International Journal of Hybrid Information Technology
Vol.9, No.2 (2016), pp. 227-238
http://dx.doi.org/10.14257/ijhit.2016.9.2.20

Fast Direction of Arrival Estimation with the Coexistence of both Uncorrelated and Coherent Signals

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Abstract

This paper proposes a novel fast direction of arrival (DOA) estimation method for scenarios when uncorrelated and coherent signals exist simultaneously. First, using uniform transformation, we change complex matrix into real matrix, then, just by exploiting real propagator method (RPM), we can attain the uncorrelated signals DOA fastly. Second, based on new spatial difference technique, we can eliminate the effection of uncorrelated signals and make coherent signals become uncorrelated. Finally, utilizing RPM technique, coherent signals DOA can be estimated swiftly. Simulation results validate the performance improvement of the proposed algorithm.

***Keywords**: direction of arrival (DOA), uncorrelated and coherent signals, uniform transformation, real propagator method (RPM), spatial difference*

1. Introduction

With the development of array signal processing, there has been swift growing interest in developing high resolution direction of arrival (DOA) estimation for narrowband planewave. Many perfect algorithms, such as multiple signal classification (MUSIC) algorithm and estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm [1-2] have been expanded and have received significant attention for their high resolution performance over the years. MUSIC and ESPRIT algorithm has satisfactory performance when the received signals are uncorrelated, however, they will be unsuccessful when signals are coherent. To deal with the coherency problem, many decorrelated methods have been proposed [3-14], in which spatial smoothing technique is the most typical technology [3-4]. Spatial smoothing technique is a kind of preprocessing technique which is based on the scheme that divides the array into overlapping subarrays, and then, through the average of the subarray covariance matrixes, coherent signals can be decorrelated. Although spatial smoothing technique can deal with coherent signals, it can reduce array aperture, so it is inconvenient in practical engineering. Based on the spatial samples of received data, matrix pencil (MP) approach is presented in [5]. The MP method can estimate coherent signals DOA conveniently without additional processing, however, the required signal-to-noise ratio (SNR) is too high to use in practice. The ML [6], Toeplitz and Improved Toeplitz algorithms [7-8] are the other significant methods that can be used for coherent signals DOA estimation, however, these approaches cannot differentiate uncorrelated signals from coherent signals. The incapability to discriminate uncorrelated signals from coherent signals lead to a grievous waste of sensors. Consequently, novel methods are proposed which can be classified as another category decorrelated technique called spatial difference technique In [9-14]. Spatial difference technique deals with uncorrelated and coherent signals separately, so it can handle more

ISSN: 1738-9968 IJHIT

signals than sensors. The Spatial difference technique is first recommended in [9], but it can only use in the scenario that there has two signals within a coherent group. Utilizing the Toeplitz and non-Toeplitz characters of array covariance matrix, the method in [10] can manage more signals. However, when the number of signals in coherent group is odd, the difference smooth matrix would be rank deficient, and it needs additional processing to recover the rank of matrix. In [11], a method for DOA estimation with fewer sensors is presented and in [12], a non Toeplitz matrix is constructed to handle coherent signals. Because the computational burdens of these methods are heavy, they are not attractive in practice. The method in [13] is also an efficiently approach for uncorrelated and coherent signals coexist, and it can avoid the cross-term effects when performing the EVD of matrix. This method is effective and efficient, but it needs to perform multiple inverses. In [14], Liu proposes new spatial differencing method to resolve more signals when uncorrelated and coherent signals coexist. After uncorrelated signals are estimated, by utilizing new spatial differencing technique, only coherent signals keep in the defined differencing matrix. New spatial differencing method not only can enhance algorithm accuracy but also can raise the number of detected signals. All the spatial difference techniques above-mentioned need two step to estimate uncorrelated and coherent signals ,so it could lead to extra computational burden which will be disadvantage for practical engineering. The main computation for DOA estimation lies in the course of subspace, and the most conventional approach for getting the subspace is eigendecomposition ,however it is time consuming .To reduce the computational cost of subspace algorithm,the propagator method(PM) is presented in [15]. PM is a linear operator based upon a partition of the steering vector, and by using propagator method; we can obtain the estimated noise subspace fast. In [16], Li expand PM into two –parallel uniform linear array (ULA) and Nizar [17] extend it to L-shape array, both of them realize 2-Dimensional DOA estimation. Unitary transform technology which is first proposed in [18] is another technique that can reduce the computation of algorithm and it is spreaded in [19-21]. It can enable the computations in the real domain, thereby substantially reducing the complexity of algorithms.

In this paper, fast effective DOA estimation method with uncorrelated and coherent signals coexist is proposed. First, we can change complex covariance matrix into real covariance matrix by using unitary transform, and then, utlizing RPM, without performing eigendecomposition, we obtain the estimated noise subspace for uncorrelated signals. Secondly, we exploit new spatial difference to eliminate uncorrelated signals contribution so that only coherent signals remain in the defined differencing matrix ,after that, through RPM technique, we can get the estimated noise subspace for coherent signals . Finally, using the orthogonality and through spectrum sweep, uncorrelated and coherent signals DOA can be acquired separately.

The paper is organized as follows. In Section 2, we introduce the narrowband signal model that will be used through the article. In Section 3, we introduce fast DOA estimation method for mixing uncorrelated and coherent signals. In Section 4, simulation results confirm the perfect performance of our proposed algorithm. Section 5 provides a conclusion to summarize the paper.

2. Data Model

Consider a ULA composed of M identical sensors， regarding first array sensor as the reference, array interspace is d which is equal to $\lambda/2$ and λ is narrowband signal wavelength.Suppose that there are L far-filed narrowband signals with distinct DOAs impinging on the array .The first L_u signals are uncorrelated and the signal that comes from direction $\theta_i(i=1,2,\cdots,L_u)$ is corresponding to the propagation of the far-field sources

$s_i(t)$ with power $\sigma_i^2, i=1,2,\ldots,L_u$ .The rest are D groups of $L_c = L - L_u$ coherent signals,which come from D statistically independent far-filed sources $s_i(t)$ with power $\sigma_i^2, i = L_u+1,\ldots,L_u+D$,and with P_i multipath signals for each source.In the ith coherent group ,the signal that comes from direction $\theta_{ip}, p=1,2,\ldots,P_i$ corresponding to the pth multipath propagation of the far-field sources $s_i(t)$,and the complex fading coefficient is α_{ip} .Suppose that coherent signals in different groups are uncorrelated with each other . The vector of received signal at time t can be modeled as follows:

$$\begin{aligned} \boldsymbol{x}(t) &= \sum_{i=1}^{L_u} \boldsymbol{a}(\theta_i)s_i(t) + \sum_{i=L_u+1}^{L_u+D}\sum_{p=1}^{P_i} \boldsymbol{a}(\theta_{ip})\alpha_{ip}s_i(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{A}_u\boldsymbol{s}_u(t) + \boldsymbol{A}_c\boldsymbol{s}_c(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \end{aligned}$$

(1)

where $\boldsymbol{x}(t) \in C^{M\times 1}, \boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$ is array output vector . $\boldsymbol{a}(\theta_i) = [1, e^{j\pi\sin(\theta_i)}, ..., e^{j\pi(M-1)\sin(\theta_i)}]^T$ is the steering vector, $\boldsymbol{A}$ is the array manifold matrix, $\boldsymbol{A}_u = [a(\theta_1),\ldots,a(\theta_{L_u})$, $\boldsymbol{A}_c = [\boldsymbol{A}_1\alpha_1, \boldsymbol{A}_2\alpha_2, \cdots, \boldsymbol{A}_D\alpha_D]$ with $\boldsymbol{A}_i = [a(\theta_{i1}),\ldots,a(\theta_{iP_i})]$.

$\alpha_i = [\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{iP_i}]^T, \alpha_i \in C^{P_i\times 1}$ is complex fading coefficient vector of the ith coherent group, $\boldsymbol{s}(t) = [\boldsymbol{s}_u^T(t), \boldsymbol{s}_c^T(t)]^T$ is signal vector in which $\boldsymbol{s}_u(t) = [s_1(t),\ldots,s_{L_u}(t)]^T$ is uncorrelated signals vector and $\boldsymbol{s}_c(t) = [s_{L_u+1}(t),\ldots,s_{L_u+D}(t)]^T$ is coherent signals vector.The $\boldsymbol{n}(t) = [n_1(t), n_2(t),\ldots,n_M(t)]^T$ is noise vector, here additional noise is Gaussian white noise with the power of each channel equal to σ_n^2 .

The operators $(.)^*$, $(.)^H$, $E\{.\}$, $|.|$ and $\arg(.)$ denote conjugate, conjugate transpose, expectation, modulus and phase angle,respectively. The symbol $diag\{r_1, r_2\}$ and blkdiag $\{\boldsymbol{Q}_1, \boldsymbol{Q}_2\}$ indicates a diagonal matrix with diagonal entries r_1, r_2 and $\boldsymbol{Q}_1, \boldsymbol{Q}_2$,respectively.All sources are uncorrelated with each other and $\boldsymbol{n}(t)$ are uncorrelated with sources . With $x(t)$, we can evaluate the array auto-covariance matrix as

$$\boldsymbol{R} = E\left\{\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right\} = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I}_M = \boldsymbol{R}_N + \boldsymbol{R}_{NT} + \sigma_n^2\boldsymbol{I}_M = \boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H + \boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H + \sigma_n^2\boldsymbol{I}_M \quad (2)$$

where $\boldsymbol{R}_s = blkdiag\{\boldsymbol{R}_u, \boldsymbol{R}_c\}$ is the signal covariance matrix with $\boldsymbol{R}_u = E\left\{\boldsymbol{s}_u(t)\boldsymbol{s}_u{}^H(t)\right\}$ and $\boldsymbol{R}_c = E\left\{\boldsymbol{s}_c(t)\boldsymbol{s}_c{}^H(t)\right\}$ respectively. And $\boldsymbol{R}_N = \boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H$ with $\boldsymbol{R}_u = diag\left\{\sigma_1^2,\ldots,\sigma_{L_u}^2\right\}$ and $\boldsymbol{R}_{NT} = \boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H$ with $\boldsymbol{R}_c = diag\left\{\sigma_{L_u+1}^2,\ldots,\sigma_{L_u+D}^2\right\}$. $\boldsymbol{I}_M$ indicates the $M\times M$ identity matrix.

3 DOA Estimation of Proposed Method

In this Section, uncorrelated and coherent signals DOA estimation are given separetely in Sections 3.1 and 3.2.

3.1 Uncorrelated Signals fast DOA Estimation

3.1.1 MUSIC Method

We first estimate the DOAs of uncorrelated signals. The coherent signals within one group can be seen as an equivalent virtual source.When $D + L_u < M$,carry out eigendecomposition of $\boldsymbol{R}$,we can acquire

$$\boldsymbol{R} = \boldsymbol{U}_s\boldsymbol{\Sigma}_s\boldsymbol{U}_s{}^H + \boldsymbol{U}_n\boldsymbol{\Sigma}_n\boldsymbol{U}_n{}^H \quad (3)$$

where $\boldsymbol{U}_s = [\boldsymbol{u}_1,\ldots,\boldsymbol{u}_{D+L_u}]$, $\boldsymbol{\Sigma}_s = diag\{\lambda_1,\ldots,\lambda_{D+L_u}\}$, $\boldsymbol{U}_n = [\boldsymbol{u}_{D+L_u+1},\ldots,\boldsymbol{u}_M]$, $\boldsymbol{\Sigma}_n = diag\{\lambda_{D+L_u+1},\ldots,\lambda_M\}$, and $\lambda_1,\lambda_2,\ldots,\lambda_M$ are the eigenvalues and $u_1,u_2,\ldots,u_M$ are their corresponding eigenvector with $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_{D+L_u} > \lambda_{D+L_u+1} = \ldots = \lambda_M = \sigma_n^2$.

The columns of $\boldsymbol{U}_s$ and the columns of $\boldsymbol{A}_u$ and $\boldsymbol{A}_c$ span the same signal subspace which is orthgonal to the noise subspace spaned by the the columns of $\boldsymbol{U}_n$ .Therefore

$$\left|(\boldsymbol{A}_i\rho_i)^H \boldsymbol{U}_n\right|^2 = 0, i = 1,\ldots,\mathrm{D} \tag{4}$$

$$g(\theta) = \left|a^H(\theta_i)U_n\right|^2 = 0\,, \quad i = 1,\ldots,L_u \tag{5}$$

where $\boldsymbol{A}_i\rho_i$ is the linear combination of $a(\theta)$,and coherent signals will not satisfy the character of uncorrelated signals shown in(5).That is to say,the effect of coherent signals in each group cannot be equivalent to a virtual source which can be confuse with the uncorrelated signals.So there is no false peak for coherent signal in the MUSIC spectrum,and only uncorrelated signals DOA estimation can be attained by the peak of $1/g(\theta)$.

3.1.2 Unitary Transform

MUSIC algorithm has perfect performance, while it is time-consuming for its complex matrix multiplication and eigende composition. Compared with complex multiplication, the computational complexity of real multiplication is less. If $\boldsymbol{R}$ is complex centro-Hermitian, by using unitary transform, we can change it into a real matrix.

For a given matrix $\boldsymbol{Q}$, we denote $\boldsymbol{J}_P$ as a $P \times P$ dimension exchange matrix with ones on its antidiagonal and zeros in rest .We call $\boldsymbol{Q}$ is a left-Π - real matrix if it satisfies $\boldsymbol{J}_p\boldsymbol{Q}^* = \boldsymbol{Q}$. Define unitary matrix as follows

$$\boldsymbol{Q}_{2n} = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_n & j\boldsymbol{I}_n \\ \boldsymbol{J}_n & -j\boldsymbol{J}_n \end{bmatrix}, \boldsymbol{Q}_{2n+1} = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_n & 0 & j\boldsymbol{I}_n \\ \boldsymbol{0}^T & \sqrt{2} & \boldsymbol{0}^T \\ \boldsymbol{J}_n & 0 & -j\boldsymbol{J}_n \end{bmatrix} \tag{6}$$

where $\boldsymbol{0} = [0,0,\ldots,0]^T$ is zero vector and $\boldsymbol{I}_n$ is a $n \times n$ dimension identity matrix. Both of $\boldsymbol{Q}_{2n}$ and $\boldsymbol{Q}_{2n+1}$ are left-Π - real matrix. We can choose $\boldsymbol{Q}_{2n}$ when sensor number is even and select $\boldsymbol{Q}_{2n+1}$ when sensor number is odd. By using $\boldsymbol{Q}_{2n}$ or $\boldsymbol{Q}_{2n+1}$, $\boldsymbol{R}$ can be changed into real matrix through follow formula

$$\boldsymbol{R}' = \boldsymbol{Q}_{2n}^H \boldsymbol{R} \boldsymbol{Q}_{2n} \ , \boldsymbol{R}' = \boldsymbol{Q}_{2n+1}^H \boldsymbol{R} \boldsymbol{Q}_{2n+1} \tag{7}$$

3.1.3 Fast MUSIC Method based on Real Propagator Method

By performing unitary transform ,we can acquired real covariance matrix $\boldsymbol{R}'$, however, the computational load of eigendecomposition for $\boldsymbol{R}'$ is also large, in order to release the computation,we can consider the real propagator method. Under the hypothesis that $\boldsymbol{A}$ has L_u rows which are linearly independent,and the other rows can be showed as a linear combination of these L_u rows. We suppose that the first L_u rows are linearly independent. The definition of the propagator is based upon the partition of $\boldsymbol{A}$.

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix} \tag{8}$$

where $\boldsymbol{A}_1$ is $L_u \times L_u$ dimension matrix, $\boldsymbol{A}_2$ is $(M - L_u) \times L_u$ matrix. Then, $\boldsymbol{A}_2$ is a linear transformation of $\boldsymbol{A}_1$.

$$\boldsymbol{P}^H \boldsymbol{A}_1 = \boldsymbol{A}_2 \tag{9}$$

where $\boldsymbol{P}$ is the propagator matrix.

Partition real matrix $\boldsymbol{R}'$ as

$$\boldsymbol{R}' = \begin{bmatrix} \boldsymbol{R}'_1 \\ \boldsymbol{R}'_2 \end{bmatrix} \tag{10}$$

where $\boldsymbol{R}'_1$ is $M \times L_u$ dimension real matrix, $\boldsymbol{R}'_2$ is $M \times (M - L_u)$ real matrix. Without consider the noise

$$\boldsymbol{R}'_2 = \boldsymbol{R}'_1 \boldsymbol{P} \tag{11}$$

Actually, there is always noise in practice, so the propagator matrix can be evaluated by the following minimization problem:

$$J_{csm}(\boldsymbol{P}) = \|\boldsymbol{R}'_2 - \boldsymbol{R}'_1 \boldsymbol{P}\|_F \tag{12}$$

where $\|.\|_F$ express Frobenius norm. The estimation of $\hat{\boldsymbol{P}}$ is via

$$\hat{\boldsymbol{P}} = (\boldsymbol{R}'^H_1 \boldsymbol{R}'_1)^{-1} \boldsymbol{R}'^H_1 \boldsymbol{R}'_2 \tag{13}$$

Define real matrix $\boldsymbol{Z}$

$$\boldsymbol{Z}^H = [\hat{\boldsymbol{P}}^H, -\boldsymbol{I}_{M-L_u}] \tag{14}$$

From above analysis,we know $\boldsymbol{Z}^H \boldsymbol{R}' = \boldsymbol{0}$ .It means that the steering vectors $a(\theta_i)$ are orthogonal to the columns of $\boldsymbol{Z}$,therefore ,we have

$$span\{\boldsymbol{Z}\} = span\{\boldsymbol{U}_n\} \tag{15}$$

The formula(15) indicates that the real propagator span the noise subspace as does the matrix $\boldsymbol{U}_n$. RPM avoids the eigendecomposition and the computation of eigendecomposition is larger than linear combination, by utilizing RPM technique,we can release the computation of subspace algorithm.We construct the following DOA estimator for uncorrelated signals:

$$P_{RPMU}(\theta) = \boldsymbol{a}^H(\theta) \boldsymbol{Z} \boldsymbol{Z}^H \boldsymbol{a}(\theta) \tag{16}$$

3.2 Coherent Signals Fast DOA Estimation

In this section, new spatial smoothing is carried out to solve coherent signals DOA estimation. In the following description, we use the equivalent subarrays to describe smoothing technique and assume the number of subarrays is p .The m th subarray covariance matrix is given by

$$\boldsymbol{R}_m = \boldsymbol{K}_m \boldsymbol{R} \boldsymbol{K}_m^H \tag{17}$$

where the selection matrix $\boldsymbol{K}_m \in C^{(M-p+1)\times M}$ is defined as follows

$$\boldsymbol{K}_m = [\boldsymbol{0}_{(M-p+1)\times(m-1)} \quad \boldsymbol{I}_{(M-p+1)} \quad \boldsymbol{0}_{(M-p+1)\times(p-m)}] \tag{18}$$

Define a p th order spatial difference matrix $\boldsymbol{D}_p$

$$\begin{aligned} \boldsymbol{D}_p &= \frac{1}{p}\sum_{k=1}^{p}[\boldsymbol{R}_1 - \boldsymbol{J}_{M-p+1}(\boldsymbol{R}_k)^* \boldsymbol{J}_{M-p+1}] \\ &= \frac{1}{p}\sum_{k=1}^{p}[\boldsymbol{W}_k + \boldsymbol{F}_k] \end{aligned} \tag{19}$$

where $\boldsymbol{R}_k = \boldsymbol{K}_k \boldsymbol{R} \boldsymbol{K}_k^H (k = 1, \ldots, p)$ in which $\boldsymbol{K}_k$ is defined in (18). $\boldsymbol{W}_k = \boldsymbol{R}_{N1} - \boldsymbol{J}_{M-p+1}(\boldsymbol{R}_{Nk})^* \boldsymbol{J}_{M-p+1}$,in which $\boldsymbol{R}_{N1} = \boldsymbol{K}_1 \boldsymbol{R}_N \boldsymbol{K}_1^H = \boldsymbol{A}_{u1} \boldsymbol{R}_u \boldsymbol{A}_{u1}^H$ with $\boldsymbol{A}_{u1} = \boldsymbol{K}_1 \boldsymbol{A}_u$, and $\boldsymbol{R}_{Nk} = \boldsymbol{K}_k \boldsymbol{R}_N \boldsymbol{K}_k^H = \boldsymbol{A}_{u1} \boldsymbol{\Phi}^{k-1} \boldsymbol{R}_u (\boldsymbol{A}_{u1} \boldsymbol{\Phi}^{k-1})^H$ with $\boldsymbol{\Phi} = diag\left\{ e^{-j\frac{2\pi d}{\lambda}\sin\theta_1}, \ldots, e^{-j\frac{2\pi d}{\lambda}\sin\theta_{L_u}} \right\}$. $\boldsymbol{F}_k = \boldsymbol{R}_{NT1} - \boldsymbol{J}_{M-p+1}(\boldsymbol{R}_{NTk})^* \boldsymbol{J}_{M-p+1}$,in which $\boldsymbol{R}_{NT1} = \boldsymbol{K}_1 \boldsymbol{R}_{NT} \boldsymbol{K}_1^H$. Because $\boldsymbol{A}_{u1}$ is a Vandermonde matrix,it is clear that $\boldsymbol{J}_{M-p+1}(\boldsymbol{A}_{u1})^* = \boldsymbol{A}_{u1} \boldsymbol{\Phi}^{M-p+1}$,then we can obtain

$$\begin{aligned} \boldsymbol{W}_k &= \boldsymbol{R}_{N1} - \boldsymbol{J}_{M-p+1}(\boldsymbol{R}_{Nk})^* \boldsymbol{J}_{M-p+1} \\ &= \boldsymbol{A}_{u1}\boldsymbol{R}_u\boldsymbol{A}_{u1}^H - \boldsymbol{J}_{M-p+1}(\boldsymbol{A}_{u1}\boldsymbol{\Phi}^{k-1}\boldsymbol{R}_u(\boldsymbol{A}_{u1}\boldsymbol{\Phi}^{k-1})^H)^*\boldsymbol{J}_{M-p+1} \\ &= \boldsymbol{A}_{u1}\boldsymbol{R}_u\boldsymbol{A}_{u1}^H - \boldsymbol{A}_{u1}\boldsymbol{\Phi}^{M-p+1}(\boldsymbol{\Phi}^{k-1}\boldsymbol{R}_u(\boldsymbol{\Phi}^{k-1})^H)^*(\boldsymbol{\Phi}^{M-p+1})^H\boldsymbol{A}_{u1}^H \\ &= \boldsymbol{A}_{u1}\boldsymbol{R}_u\boldsymbol{A}_{u1}^H - \boldsymbol{A}_{u1}\boldsymbol{\Phi}^{M-p+1}(\boldsymbol{\Phi}^{M-p+1})^H(\boldsymbol{\Phi}^{k-1}(\boldsymbol{\Phi}^{k-1})^H\boldsymbol{R}_u)^*\boldsymbol{A}_{u1}^H \\ &= \boldsymbol{A}_{u1}\boldsymbol{R}_u\boldsymbol{A}_{u1}^H - \boldsymbol{A}_{u1}\boldsymbol{R}_u^*\boldsymbol{A}_{u1}^H = \boldsymbol{0} \end{aligned} \quad (20)$$

From formula(19) and (20) we know that there is only coherent signals remaining in the spatial difference matrix $\boldsymbol{D}_p$ .In [14],Liu has proved that the rank of $\boldsymbol{D}_p$ is equal to L_c if $p > \max_k p_k$ .By using formula(19),we can perform decorrelated operation .According to the dimension of $\boldsymbol{D}_p$, utilizing unitary transform matrix,we can get real matrix $\boldsymbol{D}'_p$ by

$$\boldsymbol{D}'_p = \boldsymbol{Q}_{2n}^H\boldsymbol{D}_p\boldsymbol{Q}_{2n} \ ,\boldsymbol{D}'_p = \boldsymbol{Q}_{2n+1}^H\boldsymbol{D}_p\boldsymbol{Q}_{2n+1} \quad (21)$$

Partition $\boldsymbol{D}'_p$ as two parts

$$\boldsymbol{D}'_p = \begin{bmatrix} \boldsymbol{D}'_{p1} \\ \boldsymbol{D}'_{p2} \end{bmatrix} \quad (22)$$

where $\boldsymbol{D}'_{p1}$ is $(M-p+1)\times L_c$ dimension real matrix, $\boldsymbol{D}'_{p2}$ is $(M-p+1)\times(M-p+1-L_c)$ real matrix, utilizing $\boldsymbol{D}'_{p1}$ and $\boldsymbol{D}'_{p2}$,we can get

$$\hat{\boldsymbol{P}}_1 = ((\boldsymbol{D}'_{p1})^H\boldsymbol{D}'_{p1})^{-1}(\boldsymbol{D}'_{p1})^H\boldsymbol{D}'_{p2} \quad (23)$$

Define real matrix $\boldsymbol{Z}_1$

$$\boldsymbol{Z}_1^H = [\hat{\boldsymbol{P}}_1^H, -\boldsymbol{I}_{M-p+1-L_c}] \quad (24)$$

and

$$\boldsymbol{Z}_1^H\boldsymbol{D}'_p = \boldsymbol{0} \quad (25)$$

The formula(25) indicates that $\boldsymbol{Z}_1$ is orthogonal to $\boldsymbol{D}'_p$,and we can construct the following DOA estimator for coherent signals:

$$P_{RPMC}(\theta) = \boldsymbol{a}_1^H(\theta)\boldsymbol{Z}_1\boldsymbol{Z}_1^H\boldsymbol{a}_1(\theta) \quad (26)$$

where $\boldsymbol{a}_1(\theta)$ is $M-p+1$ dimension steering vector.

3.3 Summary the Step of Fast Algorithm

In this section, we will summarize the fast method we proposed as follows:

Step1: Calculate the covariance matrix $\boldsymbol{R}$, then, using $\boldsymbol{Q}_{2n}$ or $\boldsymbol{Q}_{2n+1}$ to get real covariance matrix $\boldsymbol{R}'$.

Step2: Calculate $\boldsymbol{R}'_1$ and $\boldsymbol{R}'_2$ and get the estimated $\hat{\boldsymbol{P}}$ by equation (13).

Step3: Construct $\boldsymbol{Z}$, using the orthogonality between $\boldsymbol{Z}$ and $a(\theta)$ to estimate the DOAs of uncorrelated signals.

Step4: Utilizing new spatial difference technique to get coherent signals covariance matrix through the equation (19), and by using $\boldsymbol{Q}_{2n}$ or $\boldsymbol{Q}_{2n+1}$ to get $\boldsymbol{D}'_p$.

Step5: Construct $\boldsymbol{Z}_1$, using the orthogonality between $\boldsymbol{Z}_1$ and $a_1(\theta)$ to estimate the DOAs of coherent signals.

3.4 Identifiability and Computational Load

(1) The number of detected signals

To correct estimate uncorrelated signals DOA, $L_u + D < M$ must be required . To correct estimate D group coherent signals DOA by spatial difference matrix $\boldsymbol{D}_p$,the standard $p \ge \max_k p_k$ and $K_c = \sum_{k=1}^{D} p_k < M - p + 1$ must be satisfied, in which p_k refer to the number of coherent signals in the k th group.The proposed algorithm can detect the maximal number of uncorrelated signals is $\left\lfloor \frac{M}{2} \right\rfloor$ and the maximal number of coherent signals is $2\times(\left\lfloor \frac{M}{2} \right\rfloor - 1)$ when $p = \max_k p_k = 2$,and the total number of detected signals is equal to $\left\lfloor \frac{3M}{2} \right\rfloor - 2$,where $\lfloor . \rfloor$ is floor operator.

(2) Computational load analysis

Through complex matrix eigendecomposition, the method in [14] attains estimated noise subspace ,and the computational load for uncorrelated signals and coherent signals is $M^2 N + O(M^3)$ and $(M-p+1)^2 N + O((M-p+1)^3)$, respectively. Our algorithm use RPM to gain estimated noise subspace and the RPM computational load is $O(LM^2)$. Contrast with complex matrix, the computational load of multiplication for real matrix is only about quarter . So, the computational load for uncorrelated and coherent signals with our proposed algorithm is $[M^2 N + +O(L_u M^2)]/4$ and $[(M-p+1)^2 N + O(L_c (M-p+1)^2)]/4$, respectively.Contrast with the method in [14],our proposed method is a low complexity algorithm .

4. Simulation Results

The performance of our fast algorothm is illustrated in this section and 8-element ULA with interelement space of half a wavelength will be used. For simplicity,we suppose that all signals are of equal power σ_s^2 and the uncorrelated signals and coherent signals come from far field at the same time. We use 600 snapshots to estimate the array covariance matrix , and the additional noise is ideal Gaussian white noise .When using MUSIC to resolve the signals DOA,the scanning is performed over $[-90^\circ, 90^\circ]$ with a step size of 0.1° .Independent Monte-Carlo research number is 100,and the root mean square error(RMSE) for uncorrelated signals and coherent signals is defined as

$$RMSE_1 = \sqrt{\frac{1}{L_u N} \sum_{i=1}^{L_u} \sum_{k=1}^{N} (\hat{\theta}_i - \theta_i)^2} \tag{27}$$

$$RMSE_2 = \sqrt{\frac{1}{L_c N} \sum_{i=1}^{L_c} \sum_{k=1}^{N} (\hat{\theta}_i - \theta_i)^2} \tag{28}$$

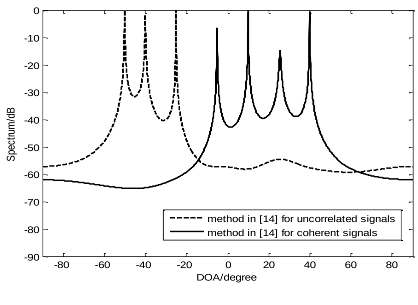

Figure 1. Method in [14] Spectrum with 10 SNR

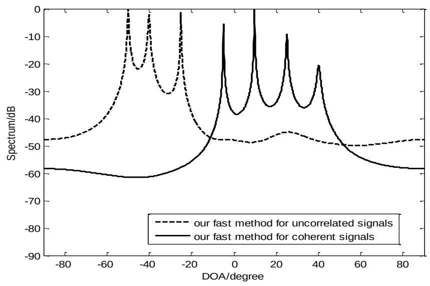

Figure 2. Our Fast Method Spectrum with 10 SNR

In the first simulation,we consider three uncorrelated signals and a group of four coherent signals are impinging on the array ,the incidence angles are $[-50^{\circ},-40^{\circ},-25^{\circ}]$ and $[-5^{\circ},10^{\circ},25^{\circ},40^{\circ}]$, respectively.The fading amplitudes and phases of the coherent signals are $[0.92,0.85,0.7,06]$ and $[108.28^{\circ},28.12^{\circ},54.03^{\circ},34.38^{\circ}]$,respectively.The SNR of each sources is $10dB$ .Simulation results of the method in [14] and our fast method are shown in Figure 1 and Figure 2, respectively .From the simulation results ,we know that both algorithms can estimate uncorrelated signals and coherent signals correctely and estimated effection both are perfect. It is noticed that the sharp difference of spectrum peak between two algorithms is not obvious. Because our fast method is based on real propagator method,its computational load for uncorrelated signals and coherent signals is $9600+O(192)/4$ and $7350+O(196)/4$. While the computational burden of method in [14] is $38400+O(512)$ and $29400+O(343)$ .The former computational load is far less than the latters. The RMSE curves of the DOA estimations versus the SNR and snapshots are shown in Figure 3 and Figure 4. The results of Figures indicate that the differenc of RMSE between two algorithms is not obvious , and because of lower computational load, our fast mthod has more wonderful prospect in practical engineering.

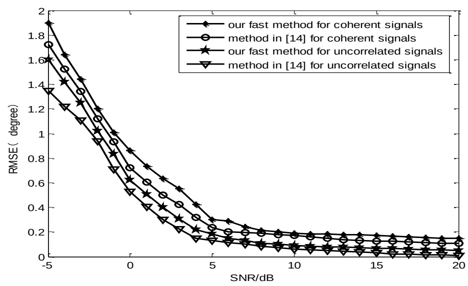

Figure 3. RMSE Versus SNR

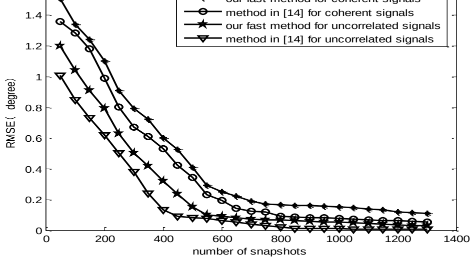

Figure 4. RMSE Versus Number of Snapshots

The second simulation consider the case that signals number accesses array elements. Four uncorrelated signals impinging from $[-43^{\circ},-22^{\circ},-10^{\circ},0^{\circ}]$ and two groups of six coherent signals impinging from $[-30^{\circ},-20^{\circ},-5^{\circ},20^{\circ},40^{\circ},60^{\circ}]$. The fading amplitudes of coherent signals are $[0.94,1,0.72,0.85,0.76,0.91]$ and $[101.21^{\circ},24.15^{\circ},58.06^{\circ},14.31^{\circ},45.21^{\circ},33.59^{\circ}]$ are fading phases, and the SNR of each sources is $10dB$. From the descrimnation in Section 3.4 we know that the maximal detected number of signals for two algorithms is the same. That is, the maximal estimated number for uncorrelated signals and coherent signals are four and six respectively when sensors are eight. The Figures of spatial spectrum for method in [14] and our fast method are shown in Figure 5 and Figure 6, and the simulation results confirm the correction of our throretical analysis. Figure 7 and Figure 8. indicate the RMSE curves of the DOA estimations versus the SNR and snapshots. From the results of Figure 7 and Figure 8, we know that our fast method has larger RMSE when the detected signals number is maximum, however, its computational burden is far less than the method in [14], so it is more available in practical engineering .

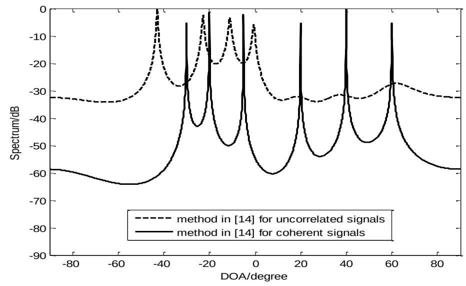

Figure 5. Method in [14] Spectrumwith 10 Signals

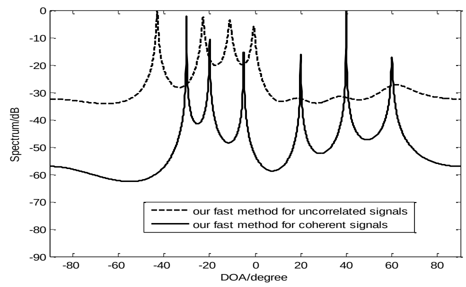

Figure 6. Our Fast Method Spectrum with 10 Signals

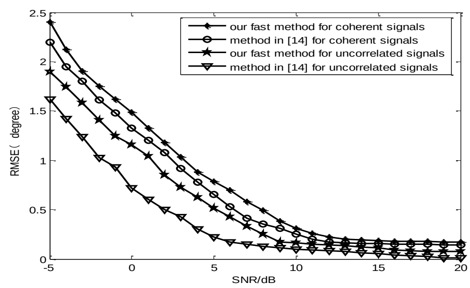

Figure 7. RMSE Versus SNR for the Second Case Simulaiton

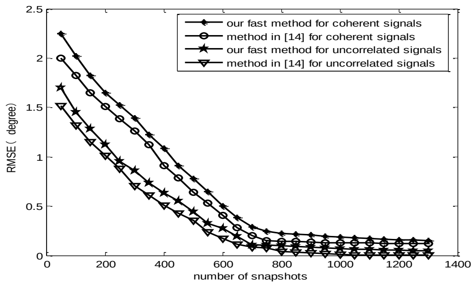

Figure 8. RMSE Versus Number of Snapshots for the Second Case Simulaiton .

In the third simulation, we take into account the situation that the effect of subarrays number p on the DOA.Suppose that two uncorrelated signals impinging from $[-50^\circ,-35^\circ]$ and two groups of four coherent signals impinging from $[-20^\circ,0^\circ]$ and $[15^\circ,35^\circ]$. The two groups coherent signals fading amplitudes are $[0.64, 0.87]$ and $[0.95, 0.71]$, and the phases are $[142^\circ,34^\circ]$ and $[53^\circ,87^\circ]$,respectively . The SNR of each source is $10dB$. The spatial spectrums with different p for our fast method are shown in Figure9, from the simulation results we know that our fast algorithm can attain better DOA estimation even when the number of subarrays is two. The RMSE curves versus the SNR for our fast method with different p are shown in Figure 10. Figure10 indicates that our fast method has more perfect accuration with the increasement of SNR.

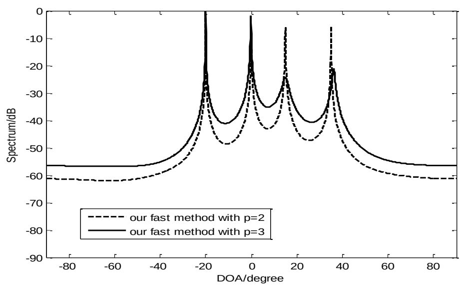

Figure 9. Our Fast Method Spectrum for Coherent Signals Versus the Number of Subarrays

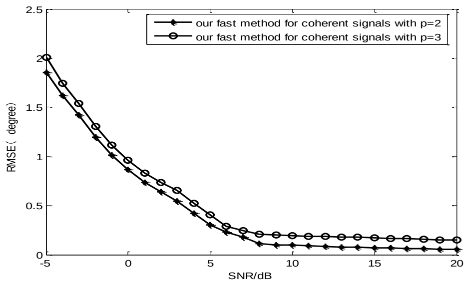

Figure 10. RMSE Versus SNR with Different p

5 Conclusions

In this paper ,we present a new fast algorithm when uncorrelated signals and coherent signals are impinging together and it is suitable for ULA no matter whether the number of array sensors is odd or even.By using real propagator method ,our algorithm can attain uncorrelated signals and coherent signals DOA fastly.The proposed algorithm has two advantages ,in which one is that it can estimate more sources than array sensors , the other is that it has lower computational complexity. The computer simulations validate the effectiveness of our proposed algorithm.

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