}(\omega) \mid \omega \in \Omega_\Gamma\} - \{\varnothing\}$

(3) Delete the element that is the proper superset of other element in S_1, and assigns it to S_2. Thta is, $S_2 = \min(S_1)$.

(4) Calculate $S_3 = \{\{a_1, \cdots, a_k\} \mid a_1 \in s_1, \cdots, a_k \in s_k\}$, $S_3 = \otimes(S_2)$, $S_2 = \{s_1, \cdots, s_k\}$

(5) Delete the element that is the proper superset of other element in S_3, and assigns it to S_4. Thta is, $S_4 = \min(S_3)$. S_4 is the final result.

Example 5. Suppose that $S = \{p, q\}$, $\Gamma = \{A, B, C, D, E\}$, where $A = p$, $B = q$, $C = (p \to q)$, $D = (q \to p)$, $E = (p \vee q)$. Then the inducible context $(\Omega_\Gamma, \Gamma, I)$ is shown as Figure 3, and the anti-inducible context is shown as Figure 4.

Table 3. Inducible Context

	p	*q*	*A*	*B*	*C*	*D*	*E*
ω_1	**0**	**0**			×	×	
ω_2	**0**	**1**		×	×		×
ω_3	**1**	**0**	×			×	×
ω_4	**1**	**1**	×	×	×	×	×

Table 4. Anti-Inducible Context

	p	*q*	*A*	*B*	*C*	*D*	*E*
ω_1	**0**	**0**	×	×			×
ω_2	**0**	**1**	×			×	
ω_3	**1**	**0**		×	×		
ω_4	**1**	**1**					

Thus, $S_1 = \{ABE, AD, BC, \varnothing\} - \{\varnothing\} = \{ABE, AD, BC\}$, $S_2 = S_1 = \{ABE, AD, BC\}$, $S_3 = \{\{a_1, a_2, a_3\} \mid a_1 \in s_1, a_2 \in s_2, a_3 \in s_3\}$ $= \{\{A, B\}, \{A, C\}, \{A, D, B\}, \{A, D, C\}$ $, \{B, A, C\}, \{B, D\}, \{B, D, C\}, \{E, A, B\}, \{E, A, C\}, \{E, D, B\}, \{E, D, C\}\}$

Delete the element that is the proper superset of other element in S_3, and assigns it to $S_4 = \{\{A, B\}, \{A, C\}, \{B, D\}, \{E, D, C\}\}$. Therefore, there are 4 reductions of Γ : $\{A, B\}, \{A, C\}, \{B, D\}, \{E, D, C\}$.

For Example 5, the calculation steps number is 77, while the calculation steps number of [10] is $40\times 5!=4800$.

Suppose that $S=\{p,q,r\}$ and $\Gamma=\{A,B,C,D,E,F\}$, where $A=p$, $B=q\to r$, $C=(p\to q)\to r$, $D=\neg q$, $E=(\neg q\to r)\wedge(r\to\neg q)$, $F=q\to p\wedge\neg r$. Then the inducible context (Ω_Γ,Γ,I) is shown as Figure 1, and the anti-inducible context is shown as Figure 2. Using algorithm of this paper, we need calculate 41 superset judgements, while [10] needs calculate $20\times 6!=14400$ superset judgements.

The time complexity of the algorithm is $O(n)+O(n)+O(2^n)+O(4^n)=O(4^n)$, which is not a polynomial form but a exponential form. However, this is inevitable because computing the minimal transversal of waned value hypergraph is a NP-complete problem. As everyone knows, NP complete problem cannot find the polynomial algorithm so far. By the algorithm of [10], the time complexity is $O(n^2\times n!)$. Although two algorithms are exponential, the time complexity $O(n^2\times n!)$ and $O(4^n)$ is quite a difference, and the algorithm has obvious advantages in this paper.

6. Conclusion

This paper presents a new method which can compute all proposition reductions at once. There are many equivalent methods to compute the minimal transversal, such as the method that get minimal disjunctive normal form equivalent to conjunctive normal form $L=\underset{v\in V}{\wedge}(\underset{p\in v}{\vee} p)$. Therefore, there is a lot of improvement space for algorithm to reduce the number of calculation. We will give some tips to simplify the process, and the algorithm has further research value. The algorithm of this paper can be used to find all reductions of two valued logic proposition set, and there is no repeat steps. In this paper, the algorithm can be extended to get all reductions of the N-valued logic proposition set, which is of great significance to the research on the reduction of proposition set.

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