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Backstepping Iterative Learning Control for Wiener System

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Abstract

In this paper, the iterative learning control (ILC) scheme combined with the backstepping controller is applied to the Wiener system, which is a typical nonlinear and non-Lipschitz one. The ILC scheme as a feed forward control can improve the convergence speed, and the perfect tracking can be achieved as the system is repeatable. The backstepping control is a feedback control which improves the robustness of the control system, especially for the existence of non-repeatable noises. The backstepping part also guarantees the asymptotic stability, which further improves the convergence speed. The design of backstepping controller is based on the error information in the current control process, and the Lyapunov method. The convergence condition is achieved and the convergence speed is analyzed as well. It can be seen that the combination of ILC scheme and backstepping method can improve the system performance to a large extant. Numerical simulations validate the above conclusions.

Keywords: *ILC, backstepping, Wiener system, convergence condition, convergence speed, system performance*

1. Introduction

The ILC scheme which seeks to construct an inverse of the plant dynamics over multiple trials [1] is proposed to compensate for output tracking error. The perfect tracking is one of the most important advantages from ILC scheme if certain requirements are satisfied. One of these requirements is the repeatability of system. However, because the basic ILC strategy is essentially a feedforward control approach that fully utilizes the past control information [2], the factors such as non-repeatable disturbances may destroy the repeatability of system seriously. Improve the robust of the ILC system is an urgent task. In the literature, there are many ILC laws proposed for different classes of systems: linear or nonlinear systems, time delay systems, cascade systems, stochastic systems *etc*., [3]. The ILC can also be applied in continuous time systems [4], [5] and discrete time systems [6]6-11].

The main characteristic of the continuous time system is that the input and output signals are continuous function of the time. The state of the continuous system can be described by differential equation. The discrete time system which is also called sampling control system transfers information by discrete digital sequence. The relationship between different variables of the discrete time system is described by difference equation. Sample time is the main difference of the two systems. Many methods, such as Tustin [12][13] rule or backward difference method [14], are applied to the discretization of the continuous system. In this paper, the ILC is applied in the continuous Wiener system.

In many cases block structured models, series and parallel arrangements comprising alternating linear dynamic and nonlinear systems, may provide an acceptable compromise [15]. Generally speaking, the basic block structured models are the Wiener system and the Hammerstein system. In this paper, the Wiener system is applied. The Wiener system is

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composed of a linear dynamical system followed by a nonlinear system which was introduced by Wiener [16]. Nowadays, more and more scholars focus on the Wiener system because the Wiener system can be used to model the majority of practical systems, for example, distillation column, pH process, biological cybernetics, power amplifier and others [17]. Any time-invariant system with fading memory may be approximated by a Wiener system [18]. In this paper, the linear part in the Wiener system is time-invariant linear dynamics with relative degree one while the nonlinear part is an affine system. The backstepping method can solve the problems of the system not only with relative degree one but alsowith higher relative degree.

In this paper, the backstepping method is applied to improve the robust of the system. This is an effective method to solve the problems caused by the non-repeatability factors in the system. In recent years, the backstepping control designtechniques have received of great attention because of itssystematic and recursive design methodology for nonlinearfeedback control [19]. This method can flexibly combinewith other control techniques, such as the adaptive control [20] and the optimal control [21].The backstepping method has been successfully applied to many practical applications such as the boiler-turbine unit control system of coal-firedpower plant [22], the single machine infinite bussystem [23], the Nuclear U-tube Steam Generator [24], the VGT Pneumatic Actuator [25], and so on[26]. In this paper, the ILC scheme is combined with the backstepping control for the reason that this strategy can improve the system performance especially the robust and adaptive to a large extent compared to the ILC scheme. In fact, the stability analysis for the ILC system is difficult so that some nonlinear feedback control schemes are proposed [27]. Backsteppingalgorithm is designed for stabilizing nonlinear systemsin the applications of tracking and regulation [28]. The main advantages of the backstepping method are included but not limited to: global stability can be achieved with ease, transient performance can be guaranteed and explicitly analyzed, and they have the flexibilityto avoid unnecessary cancellation of useful nonlinearities compared with the feedback linearization technique [29], the convenience of analyzing the stability of system because the controller is a Lyapunov-basedrecursive design controller, the stronger robustness and adaptiveness of the system improved by the feedback control property, respectively. The backstepping technique combined with the ILC mechanism is applied for developing a constructive control strategy to cope with Wiener system. To the best of our knowledge, the result we will propose is not covered by any of the scarce results, and the problem of backstepping method combined with ILC applied in the Wiener system remains open.

This paper is organized as follows: some preliminaries are introduced in Section 2. The backstepping controller is designed in Section 3. In section 4,the convergence of the Wiener system is analyzed. Section 5 shows some simulation results to valid the conclusions.In the last Section 6, the conclusions are summarized.

2. Preliminaries

In this Section, the Wiener system is considered which includes linear part and nonlinear part as shown in Figure 1. The linear part is a time invariant linear system while the nonlinear part is an affine system.

The Wiener system can be described as

$$\begin{aligned}
&\dot{\xi}_k(t) = \tilde{A}\xi_k(t) + \tilde{B}u_k(t),\\
&\omega_k(t) = C_1\xi_k(t),\\
&\dot{x}_k(t) = \tilde{f}\big(x_k(t)\big) + \tilde{g}\big(x_k(t)\big)\omega_k(t),\\
&y_k(t) = C_2x_k(t).
\end{aligned}$$

In the equations above, $\xi_k(t)$and $x_k(t)$ are the state variables.The output of the linear part is $\omega_k(t)$ while that of the nonlinear part is$y_k(t)$.$y_k(t)$is also the output of the Wiener system.The iteration number is k. The system input is $u_k(t)$.The polynomial $\tilde{f}\big(x_k(t)\big)$

and $\tilde{g}\big(x_k(t)\big)$ are related to the state variable of the nonlinear part.The parameters of the Wiener system are $\tilde{A}$, $\tilde{A}$, C_1 and C_2, respectively. The three assumptions below should be satisfied all through this paper.

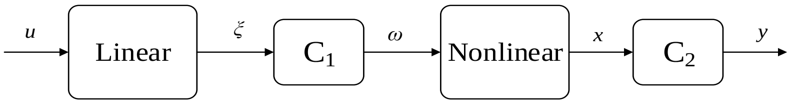

Figure 1. The Wiener System

Assumption 1. The assumption is satisfied that the linear part and nonlinear part in the Wiener system is controllable and observable.So the relationship below can be achieved
$\xi_k(t) = ({C_1}^T C_1)^{-1} {C_1}^T \omega_k(t)$,
$x_k(t) = ({C_2}^T C_2)^{-1} {C_2}^T y_k(t)$.
Then,
$\dot{\omega}_k(t) = C_1 \dot{\xi}_k(t)$
$= C_1 \tilde{A} \xi_k(t) + C_1 \tilde{B} u_k(t)$
$= C_1 \tilde{A} ({C_1}^T C_1)^{-1} {C_1}^T \omega_k(t) + C_1 \tilde{B} u_k(t)$
$= A\omega_k(t) + Bu_k(t)$,
where
$A = C_1 \tilde{A} ({C_1}^T C_1)^{-1} {C_1}^T$,
$B = C_1 \tilde{B}$.
By the same way
$\dot{y}_k(t) = C_2 \dot{x}_k(t)$
$= C_2 \tilde{f}\big(x_k(t)\big) + C_2 \tilde{g}\big(x_k(t)\big)\omega_k(t)$
$= C_2 \tilde{f}\big(({C_2}^T C_2)^{-1} {C_2}^T y_k(t)\big) + C_2 \tilde{g}\big(({C_2}^T C_2)^{-1} {C_2}^T y_k(t)\big)\omega_k(t)$
$= f\big(y_k(t)\big) + g(y_k(t))\omega_k(t)$,
where
$f\big(y_k(t)\big) = C_2 \tilde{f}\big(({C_2}^T C_2)^{-1} {C_2}^T y_k(t)\big)$,
$g\big(y_k(t)\big) = C_2 \tilde{g}\big(({C_2}^T C_2)^{-1} {C_2}^T y_k(t)\big)$.
Assumption 2. The initial values of the system are the same.
Assumption 3. The system is repeatable. This is a fundamental requirement for the ILC scheme.

Here are some definitions and one lemma which will be used in this paper.
Definition 1. Suppose A is $i * j$ matrix and the ∞-norm of matrix A is defined as

$$\parallel A \parallel_\infty = max_{0 \le i \le n} \sum_{j=1}^{n} |a_{ij}|,$$

which is simply the maximum absolute row sum of the matrix.
Definition 2. A vector norm for r-vector-valued functions $e_k(t)$ defined on$[0, T]$ as follows

$$\parallel e_k(t) \parallel_\lambda = max_{0<t<T}\, e^{-\lambda t} \parallel e_k(t) \parallel_\infty,$$

this is the λ-norm of the vector $e_k(t)$, where λ is a positive constant.
Lemma 1. In a normed vector space V, one of the defining properties of the norm is the triangle inequality,$f(x)$ and $g(x)$ are continuous functions satisfy

$$\parallel f(x) + g(x) \parallel \le \parallel f(x) \parallel + \parallel g(x) \parallel.$$

This lemma is called the triangle inequality.

3. Design of Backstepping Controller

The feedback control is a fundamental method to realize the stability of a nonlinear system, at the same time, it is also one of the effective methods as a feedback control to solve the nonlinear system problems. The idea behind backstepping is to design the control signal by using some of the statesas "virtual controls" and designing for them as intermediate control variables [30].

In this section, the backstepping controller is designed to improve the performance of the system.Some of the subscript and the variables in the description of function may be ignored for the convenience of the introduction. The Wiener system is considered as

$\dot{\omega}_k(t) = A\omega_k(t) + Bu_k(t),$

$\dot{y}_k(t) = \big(y_k(t)\big) + g(y_k(t))\omega_k(t).$

In this system, A and B are the parameters of the linear part in system, $f\big(y_k(t)\big)$ and $g(y_k(t))$ are the polynomial of nonlinear part which are related to the output of the system $y_k(t)$.

Define $\omega = \Phi(y)$, where$\Phi(y)$ is a virtual input of the nonlinear part. And

$$\dot{V}_1(y) = \frac{\partial V_1}{\partial y^T}[f(y) + g(y)\Phi(y)] \le -V_a(y) \le 0,$$

Where $V_1(y) > 0$and $V_a(y) > 0$. It's obvious that

$$\dot{y} = f(y) + g(y)\Phi(y) + g(y)[\omega - \Phi(y)].$$

Then it's defined that $z = \omega - \Phi(y)$,so$\dot{z} = \dot{\omega} - \dot{\Phi}(y)$. Next, define the vector v equals to$v = \dot{z}$. So the nonlinear equation in the Wiener system is transferred as

$$\dot{y} = f(y) + g(y)\Phi(y) + g(y)z.$$

The Lyapunov function $V(y)$ is designed as

$$V(y) = V_1(y) + \frac{1}{2}z^2,$$

so that $V(y) > 0$ and the derivative of the function $V(y)$ is

$$\dot{V}(y) = \frac{\partial V_1}{\partial y^T}[f(y) + g(y)\Phi(y) + g(y)z] + z\dot{z}$$

$$= \frac{\partial V_1}{\partial y^T}f(y) + \frac{\partial V_1}{\partial y^T}g(y)\Phi(y) + \frac{\partial V_1}{\partial y^T}g(y)z + zv.$$

Then define$v = -(\frac{\partial V_1}{\partial y^T}g(y) + mz)$, $m > 0$.So

$$\dot{V}(y) = \frac{\partial V_1}{\partial y^T}f(y) + \frac{\partial V_1}{\partial y^T}g(y)\Phi(y) - mz^2 \le -V_a(y) - mz^2,$$

which means that

$$\dot{V}(y) < 0.$$

The conclusion is achieved that the system with backstepping controller as a feedback controller is asymptotic stability.The backstepping controller is designed as

$$u_k(t) = \left\{\frac{\partial \Phi}{\partial y^T}[f(y_k) + g(y_k)\omega] - \frac{\partial V_1(y_k)}{\partial y^T(y_k)}g(y_k) - m[\omega - \Phi(y_k) - A\omega]\right\}[(B^TB)^{-1}B^T].$$

This backstepping design is toselect recursively some appropriate functions of statevariables as pseudo-control inputs for lower dimensionsubsystems of the overall system [26]. The condition of the Lyapunov function should be satisfied so that the system is asymptotic stability.

Remark 1The backstepping controller as a feedback control process does not influence the convergence performance (which will be analyzed in next section),however, the convergence speed can be influenced. The robust and adaptive of the system are improved by this backstepping controller.

4. Convergence Analysis

Still the same Wiener system is considered as

$\dot{\omega}_k(t) = A\omega_k(t) + Bu_k(t),$

$\dot{y}_k(t) = f\big(y_k(t)\big) + g(y_k(t))\omega_k(t).$

And the ILC scheme is

$u_{k+1}(t) = u_k(t) + \Gamma e_k(t) + \Omega\dot{e}_k(t).$

The learning law is PD type with the learning gainsΓandΩ, respectively.The input of the nonlinear part equals to the output of linear part. As mentioned before, the output of the linear part $\omega(t)$ tends to the reference input of the nonlinear part, the output of the Wiener system tracks the reference output.

The solution of $\dot{\omega}_k(t) = A\omega_k(t) + Bu_k(t)$ can be achieved as

$$\omega_k(t) = e^{At}\omega_k(0) + \int_0^t e^{A(t-\tau)}Bu_k(\tau)\,d\tau.$$

Define the output tracking error of the linear part is$e_k(t) = \omega_d(t) - \omega_k(t)$. The convergence condition can be gained by the method of contraction mapping method. Then the following relationships can be achieved

$$e_k(t) = \omega_d(t) - \omega_k(t)$$

$$= \omega_d(t) - e^{At}\omega_k(0) - \int_0^t e^{A(t-\tau)}Bu_k(\tau)\,d\tau$$

$$= \omega_d(t) - e^{At}\omega_{k-1}(0) - \int_0^t e^{A(t-\tau)}Bu_{k-1}(\tau)\,d\tau + e^{At}\omega_{k-1}(0) + \int_0^t e^{A(t-\tau)}Bu_{k-1}(\tau)\,d\tau - e^{At}\omega_k(0) - \int_0^t e^{A(t-\tau)}Bu_k(\tau)\,d\tau.$$

Then

$$e_k(t) = e_{k-1}(t) + e^{At}\omega_{k-1}(0) + \int_0^t e^{A(t-\tau)}Bu_{k-1}(\tau)\,d\tau - e^{At}\omega_k(0) - \int_0^t e^{A(t-\tau)}Bu_k(\tau)\,d\tau.$$

From **Assumption 2.** The initial values of the system are the same., it's known that the initial value of the system is the same which meansthat$\omega_{k-1}(0) = \omega_k(0)$. So

$$e_k(t) = e_{k-1}(t) + \int_0^t e^{A(t-\tau)}B[u_{k-1}(\tau) - u_k(\tau)]\,d\tau.$$

Then we have

$$e_k(t) = e_{k-1}(t) - \int_0^t e^{A(t-\tau)}B[\Gamma e_{k-1}(\tau) + \Omega\dot{e}_{k-1}(\tau)]d\tau$$

$$= (I - B\Omega)e_{k-1}(t) - \int_0^T Ae^{A(t-\tau)}B\Omega e_{k-1}(\tau)]d\tau - \int_0^t e^{A(t-\tau)}B\Gamma e_{k-1}(\tau)d\tau.$$

Applying λ-norm to both sides of the equation above, yields the convergence condition in theλ-norm,

$$\| I - B\Omega \|_\lambda < 1.$$

The conclusion can be achieved that the convergence condition is only related to the feedforward control, however, the feedback control dose not influence the convergence condition in theλ-normin this paper.

In paper [31] the similar idea was also published that the convergence condition of the learning control in the feedback configuration does not change from the condition in an open-loop configuration, moreover, the proportion learning gain does not influence the convergence condition in theλ-norm, either. The learning speed is changed by the P type learning gain.

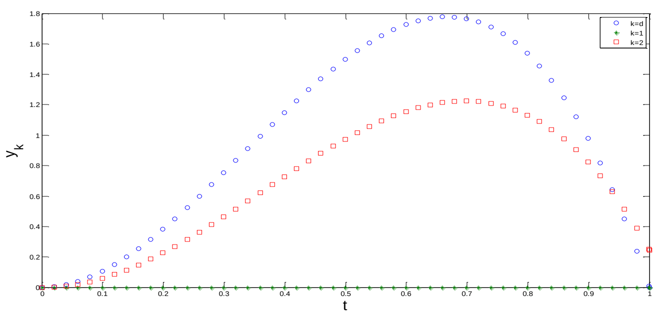

Figure 2. The Tracking Trajectory of the Outputs by the First Experiment. The Linear System is $\dot{x} = Ax_k + bu_k$, $y_k = x_k$ with PD type Learning Law, where $A = -1$, $C = 1$, $\Gamma = 3$ and $\Omega = 2$, Respectively. The Control Direction is Chosen as $CB = 0.3$

Remark 2 The learning gains can be efficiently chosen not only by the convergence condition, but also can be achieved properly by small gain approach.The control direction of the system can be decided by experiments as shown in Figure 2 and Figure 3. In Figure 2, the reference output is the middle red line, the output of iteration 1 is zero and that of iteration 2 is the blue line. It's obvious that the result of the first experiment is what we want. The result of the second experiment is obvious that the control direction is wrong as shown in Figure 3. Then the direction can be confirmed. Next, learning gains can be chosen small enough to meet the convergence condition. The learning gains which make the system satisfy the convergence condition can also be achieved by this small gain approach.

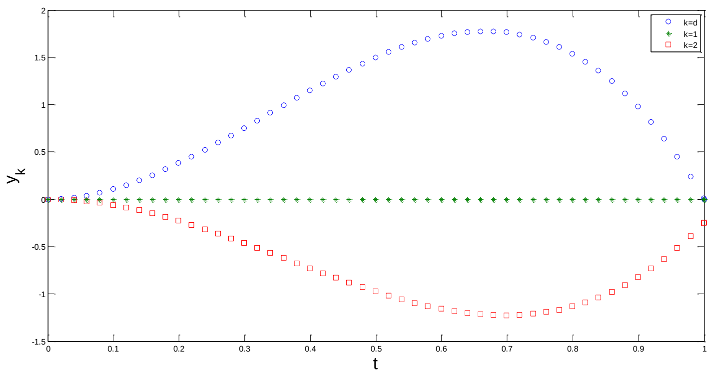

Figure 3. The Tracking Trajectory of the Outputs by the First Experiment. The Linear System is $\dot{x} = Ax_k + bu_k$, $y_k = x_k$ with PD Type Learning Law, where $A = -1$, $C = 1$, $\Gamma = 3$ and $\Omega = 2$, respectively. The Control Direction is Chosen as $CB = -0.3$

5. Simulation Results

In this Section, 3 groups of simulation results are achieved to valid the correction of the conclusion in this paper. Only the backstepping controller applied in subsection 5.1. Backstepping Control, in subsection 5.2.ILC Scheme only the ILC scheme is used, in the end, the ILC scheme combined with backstepping control is applied in subsection 5.3 ILC Scheme Combined with Backstepping Control.

The Wiener system is considered as

$$\dot{\omega}_k(t) = A\omega_k(t) + Bu_k(t),$$
$$\dot{y}_k(t) = f\big(y_k(t)\big) + g(y_k(t))\omega_k(t).$$

In this system, $\omega_k(t)$ is the state of the linear part while $u_k(t)$ is the input. The output of the nonlinear part is $y_k(t)$. The convergence speed by ILC scheme is fast, however, the control effect is terrible as the non-repeatable noises are added. While only the backstepping control is applied, the Wiener system is asymptotic stability as certain requirements are satisfied. In the last case, the ILC scheme combined with backsteppingcontrol is used. The convergence speed is a little slower, but strong robustness of the system is achieved. This combined method improve the system performance.

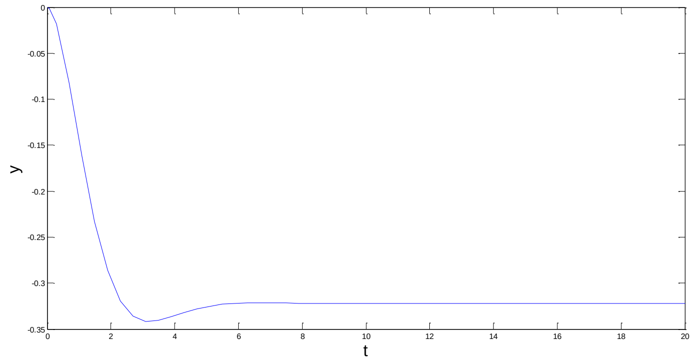

Figure 4. The Backstepping Control is Applied in the Wiener System. The Parameters are $A = 0.5$, $a = 0.9$, $B = 1$ and $m = 0.5$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$, $g = 1$

5.1. Backstepping Control

In this subsection, only backstepping control is utilized as a feedback control.The backstepping controller is applied as

$$u_k(t) = \left\{\frac{\partial\Phi}{\partial {y_k}^T}[f(y_k) + g(y_k)\omega] - \frac{\partial V_1(y_k)}{\partial y^T(y_k)}g(y_k) - m[\omega - \Phi(y_k) - A\omega]\right\}[(B^T B)^{-1}B^T].$$

The simulation result is shown in Figure 4. The system is asymptotic stability by backstepping control as a feedback control. The parameters are $A = 0.5$, $a = 0.9$, $B = 1$ and $k = 0.5$, respectively. And the Lyapunov function is $V_1 = 0.5y^2$. The system can be chosen as $f = ay^2 - y^3$, $g = 1$. The x-axis is time while the y-axis is output. It's shown that the system is asymptotic stability by backstepping control as a feedback control.

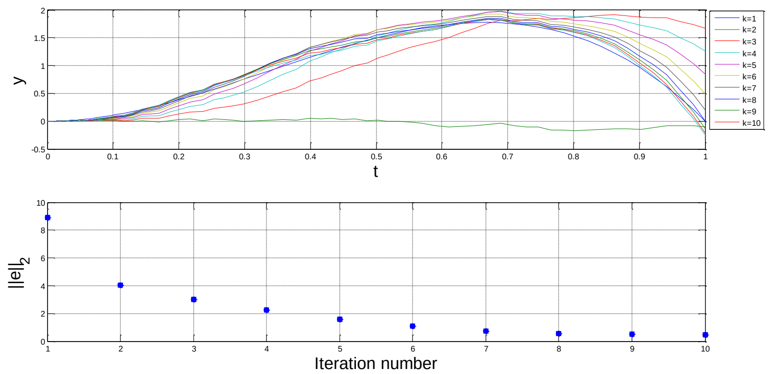

Figure 5. The ILC Scheme is Applied in the Wiener System. The Parameters are $A = 0$, $B = 1$, $a = 0.1$, $\Omega = 3.9$, $\Gamma = 0.5$, $f = ay^2 - y^3$ and $g = 1$, Respectly

5.2.ILC Scheme

In this subsection, only the ILC scheme is applied as a feedforward control. The ILC scheme is

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t) + \Omega\dot{e}_k(t).$$

The simulation result is shown in Figure 5. The parameters of the system are $A = 0$, $B = 1$ and $a = 0.1$, respectively. The learning gains in the ILC scheme are $\Omega = 3.9$ and $\Gamma = 0.5$. The polynomial in the nonlinear part is $f = ay^2 - y^3$, $g = 1$. It's shown that the system is monotonically convergent and the convergence speed is fast. That is one of the advantages of the ILC scheme. The output tracking errors are $\| e_k(t) \|_6 = 1.1236$and $\| e_k(t) \|_{10} = 0.4168$, respectively.

However, the system is very sensitive to noises. The random noises are added. The variance of the noises is 1 and the noises are non-repeatable. They are added in the output of the linear part in the Wiener system. The result is shown in Figure 6 that the output tracking errors are large.

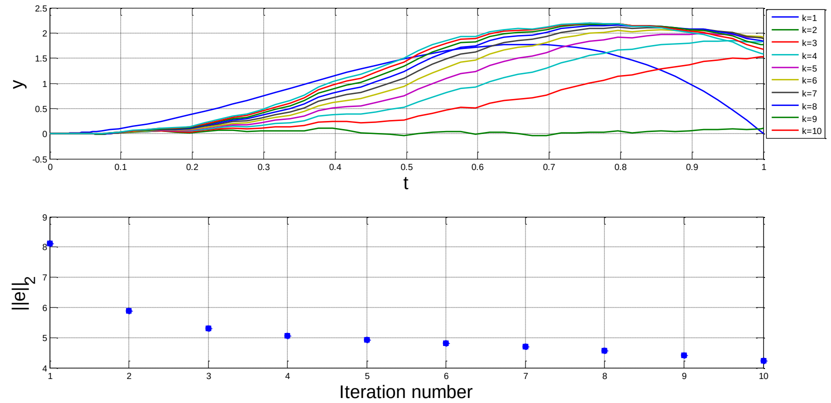

Figure 6. The ILC Scheme is Applied in the Wiener System with Non-Repeatable Noises. The Parameters are $A = 0$, $B = 1$, $a = 0.1$, $\Omega = 3.9$, $\Gamma = 0.5$, $f = ay^2 - y^3$ and $g = 1$, Respectly

5.3 ILC Scheme Combined with Backstepping Control

The ILC scheme combined with backstepping control is applied in this subsection as

$$u_k(t) = u_{k-1}(t) + \Gamma e_{k-1}(t) + \Omega \dot{e}_{k-1}(t) + \left\{ \frac{\partial \Phi}{\partial {y_k}^T}[f(y_k) + g(y_k)\omega] - \frac{\partial V_1(y_k)}{\partial y^T(y_k)} g(y_k) - m[\omega - \Phi(y_k) - A\omega] \right\} [(B^T B)^{-1} B^T].$$

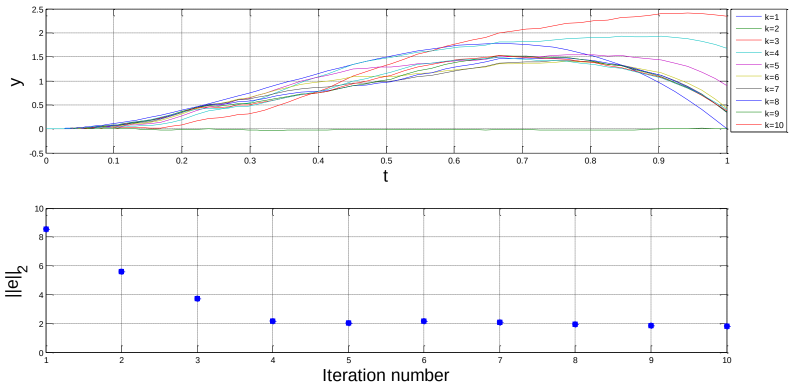

Figure 7. The ILC Scheme Combined with Backstepping Control is Applied in the Wiener System. The Parameters are $A = -0.9$, $B = 2.1$, $a = 0.7$, $\Omega = 1.7$, $\Gamma = 0.5$, $m = 0.2$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$,$g = 1.8$

The simulation result is shown in Figure 7. The parameters of the system are $A = -0.9$, $B = 2.1$, $a = 0.7$. The P-type and D-type learning gains are $\Omega = 1.7$, $\Gamma = 0.5$. The

Lyapunov function is achieved as $V_1 = 0.5y^2$. The nonlinear part is definedas $f = ay^2 - y^3$,$g = 1.8$. Somefactors in the feedback part are$\Phi = -y - ay^2$, $m = 0.2$. The random noises which are non-repeatable are added in this part to demonstrate the robust of the system. The variance of the noises is 1and the noises are added in the output of the linear part in the Wiener system. It's shown that the output tracking errors decreasing monotonically by iterations. The convergence speed is not as fast as that in subsection5.2.ILC Scheme but the control process in this subsection satisfies strong robustness.

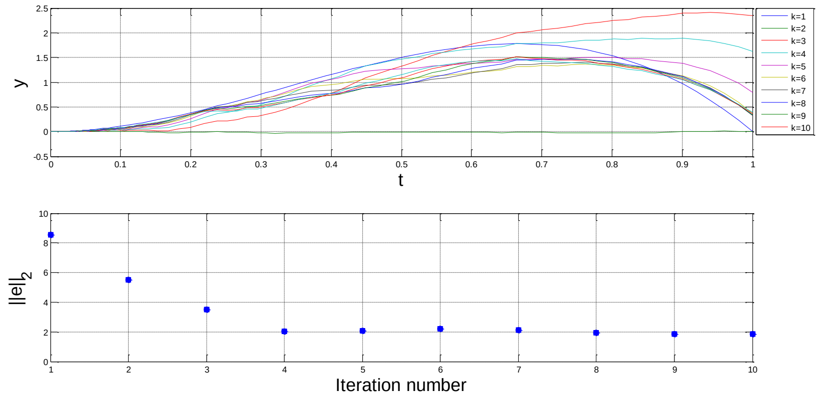

Figure 8. The ILC Scheme Combined with Backstepping Control is Applied in the Wiener System. The Parameters are $A = -0.8$, $B = 2.1$, $a = 0.7$, $\Omega = 1.7$, $\Gamma = 0.5$, $m = 0.2$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$,$g = 1.8$

This controller is also adaptive. The cases of $A \pm 0.1$ and $B \pm 0.1$ are applied to the previous simulations in this subsection to prove the adaption of the controller. From Figure 8, Figure 9, Figure 10, Figure 11, it is known that the simulation results are almost the same in the cases of changing parameters. So, this ILC scheme combined with backstepping control is adaptive for the Wiener system.

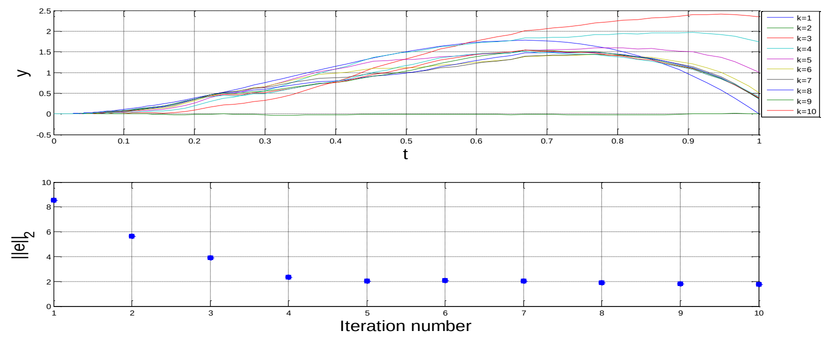

Figure 9. The ILC Scheme Combined with Backstepping Control is Applied in the Wiener System. The Parameters are $A = -1$, $B = 2.1$, $a = 0.7$, $\Omega = 1.7$, $\Gamma = 0.5$, $m = 0.2$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$,$g = 1.8$

Remark 3 The complement of backstepping control sacrifices little convergence speed, however, much stronger robust and adaptive performance are achieved for the Wiener system. In the case of non-repeatable noises exist, the backstepping control improves the system performance to a large extent obviously. The following Table 1 summarizes the differences of the ILC scheme and the ILC scheme combined with backstepping control.

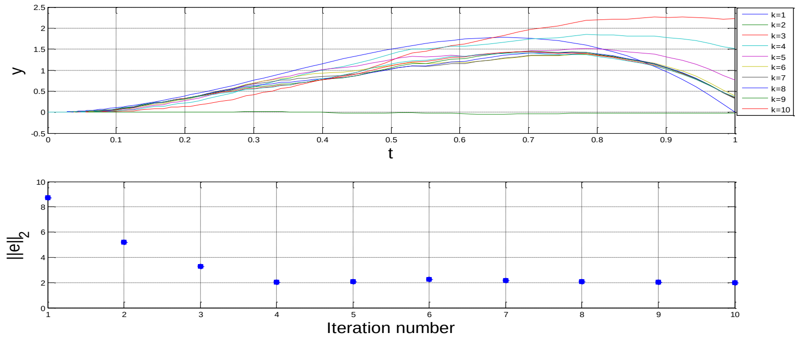

Figure 10. The ILC Scheme Combined with Backstepping Control is Applied in the Wiener System. The Parameters are $A = -0.9$,$B = 2$, $a = 0.7$, $\Omega = 1.7$, $\Gamma = 0.5$, $m = 0.2$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$,$g = 1.8$

6. Conclusions

In this paper, the ILC scheme combined with the backstepping control is applied in the Wiener system. The Wiener system is consist of a time invariant linear system followed by an affine nonlinear system. The convergence speed of the ILC scheme is fast, however, it's very sensitive to the non-repeatable noises. As the repeatability of the system is destroyed, the effect of ILC scheme is terrible. The backstepping control as a feedback control can improve the robust and adaptive of the system and make the system be asymptotic stability without changing the convergence condition. Both of the two methods above are effect in solving the problem of nonlinear systems. The controller designed in this paper combined with ILC scheme and backstepping control can improve the system performance of Wiener system to a large extent. The information in the current control process, the Lyapunov function and the virtual input are applied in designing the backstepping controller. The PD type ILC scheme is applied in the Wiener system. The convergence condition is achieved by the contraction mapping method and the convergence speed is analyzed. The P type learning gain dose not influence the convergence condition in the λ-norm which only influences the convergence speed. A numerical simulation results are shown to valid the conclusion in this paper.

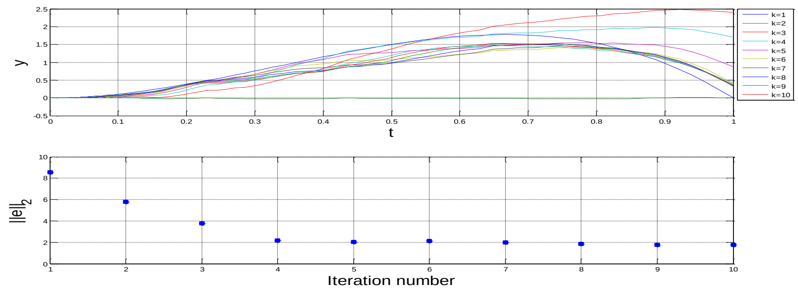

Figure 11. The ILC Scheme Combined with Backstepping Control is Applied in the Wiener System. The Parameters are $A = -0.9$, $B = 2.2$, $a = 0.7$, $\Omega = 1.7$, $\Gamma = 0.5$, $m = 0.2$, Respectively. And $V_1 = 0.5y^2$, $\Phi = -y - ay^2$, $f = ay^2 - y^3$,$g = 1.8$

Table 1 Comparing the ILC Scheme and the ILC scheme with Backstepping Control

Controller / Performance	ILC	ILC and backstepping
Convergence speed	faster	fast
Robustness	Poor robust	Strong robust, adaptive

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