International Journal of Hybrid Information Technology
Vol. 9, No.6 (2016), pp. 135-142
http://dx.doi.org/10.14257/ijhit.2016.9.6.12

Combined Forecasting Model of Subgrade Settlement Based on Least Square Twin Support Vector Regession

GAO Hui[1], Song Qi-chao[1] and Huang Jun[2]

1. School of Electrical and Information Engineering, Heilongjiang institute of Technology, Harbin, Heilongjiang, China 150050
2. Faculty of Geosciences and Environmental Engineering, Southwest Jiantong University, Chengdu, Sichuan, China 610031
E-mail:Gaohui_dl@sina.com

Abstract

Due to the normal forecasting methods for subgrade settlement using observation data have different applicabilities, and the predicting results has bigger volatility and lower accuracy. The Combined forecasting model of subgrade settlement based on Least Square twin support vector regession is put forward in this paper. At the first, according to the basic settlement law of subgrade and characteristics of settlement curve, the growth curve with the S-type characteristics are choosed as single forcasting model; Then taking prediction results of each individual model as the least square support vector regression model input and to construct the combined forecasting model of subgrade settlement. The result of engineering practice shows that the proposed method has better prediction accuracy and stability.

Keywords: *Subgrade Settlement prediction, Combination forecast model, Least Square twin support regession*

1. Introduction

The development of subgrade settlement has complex characteristics, such as non-linear, non-stabilization, and including numerous uncertain information, so it is very difficult to forecast the subgrade settlement accurately. At present, the methods of subgrade settlement prediction mainly include layer-wise summation method (LWSM), numerical analysis method (NAM) and modeling method based on the observed data (MMBD)[1].

LWSM and NAM demand the precise geotechnical parameters and model of elasto plastic constitutive relation, but the geotechnical parameters and the model of elasto plastic constitutive relation is difficult to be acquired, so their application is limited. The observed data of subgrade settlement is an integrated reaction of all kinds of factors, and contains abundant information, then MMBD is paid more attention. MMBD includes experience formula method (EFM) (such as hyperbolic method, exponential curve method, growth-curve approach and parabolic method), system analysis and control theory method (grey system method and nerve network method). In a word, each forecasting model has their own advantages and their scopes of application, but there are some shortcomings too[2,3].

Combined forecast makes full use of the advantages of single-phase prediction model and overcomes its shortcomings, improving the forecasting precision and reducing the forecasting risk[4,5]. Combined forecasting method combines different single-phase prediction models through considering the characteristics of each single-phase prediction model, so the predicted results could make full use of the obtained information from each single-phase prediction model, it has strong

ISSN: 1738-9968 IJHIT

adaptability and good stability[6,7]. We use machine learning and optimized calculation to establish combined forecasting model of subgrade settlement. Firstly, we lead Usher, Logistics, Gompertz of S-type single-phase prediction model into combined forecast according to the development law and sedimentation curve characteristic of subgrade settlement, and calculating the corresponding prediction results; then taking prediction results of each individual model as the least square support vector regression model input, and establishing combined forecasting model of subgrade settlement based least square support vector regression machine.. Engineering case analysis shows that the combined forecasting model has better prediction accuracy and stability, it has important theory and engineering practical value[8].

2. Least Square Twin Support Vector Regession

2.1. Linear Regression

Considering linear regression, the original optimization problem of Least Square twin support vector regession is:

$$\min \frac{1}{2}\left(Y-\left(Xw_1+eb_1\right)\right)^{\mathrm{T}}\left(Y-\left(Xw_1+eb_1\right)\right)+\frac{C_1}{2}\xi^{\mathrm{T}}\xi \tag{1}$$

$$\text{s.t.}\, Y-\left(Xw_1+eb_1\right)=-e\varepsilon_1-\xi \tag{2}$$

$$\min \frac{1}{2}\left(Y-\left(Xw_2+eb_2\right)\right)^{\mathrm{T}}\left(Y-\left(Xw_2+eb_2\right)\right)+\frac{C_2}{2}\eta^{\mathrm{T}}\eta \tag{3}$$

$$\text{s.t.}\, Y-\left(Xw_2+eb_2\right)=-e\varepsilon_2-\eta \tag{4}$$

Among formulas: $C_1>0$, $C_2>0$ are penalty parameters, $\varepsilon_1>0$, $\varepsilon_2>0$ are the parameters of insensitive loss function, ξ, η are relaxation factors. The aim of Least Square twin support vector regession is to find $f_1(x)=w_1^T x+b_1$ and $f_2(x)=w_2^T x+b_2$ to construct the final regression function. Geometric interpretation of Least Square twin support vector regession is shown as Figure 1.

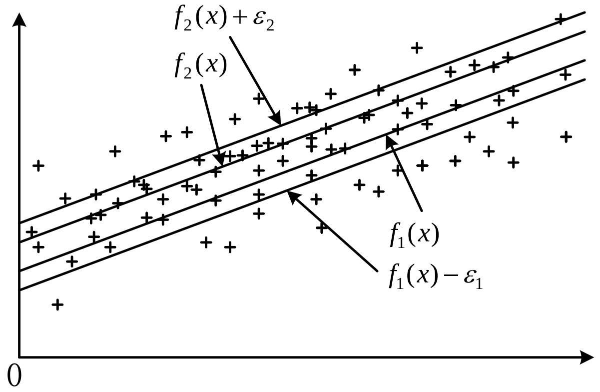

Figure 1. Geometric Interpretation of Least Square Twin Support Vector Regession

In order to solve the problems of (1) and (2), taking the constraint conditions into the objective function to obtain Lagrange function as follows:

$$L=\frac{1}{2}\left\|Y-\left(Xw_1+eb_1\right)\right\|^2+\frac{C_1}{2}\left\|\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right\|^2 \tag{5}$$

Partial derivatives are as follows:

$$\frac{\partial L}{\partial w_1} = -X^T\left(Y-\left(Xw_1+eb_1\right)\right)+C_1X^T\left(\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right) \tag{6}$$

$$\frac{\partial L}{\partial b_1} = -e^T\left(Y-\left(Xw_1+eb_1\right)\right)+C_1e^T\left(\left(Xw_1+eb_1\right)-Y-e\varepsilon_1\right) \tag{7}$$

Making the formulas (6) and (7) equal to zero, we could obtain:

$$\begin{bmatrix} X^TX+C_1X^TX & X^Te+C_1X^Te \\ e^TX+C_1e^TX & e^Te+C_1e^Te \end{bmatrix}\begin{bmatrix} w_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} X^T+C_1X^T & C_1X^Te \\ e^T+C_1e^T & C_1e^Te \end{bmatrix}\begin{bmatrix} Y \\ \varepsilon_1 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} w_1 \\ b_1 \end{bmatrix} = \left(1+C_1\right)\left(G^TG\right)^{-1}G^T\left[\left(1+C_1\right)Y+C_1\varepsilon_1 e\right] \tag{9}$$

Among it, $G=\left[X,e\right]$.

Similarly, we also could get:

$$\begin{bmatrix} w_2 \\ b_2 \end{bmatrix} = \left(1+C_2\right)\left(G^TG\right)^{-1}G^T\left[\left(1+C_2\right)Y+C_2\varepsilon_2 e\right] \tag{10}$$

The final regression function is:

$$f(x)=\frac{1}{2}\left(f_1(x)+f_2(x)\right)=\frac{1}{2}\left(w_1+w_2\right)^Tx+\frac{1}{2}\left(b_1+b_2\right) \tag{11}$$

Linear regression solving process of Least Square twin support vector regession is shown as Figure 2.

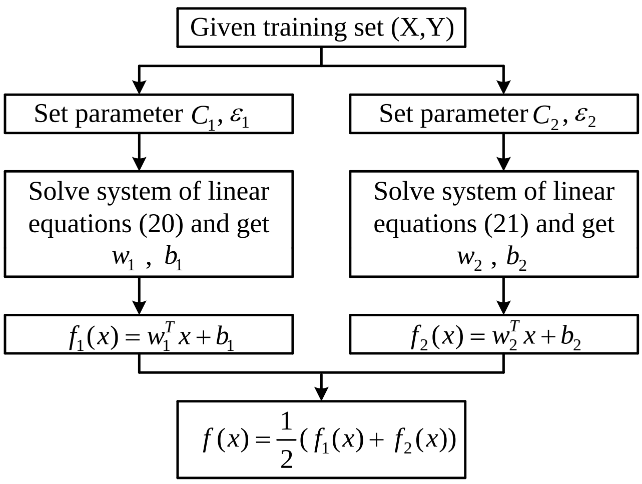

Figure 2 Flow Chart of Least Square Twin Support Vector Regession

2.2. Nonlinear Regression

Considering nonlinear regression, the task is to solve two nonlinear hyper planes $f_1(x)=u_1^TK(X,x)+\gamma_1$ and $f_2(x)=u_2^TK(X,x)+\gamma_2$. In order to get these two hyper planes, two optimization problems are established:

$$\min\frac{1}{2}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)\right)^{\mathrm{T}}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)\right)+\frac{C_1}{2}\xi^{\mathrm{T}}\xi \tag{12}$$

$$\text{s.t.}\,Y-\left(K\left(X,X^{\mathrm{T}}\right)u_1+e\gamma_1\right)=-e\varepsilon_1-\xi \tag{13}$$

$$\min\frac{1}{2}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)\right)^{\mathrm{T}}\left(Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)\right)+\frac{C_2}{2}\eta^{\mathrm{T}}\eta \tag{14}$$

$$\text{s.t.}\,Y-\left(K\left(X,X^{\mathrm{T}}\right)u_2+e\gamma_2\right)=-e\varepsilon_2-\eta \tag{15}$$

Among formulas: $C_1 > 0$, $C_2 > 0$ are penalty parameters, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ are the parameters of insensitive loss function, ξ, η are relaxation factors.

In order to solve the problems of (12) and (13), taking the constraint conditions into the objective function to obtain Lagrange function as follows:

$$L = \frac{1}{2}\left\| Y - \left(K\left(X, X^T\right)u_1 + e\gamma_1\right)\right\|^2 + \frac{C_1}{2}\left\|\left(K\left(X, X^T\right)u_1 + e\gamma_1\right) - Y - e\varepsilon_1\right\|^2 \tag{16}$$

Partial derivatives are as follows:

$$\frac{\partial L}{\partial u_1} = -K\left(X, X^T\right)\left(Y - \left(K\left(X, X^T\right)u_1 + e\gamma_1\right)\right) + C_1 K\left(X, X^T\right)^T\left(\left(K\left(X, X^T\right)u_1 + e\gamma_1\right) - Y - e\varepsilon_1\right) \tag{17}$$

$$\frac{\partial L}{\partial b_1} = -e^T\left(Y - \left(K\left(X, X^T\right)u_1 + e\gamma_1\right)\right) + C_1 e^T\left(\left(K\left(X, X^T\right)u_1 + e\gamma_1\right) - Y - e\varepsilon_1\right) \tag{18}$$

Making the formulas (17) and (18) equal to zero, we could obtain:

$$\begin{bmatrix} K^T K + C_1 K^T K & K^T e + C_1 K^T e \\ e^T K + C_1 e^T K & e^T e + C_1 e^T e \end{bmatrix}\begin{bmatrix} w_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} K^T + C_1 K^T & C_1 K^T e \\ e^T + C_1 e^T & C_1 e^T e \end{bmatrix}\begin{bmatrix} Y \\ \varepsilon_1 \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} u_1 \\ \gamma_1 \end{bmatrix} = \left(1 + C_1\right)\left(H^T H\right)^{-1} H^T\left[\left(1 + C_1\right)Y + C_1\varepsilon_1 e\right] \tag{20}$$

Similarly, we also could get:

$$\begin{bmatrix} u_2 \\ \gamma_2 \end{bmatrix} = \left(1 + C_2\right)\left(H^T H\right)^{-1} H^T\left[\left(1 + C_2\right)Y + C_2\varepsilon_2 e\right] \tag{21}$$

The final regression function is:

$$f(x) = \frac{1}{2}\left(f_1(x) + f_2(x)\right) = \frac{1}{2}\left(u_1 + u_2\right)^T K(X, x) + \frac{1}{2}\left(\gamma_1 + \gamma_2\right) \tag{22}$$

From the above to the process can be seen, for nonlinear regression, the solution of the problem can be obtained by introducing the kernel function, the flow chart of nonlinear regression is different from that of linear regression, which is different from the linear equations.

3. Combined Forecasting Model of Subgrade Settlement Based on LSTSVR

3.1. Selecting Single Forecasting Model

By researching the basic settlement law of subgrade and characteristics of settlement curve, we can know that the developing process of subgrade settlement presents 'S' type curve in linear loading, then the growth curve with the S-type characteristics are adopted as single forcasting model. In this paper, single forecasting model contains Usher model, Logistics model and Gompertz model , the expressions of each model are:

Usher model:

$$S(t) = \frac{k}{\left(1 + ae^{-bt}\right)^{\frac{1}{c}}} \tag{22}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b, c are the parameters to be estimated.

Gompertz model:

$$S(t) = ke^{-ae^{-bt}} \tag{23}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

Logistic model:

$$S(t)=\frac{k}{1+ae^{-bt}} \tag{24}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

3.2. Basic Structure of Combined Forecasting Model

Combined forecasting model consists of LSTSVR and three single forecast models of Logistics model, Usher model and Gompertz model, the basic idea is: first, for each point, calculating the predicted value of each single forecast models, and treating it as various components of the input vector of LSTSVR, while the observed value of the corresponding point as the output vector of LSTSVR. Through training and optimizing LSTSVR, the LSTSVR of combined forecasting model is established. Calculating the predicted value of the corresponding point of each single forecast model, and using the LSTSVR of combined forecasting model created before, calculate and output the final predicted value.

The structure combined forecasting model is shown as Figure 3.

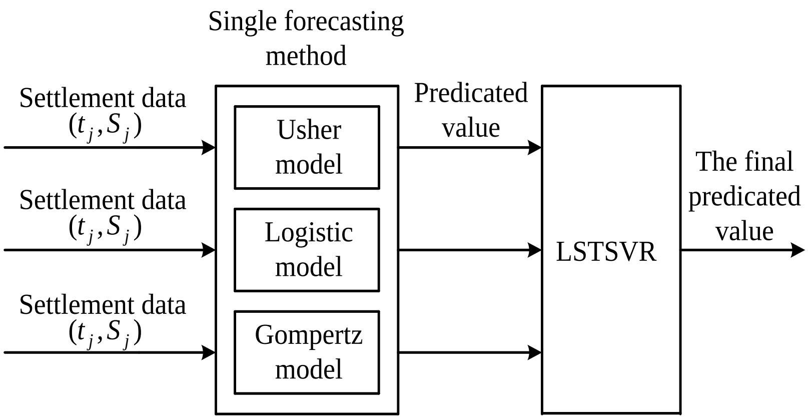

Figure 3 Basic Structure of Combined Forecasting Model Base on LSTSVR

4. Engineering Application which Based on Least Square Twin Support Vector Regession

In this paper, choose Ning-Hang highway subgrade NH standard K095+520 section as observation point, select 15 group settlement observation data from 2001-08-09 to 2002-12-14 as research object. Using Usher model, Logistic model, Gompertz model and combined forecasting model to model and forecast, the first 11 data are used to model and after 4 data are used to forecast.

Each single forecasting model parameter values are shown in Table 1. Forecast results and relative errors of each single forecasting model and combined forecasting model are shown in Table 2. Forecast results contrast of each single forecasting model and combined forecasting model is shown in Figure 1, relative error contrast is shown in Figure 2.

Table 1. Parameter Value of Each Single Forecasting Model

Model	Parameter estimation			
	a	*b*	*c*	*k*
Usher	5.5590	-3.9067	5.6292	-5.2187
Logistic	6.1583	-4.7323	6.2051	-5.5293
Gompertz	6.3039	3.9047	6.3465	3.2549

Table 2. Four Kinds of Forecast Methods Comparison Of Forecast Results

NO.	Measured values (cm)	Usher Model		Logistics model		Gompertz model		Combined model	
		Predicted values (cm)	Relative error （%）	Predicted values (cm)	Relative error （%）	Predicted values (cm)	Relative error （%）	Predicted values (cm)	Relative error （%）
1	5.35	5.5590	-3.9067	5.6292	-5.2187	5.6878	-6.3139	5.4012	-0.9570
2	5.88	6.1583	-4.7323	6.2051	-5.5293	6.2218	-5.8124	5.7834	1.6428
3	6.56	6.3039	3.9047	6.3465	3.2549	6.3518	3.1736	6.5345	0.3887
4	7.48	6.9992	6.4282	7.0282	6.0012	6.9780	6.7119	7.5548	-1.0000
5	7.82	7.6306	2.4221	7.6540	2.1229	7.5595	3.3312	7.7408	1.0127
6	8.05	7.9754	0.9271	7.9968	0.6614	7.8854	2.0452	8.0310	0.2360
7	8.38	8.1905	2.2608	8.2105	2.0227	8.0927	3.4282	8.4605	-0.9606
8	8.59	8.6772	-1.0156	8.6915	-1.1821	8.5763	0.1599	8.5769	0.1525
9	8.84	9.0932	-2.8643	9.0973	-2.9103	9.0107	-1.9309	8.8189	0.2386
10	9.01	9.2409	-2.5629	9.2394	-2.5464	9.1711	-1.7880	9.0689	-0.6537
11	9.18	9.4438	-2.8735	9.4324	-2.7499	9.3979	-2.3735	9.2299	-0.5435
12	**9.81**	**9.7359**	**0.7554**	**9.7046**	**1.0744**	**9.7407**	**0.7069**	**9.7712**	**0.3955**
13	**10.04**	**9.9770**	**0.6274**	**9.9225**	**1.1699**	**10.0423**	**-0.0234**	**9.9989**	**0.4093**
14	**10.06**	**10.0198**	**0.3999**	**9.9604**	**0.9899**	**10.0981**	**-0.3784**	**10.1009**	**-0.4065**
15	**10.11**	**10.1745**	**-0.6382**	**10.0951**	**0.1472**	**10.3066**	**-1.9443**	**10.1378**	**-0.2749**

From Table 2, we can see that the relative error maximum of fitting value of combined forecasting model is 1.6428%, it is better than one of Usher model which is 6.4282%, Logistics model which is 6.0012% and Gompertz model which is 6.7119%, the elative error maximum of predicting value of combined forecasting model is 0.4093%, it is lower than the one of Usher model is 0.7554%, Logistic model is 1.0744% and Gompertz model is 1.9443%. So the fitting and forecasting precision of combined forecasting model is more superior to each single forecasting model.

In Figure 4, the settlement development curves from each forecasting model are describes repectively. We could know that combined forecasting model can reflects the characteristics and law of settlement development curve in both fitting period and forecasting period, but each single forecasting model has good performance at some time.

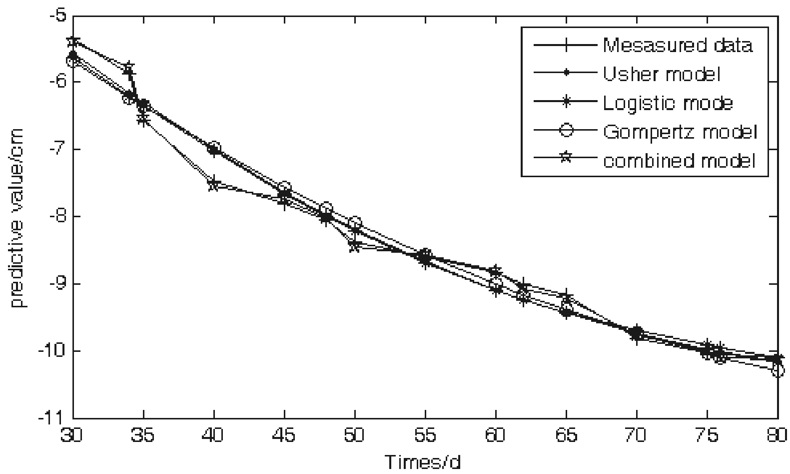

Figure 4. Predictive Effects of Four Kinds of Forecasting Models

In Figure 5, the error curves of each forecasting model are illustrates repectively. We could see that the relative error of single forecasting model appears larger fluctuations at the beginning of the model period, and the relative error is also larger during the forecast period, but the relative error of predicted results of combined forecasting model is relatively stable, it reduces the risk of forecasting.

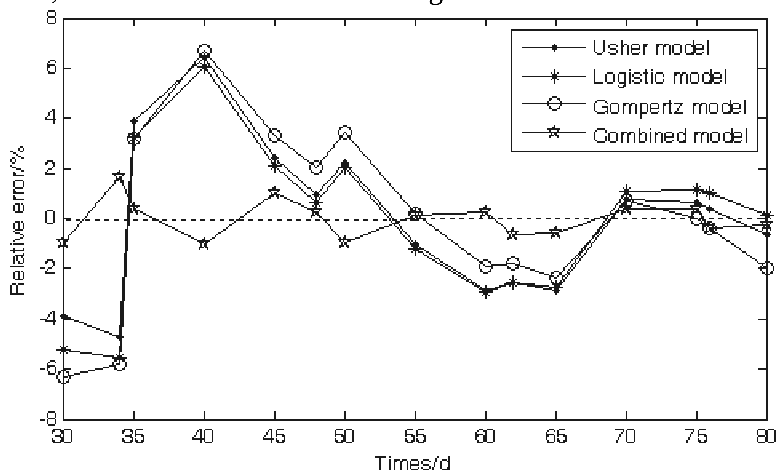

Figure 5. Relative Errors of Four Kinds of Forecasting Model

5. Conclusion

According to the development law of subgrade settlement and the characteristics of settlement curve, selecting growth curve with S-type characteristics as a single phase prediction model, using the least square support vector regression machine and prediction results of each single phase prediction model to construct combined forecasting model, and it provides an effective tool for subgrade settlement prediction. Engineering examples show that the proposed combined forecasting model could improve the predicting accuracy, reduce the predicting risk, and it has some practical value.

Acknowledgements

This work was supported by the Natural Science Foundation of Heilongjiang Province, China (Grant No. E201326).

References

[1] M.H. Zhao, L. Zhao and X.J. Zou, “Prediction of roadbed settlement by user model”, Rock and Soil Mechanics, vol. 29, no. 11, **(2008)**, pp. 2973-2976.
[2] G.X. Mei, J.M. Zai, Z.Z. Yin, W.B. Zhao and J.H. Yin, “Proof of s-t curve appearing “S” shape based on one - dimensinal consolidation theory”, Rock and Soil Mechanics, vol. 25, no. 1, **(2004)**, pp. 20-22.
[3] S.X. Chen, X.Y. Wang, X.C. Xu and X.G. Wang “[J]”, Rock and Soil Mechanics, vol. 31, no. 2, **(2010)**, pp. 478-481.
[4] W.M. Leng, Q. Yang, R.S. Nie and J. Yue, “Study of post-construction settlement combination forecast method of high-speed railway bridge pile foundation”, Rock and Soil Mechanics, vol. 32, no. 11, **(2011)**, pp.3341-3347.
[5] S. Dou and Q. Xia, “Study on variable-weight combination forecasting for settle of loess subgrade”, Sugrade Engineering, no.4, **(2012)**, pp.30-33.
[6] K.M. Arun and M. Gopal, “Least squares twin support vector machines for pattern classificaton”, Expert Systems with Applications, vol. 4, no. 36, **(2009)**, pp.7535-7543.
[7] X. Peng, “TSVR: An efficient twin support vector machine for regression”, Neural Networks, vol. 3, no. 23, **(2010)**, pp.365-372.
[8] Z.X. Lu, Z.X. Yang and X.Y. Gao, “Least square twin support vector regression”, Computer Engineering and Applications, vol. 50, no. 23, **(2014)**, pp.140-144.