International Journal of Hybrid Information Technology
Vol. 9, No.12 (2016), pp. 231-242
http://dx.doi.org/10.14257/ijhit.2016.9.12.21

BLDC Multi-Motor Speed Synchronization by Leader Following Multi-Agent and Hybrid Control

Suhaib Masroor[1], Chen Peng[2], Muhammad Aamir[3] and Zain Anwar Ali[4]
[1, 2]*School of Mechatronics Engineering and Automation, Shanghai University, Shanghai, China*
[3]*Department of Electronics Engineering, Sir Syed University of Engineering and Technology, Karachi, Pakistan*
[4]*College of Automation Engineering, Nanjing University of Aeronautics and Astronauts, Nanjing, China*
suhaibmasroor1@gmail.com, c.peng@shu.edu.cn, muaamir5@yahoo.com, zainanwar86@hotmail.com.

Abstract

In this article, a unique and innovative design for addressing the speed synchronization problem in a multi-motor system using Brush less DC (BLDC), motors is presented by utilizing motor back emf equation and rotor position commonly called sensor less technique. In the proposed method, the system is modelled as a consensus problem of leader following multi-agent system (MAS), and a hybrid controller is designed by using Model Reference Adaptive Control (MRAC) along with Variable Structure (VS) control technique of Sliding Mode Control (SMC), herein called Variable Structure Model Reference Adaptive Control (VS-MRAC). The consensus algorithm of MAS (reformed w.r.t motor speed data), is fused with the hybrid controller to reach consensus on speed regulated by proposed hybrid controller. The stability of a system is endorse by designing a Lyapunov function. The efficiency of the proposed methodology is proven by simulations performed in MATLAB and the acquired results validate the success of proposed design methodology.

Keywords*: BLDC motor, Hybrid control, Consensus control, Leader following MAS, Lyapunov stability*

1. Introduction

In the preceding era, the notable area of research in control academics is Multi-agent Systems (MAS). MAS provide a new and novel approach to model and control complex systems. When dealing with autonomous MAS, the main problem that occurred mostly is the consensus. There are two main settings in which MAS can be organized, first is leader following MAS while the other one is leader-less MAS. Therefore, consensus problem can also be modeled as either leader follower or leader-less respectively. The main difference in the architecture of the two MAS categories is the presence of leader which dictates desired or reference path for the following agents to reach consensus while in the leader-less scenario, the agents interchange information with each other to achieve consensus. Moreover, in leader following architecture, the consensus or preferred task is accomplished by regulating the leader such that the leader is keep connected with the followers. The main benefit rest with this architecture of MAS is that, leader describe reference course and the control protocol of every distinct agent ensure stability.

Recent advancement in control automation theory and with the introduction of a communication network in the complex control systems devised the notion of Network

ISSN: 1738-9968 IJHIT

Control System (NCS). The classical concept of controls is reformed by NCS, enabling a new area of research by integrating NCS in different systems particularly NCS centered MAS, NCS based smart grids, NCS based motion control systems, and etc. In the case of electric motors, Nirali in [1], present speed control problem with three phase indirect vector controlled induction motor using agent based approach. In this approach, MAS is implemented using PI, Fuzzy logic, and neural network, and the resultant speed responses of controllers and MAS is compared. Furthermore, in [2], a problem of synchronous control in the networked multi-motor system is addressed using leader following MAS theory by assuming each motor as single agent sharing its output data with the neighboring agent. The networked is supposed to be switching and undirected while networked delay is addressed by proposing a delay consistency protocol whereas system stability is proved by lyapunov theory considering the fact that, if a single agent is controllable and observable than the multi-motor system can attain leader following stability. The method of MAS based on NCS are adopted by various researchers to address different problems related to control of MAS including [3-4].

Since, the synchronous multi-motor system exists in every industrial setting, facing the problem of synchronous speed due to the uneven speed of motors in the system. Synchronization problem mostly occurs due to the sudden change in motor load causing acceleration or deceleration of motor unevenly. Although, at the stage of system design, such problem is considered but variation in system parameters effect synchronization. In [5], the problem of over speeding and disturbance rejection in multi-motor speed synchronization system is addressed by using compensation theory and internal model control such that precise synchronization is reached. In [6], real time speed synchronization of the multi-motor system is presented using Controller Area Network. Furthermore, in [7], synchronization problem with two induction machines (multi-motor), using an inverter for the electric vehicle is presented using Electronic Virtual Line Shafting technique.

So, above studies encourage us to address the problem of speed synchronization in BLDC motors using the MAS consensus algorithm along with Variable Structure Model Reference Adaptive Control (VS-MRAC). It is obvious that every network offers some delay, therefore it is worthy to explore the aforementioned consensus problem with the network induced delay. In such condition, the consensus algorithm is altered in such a way that each agent compares its present state with the adjoining agents delayed one. We take on the aforesaid problem with BLDC motor because it offers better efficiency, speed, noise suppression, control, and linearity concerning speed and torque. Most importantly, they are used in a lot of application from automotive vehicles to aerospace, UAV's, robotics motion control and etc. In [8], the problem of formation control of spacecraft using VS-MRAC methodology with the leader-following approach for satellite is addressed. Stability of formation is proved by developing lyapunov function. In this approach, the control protocol gives better robustness and disturbance rejection. In [9], tracking problem with SISO system is presented using MRAC control by developing error model to design variable structure controller. A neural network is also incorporated in the design to estimate online the state and desired trajectory while tracking stability is proved by lyapunov stability theorem. Formation and tracking control problem in MAS using adaptive controller are addressed by various authors in [10-11].

The paper addresses the problem of speed synchronization with BLDC motors using the concept of leader following MAS consensus & VS-MRAC algorithm, connected in such a way that communication topology among agents is supposed to be fixed. The aim of this paper is to, (1) design a unique controller by utilizing the MAS consensus protocol along with hybrid controller compose of MRAC and VS controllers, (2) model BLDC motor, with the aim of transforming into larger MAS system, and (3) formulate and solve a synchronous speed problem in the network connected BLDC motors using the proposed unique integration of two different control methods (VS-MRAC & MAS), and ensure system stability by developing a lyapunov function.

The paper is systematized into VII sections. After Introduction in section I, the basics of graph theory and MAS is given in section II while section III provide modeling of BLDC motor. In section IV, a consensus protocol of MAS w.r.t BLDC motor speed is presented whereas section V provide proposed controller design. Furthermore, the results are given in section VI and finally conclusion is given in section VII.

2. Preliminaries

When dealing with MAS, it is necessary to connect agents using a communication network to make coordination possible among the entire system to reach a common goal or simply consensus. This communication is modeled using the notion of graph theory. A graph $\mathcal{G}\ (\alpha, \beta)$, consist of nodes $\alpha = \{\alpha_1, \dots \alpha_N\}$ and edges $\beta \subseteq \alpha\ \times \alpha$ such that each i_{th} agent is considered to be a node and information exchange between the pair of nodes are modeled by edges. Moreover, the set of a neighbor of the node is given as N_i i-e $N_i = \{\mathrm{j}\colon \mathrm{ij} \in \beta\}$. In order to determine the number of neighbor of any node, a matrix called degree matrix is proposed as $\mathcal{D} = diag\{d_1, \dots. d_N\}$, such that $\deg(\alpha_i)\ if\ i = j\ or\ else\ \deg(0)$. Furthermore, another useful matrix of graph theory is adjacency matrix $\mathcal{A}$ that define the adjacency of graph $\mathcal{A} = [\mathrm{a}_{ij}] \in \mathbb{R}^{N\times N}$ as positive if $(\alpha_j, \alpha_i) \in \beta$ otherwise it is 0. Moreover, in the case of undirected graphs $\mathrm{a}_{ij} = \mathrm{a}_{ji}$. Lastly, the most important matrix of graph theory is laplacian $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ define as $\mathcal{L} = \mathcal{D} - \mathcal{A}$ i-e $l_{ii} = \sum_{j\, \neq \mathrm{i}} \mathrm{a}_{ij}$ and $\mathrm{l}_{ij} = -\mathrm{a}_{ij}, \mathrm{i} \neq j$. The key property of laplacian is its row sum which is always zero and the matrix has the simple eigenvalue of 0 with eigenvector 1. Additionally, laplacian matrix is positive definite for undirected graphs as shown by equation (01)

$$\Pi(x) = x^T \mathcal{L} x = \frac{1}{2} \sum_{i,j} \mathrm{a}_{ij} \left(x_j - x_i\right)^2 \tag{01}$$

Where equation (1) is laplacian potential equation.

Lemma 1. [12], If $\mathcal{G}$ is connected, than the rank of matrix $\mathcal{L}$ is given as

$$rank\ (\mathcal{L}) = n - c \tag{02}$$

Where c is connected components in $\mathcal{G}$.

Lemma 2. [13], If $\mathcal{G}$ is connected, than $\mathcal{L}$ has simple eigenvalue 0. Moreover, the smallest positive eigenvalue (λ_2), holds $\min_{x\neq 0, 1^T x = 0} \frac{x^T \mathcal{L} x}{x^T x}$

Lemma 3. [14], If $\lambda_2(\mathcal{G}) > 0$, than $\mathcal{G}$ is connected.

In leader following MAS, leader communicates with that follower which are its immediate neighbors. Usually, node 0 act as a leader, therefore, in this paper, we assumed the same. With this assumption, a new sub-graph $\overrightarrow{\mathcal{G}}$ came in, defining the connectivity between leader and neighboring followers. A diagonal matrix $\xi = diag\ \{\xi_1, \xi_2, \dots \dots \xi_N\}$ defines this connectivity as $\xi_i > 0$ if there exists a following agent i in the neighborhood of leader else it is 0.

Thus, in the proposed networked arrangement, we aim to design a controller using the MAS consensus protocol and VS-MRAC algorithm such that all the agents in the network will follow the reference trajectory as described by the leader. Moreover, using the tools of aforesaid graph theory, we present the effectiveness of the proposed method.

Definition: For leader following MAS, if there exists a controller u_i such that $\{w_j : \mathrm{j} \in \mathrm{N}_i\}$ for every agent $i \in \{1, \ldots, \mathrm{N}\}$ satisfying $\lim_{t \to \infty} \| w_i(t) - w_0(t) \| = 0,\ i = 1, \ldots, \mathrm{N}$, than it is said that consensus is reached.

Lemma 4. [15], If $\mathcal{G}$ and $\overrightarrow{\mathcal{G}}$ is connected and undirected, a matrix $\Xi = \mathcal{L} + \xi$ exists which is symmetric and positive definite providing sub-graph $\overrightarrow{\mathcal{G}}$ is connected.

Lemma 5. [16], If the leader's node (α_0), is reachable in $\overrightarrow{\mathcal{G}}$, than matrix Ξ is said to be stable.

3. Motor Modeling

In BLDC motor, a magnet is located adjacent to its rotor while windings are located on stator side energized by fixed serialized DC power source, a process termed as commutation. A commutation setting for idealized six phase BLDC motor is given in Figure 1, such that creating a rotary magnetic field. Furthermore, back EMF is produced in stator windings whenever the rotating magnetic field intermingles with stator pole. Three phase star connected model of BLDC motor is given by [17].

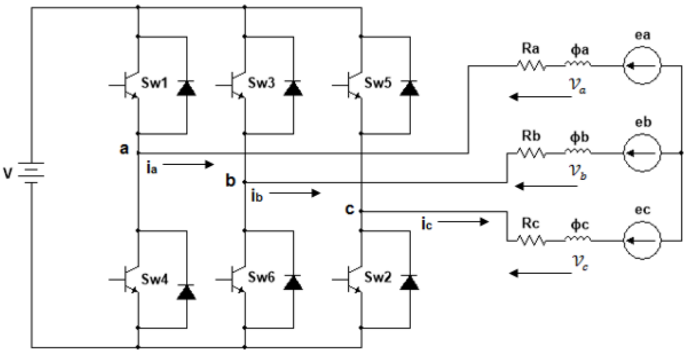

Figure 1. Model of BLDC Motor

$$\mathcal{V}_a = \mathcal{R}_a\, i_a + \phi_a \frac{d i_a}{dt} + e_a \tag{03}$$

$$\mathcal{V}_b = \mathcal{R}_b\, i_b + \phi_b \frac{d i_b}{dt} + e_b \tag{04}$$

$$\mathcal{V}_c = \mathcal{R}_c\, i_c + \phi_c \frac{d i_c}{dt} + e_c \tag{05}$$

$$\begin{bmatrix} \mathcal{V}_a \\ \mathcal{V}_b \\ \mathcal{V}_c \end{bmatrix} = \begin{bmatrix} \mathcal{R}_a & 0 & 0 \\ 0 & \mathcal{R}_b & 0 \\ 0 & 0 & \mathcal{R}_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \phi_a & \xi_{ab} & \xi_{ac} \\ \xi_{ba} & \phi_b & \xi_{bc} \\ \xi_{ca} & \xi_{cb} & \phi_c \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{06}$$

Let's suppose resistors and inductors to be $\mathcal{R}$ and ϕ while flux linkages ξ are assumed to be 0, then

$$\begin{bmatrix} \mathcal{V}_a \\ \mathcal{V}_b \\ \mathcal{V}_c \end{bmatrix} = \mathcal{R} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \phi \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{07}$$

Motor back EMF is given as,

$$e_a = w_r . \mathcal{K}_r\, \mathbb{f}(\theta_r) \tag{8}$$

$$e_b = w_r . \mathcal{K}_r\, \mathbb{f}(\theta_r - \tfrac{2\pi}{3}) \tag{9}$$

$$e_c = w_r . \mathcal{K}_r\, \mathbb{f}(\theta_r + \tfrac{2\pi}{3}) \tag{10}$$

Where w_r is angular velocity of the rotor, θ_r is a position of the rotor and $\mathcal{K}_r$ magnetic flux constant of the rotor. Torque generated by motor is given as

$$\mathbb{T}_e = \frac{e_a i_a + e_b i_b + e_c i_c}{w_r} \tag{11}$$

The torque generated by motor can also be described in the term of machine factors as

$$\mathbb{T}_e = \mathbb{T}_L + \mathcal{J}\frac{dw_r}{dt} + \mathfrak{B}\, w_r \tag{12}$$

Where $\mathfrak{B}$ is motor co-efficient of friction, $\mathcal{J}$ is inertia and $\mathbb{T}_L$ is loaded torque of the motor.

Since we are dealing with MAS, it is necessary to consider each motor model as the single autonomous agent working in coordination with other identical N agents and a virtual leader, defining the reference speed for followers. So, the complete state space model of BLDC motor, which is said to be dynamics of agent i, is given as

$$\dot{x}_i = Ax_i + Bu_i \tag{13}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ \dot{w}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} {-\mathcal{R}}/{\phi} & 0 & 0 & (\mathcal{K}_r\, \mathbb{f}(\theta_r))/\mathcal{J} & 0 \\ 0 & {-\mathcal{R}}/{\phi} & 0 & (\mathcal{K}_r\, \mathbb{f}(\theta_r - \frac{2\pi}{3}))/\mathcal{J} & 0 \\ 0 & 0 & {-\mathcal{R}}/{\phi} & \left(\mathcal{K}_r\, \mathbb{f}\left(\theta_r + \frac{2\pi}{3}\right)\right)/\mathcal{J} & 0 \\ (\mathcal{K}_r\, \mathbb{f}(\theta_r))/\mathcal{J} & (\mathcal{K}_r\, \mathbb{f}(\theta_r - \frac{2\pi}{3}))/\mathcal{J} & \left(\mathcal{K}_r\, \mathbb{f}\left(\theta_r + \frac{2\pi}{3}\right)\right)/\mathcal{J} & {-\mathfrak{B}}/{\mathcal{J}} & 0 \\ 0 & 0 & 0 & \mathcal{P}/2 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ w_r \\ \theta_r \end{bmatrix} + \begin{bmatrix} 1/\phi & 0 & 0 & 0 \\ 0 & 1/\phi & 0 & 0 \\ 0 & 0 & 1/\phi & 0 \\ 0 & 0 & 0 & -1/\mathcal{J} \end{bmatrix} \begin{bmatrix} \mathcal{V}_a \\ \mathcal{V}_b \\ \mathcal{V}_c \\ \mathbb{T}_L \end{bmatrix} \tag{14}$$

Motor parameters includes $50\, V\, dc$, $\mathcal{R} = 8\, \Omega$, $\phi = 13\, mH$, $\mathcal{J} = 0.25\, kg.cm^2$, $\mathbb{T}_L = 60\, Nm$ and $w_{ref} = 370\, rpm$

4. Consensus Algorithm for BLDC Motors

The aim of this work is to integrate MAS consensus protocol (leader following MAS), with BLDC motors model in such a way that the speed of all motors follows the reference speed, in this case, the reference speed is that of a leader (w_0). The proposed structure of communication is fixed and shown in figure 4. Since, there exists a communication structure to interconnect every agent, therefore it is necessary to consider the delay in the system induces by the network. So, the proposed consensus protocol is used in two different scenarios i-e with respect to delay and without delay. Furthermore, at any initial speed, the dynamics of error must hold the condition that $\lim_{t\to\infty} \varepsilon(t) = 0$ and $\varepsilon_i = w_i - w_0$ where $\varepsilon = [\varepsilon_1{}^T, \dots \varepsilon_N{}^T]^T$. So, the consensus protocol for speed follower i_{th} agent is given as

$$u_i(t) = \textstyle\sum_{j \in N_i} \left(w_i - w_j\right) + d_i(w_0 - w_i) \tag{15}$$

In case of communication induced delay, the consensus protocol is reformed as

$$u_i(t) = \textstyle\sum_{j \in N_i}\left(w_i(t-\tau) - w_j(t-\tau)\right) + d_i(w_0(t-\tau) - w_i(t-\tau)) \tag{16}$$

To reach consensus with delay, it is necessary that the delay must confine by $\pi/_{2\lambda_{max}}$ or there were zero encirclements of Nyquist plot at $-1/_{\lambda_k}, \forall k > 1$, as proved in [18]. Figure 2 shows the networked connected schematic model of the motors.

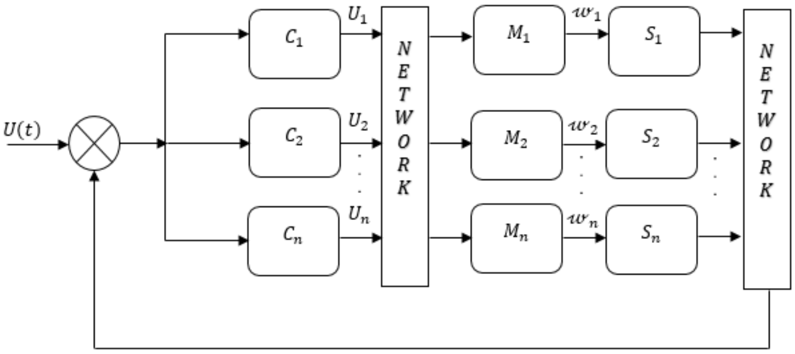

Figure 2. Schematic of Network Connected Motors

5. Controller Architecture

For complex systems, the adaptive control method is far most studied specifically Model Reference Adaptive Control (MRAC) systems such that actual output of the system must follow the model or reference system output. In [19], instantaneous and steady state powers are used do design MRAC controller to control the rotor speed of PM synchronous generator for sensor less operation of wind turbines. Moreover, in [20], the translational and rotational movement of Unmanned Aerial Vehicle (UAV) is controlled by hybrid controller composed of MRAC and RST controllers. Moreover, when it comes to addressing disturbances and parametric variations in the system, sliding mode control (SMC) gain more rapid attention particularly variable structure (VS). In [21], a SMC based ANN is deigned to control the stator voltage of IM thereby providing energy conservation. In this paper, a novel hybrid controller consisting of MRAC and VS controller is presented (VS-MRAC), to attain robust control of BLDC motor speed in the multi-motor system such that all motors follows the reference speed and drive the error between desired and actual speed to zero. The schematic of proposed controller is given by Figure 3.

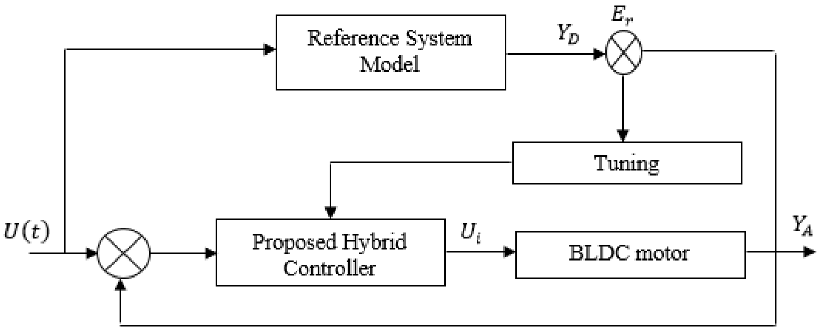

Figure 3. Proposed Controller Architecture

The desired response of the system is written as

$$\dot{X}_D(t) = A_D\, X_D(t) + B_D\, R \tag{17}$$

$$Y_D(t) = C_D\, X_D(t) \tag{18}$$

The actual system response is defined as

$$\dot{X}_A(t) = A_A\, X_A(t) + B_A\, U_A(t) \tag{19}$$

$$Y_A(t) = C_A\, X_A(t) \tag{20}$$

The error between actual and desired system responses are given as

$$E_r(t) = Y_A(t) - Y_D(t) \tag{21}$$

Likewise other MRAC schemes, the objective of proposed scheme is to force $Y_A(t)$ to follow the reference output $Y_D(t)$ such that the error gets zero. To accomplish such task, the control protocol is designed as

$$U_A(t) = \emptyset_1\, Y_A + \emptyset_2\, R \tag{22}$$

Where $\emptyset_1$ & $\emptyset_2$ are ideal controller parameters and can be found by the given equation

$$\frac{Y_A}{R} = \frac{Y_D}{R} \tag{23}$$

And the control protocol is redefined as

$$U_A(t) = {\emptyset_1}'\, Y_A + {\emptyset_2}'\, R \tag{24}$$

The actual and desired outputs are redefined as

$$\dot{Y}_A(t) = -A_A\, Y_A + K_A\, U_A \tag{25}$$

$$\dot{Y}_D(t) = -A_D\, Y_D + K_D\, R \tag{26}$$

By comparing equation (25 & 26) with equation (22), we get

$$U_A(t) = \emptyset_1\, Y_A + \emptyset_2\, R - {\emptyset_1}'\, Y_A - {\emptyset_2}'\, R + \emptyset_1\, Y_A + {\emptyset_2}'\, R$$

$$U_A(t) = (\emptyset_1 - {\emptyset_1}')\, Y_A + (\emptyset_2 - {\emptyset_2}')\, R + {\emptyset_1}'\, Y_A + {\emptyset_2}'\, R$$

$$U_A(t) = \overline{\emptyset_1}\, Y_A + \overline{\emptyset_2}\, R + {\emptyset_1}'\, Y_A + {\emptyset_2}'\, R \tag{27}$$

Put in equation (27) into equation (25) yields

$$\dot{Y}_A(t) = -A_A\, Y_A + B_A\, (\overline{\emptyset_1}\, Y_A + \overline{\emptyset_2}\, R + {\emptyset_1}'\, Y_A + {\emptyset_2}'\, R)$$

$$\dot{Y}_A(t) = -A_A\, Y_A + B_A\, (\overline{\emptyset_1}\, Y_A + \overline{\emptyset_2}\, R) + B_A\, {\emptyset_1}'\, Y_A + B_A\, {\emptyset_2}'\, R)$$

$$\dot{Y}_A(t) = -(A_A - B_A\, {\emptyset_1}')\, Y_A + B_A\, (\overline{\emptyset_1}\, Y_A + \overline{\emptyset_2}\, R) + B_A\, {\emptyset_2}'\, R \tag{28}$$

From equation (24), we have

$$R = \frac{U_A - {\emptyset_1}'\, Y_A}{{\emptyset_2}'} \tag{29}$$

By putting in Equation (27 and 29) into equation (26) gives

$$\dot{Y}_D(t) = -A_D\, Y_D + B_D\, R + \frac{B_D}{{\emptyset_2}'}\, (\overline{\emptyset_1}\, Y_A + \overline{\emptyset_2}\, R) \tag{30}$$

Now, by comparing equation (28) and (30), as required by condition of equation (23), yields desired parameters of the controller.

$${\emptyset_1}' = \frac{A_A - A_D}{B_A} \tag{31}$$

$${\emptyset_2}' = \frac{B_D}{B_A} \tag{32}$$

The parameters in equation (31) and (32) guarantees the output to follow the reference model. Now, the error time derivative is given as

$$\dot{E_r}(t) = \dot{Y_A}(t) - \dot{Y_D}(t)$$

$$\dot{E_r}(t) = -A_D Y_A + B_D R + \frac{B_D}{\emptyset_2'} \left(\overline{\emptyset_1} Y_A + \overline{\emptyset_2} R\right) - (-A_D Y_D + B_D R)\, \dot{E_r}(t)$$

$$= -A_D(Y_A - Y_D) + \frac{B_D}{\emptyset_2'} \left(\overline{\emptyset_1} Y_A + \overline{\emptyset_2} R\right)$$

$$\dot{E_r}(t) = -A_D E_r + \frac{B_D}{\emptyset_2'} \left(\overline{\emptyset_1} Y_A + \overline{\emptyset_2} R\right) \quad (33)$$

To ensure stability, taking Lyapunov candidate

$$V(E_r) = 0.5\, E_r^2 > 0 \quad (34)$$

$$V(\dot{E}_r) = E_r \dot{E}_r$$

$$V(\dot{E}_r) = \left[- A_D E_r + \frac{B_D}{\emptyset_2'} \left(\overline{\emptyset_1} Y_A + \overline{\emptyset_2} R\right)\right] E_r$$

$$V(\dot{E}_r) = -A_D E_r^2 + \frac{B_D}{\emptyset_2'} \left[(\emptyset_1 - \emptyset_1')\, E_r Y_A + (\emptyset_2 - \emptyset_2')\, E_r R\right] \quad (35)$$

Now, by using the law of switching, we gets

$$\emptyset_1 = -\widetilde{\emptyset_1}\, sgn\,(E_r Y_A) \quad (36)$$

$$\emptyset_2 = -\widetilde{\emptyset_2}\, sgn\,(E_r R) \quad (37)$$

$$V(\dot{E}_r) = -A_D E_r^2 + \frac{B_D}{\emptyset_2'}[(\widetilde{\emptyset_1} \mid E_r Y_A \mid + \emptyset_1' E_r Y_A) + (\emptyset_2 \mid E_r R \mid + \emptyset_2' E_r R)] \quad (38)$$

So, $V(\dot{E}_r) \leq -A_D E_r^2 < 0$, ensure $E_r = 0$, is globally asymptotically stable.

6. Results and Simulations

This part of the paper gives an insight of the theory that we have established in former sections by presenting simulation results and mathematical calculations. For simulation purpose, the multi-motor leader following system consist of four agents with same dynamics connected in fixed topology with one leader and three followers as shown in Figure 4. Now, by using the graph theoretical concept, the laplacian ($\mathcal{L}$) and leader adjacency (ξ), matrices w.r.t to the communication topology of Figure 3 are calculated and are given below thereby authenticating Lemma 2 and 3 with eigenvalue of laplacian matrix 0 (min), & 2 (max) respectively while the smallest positive eigenvalue of Ξ is 0.3820, thereby endorsing lemma 4 & 5.

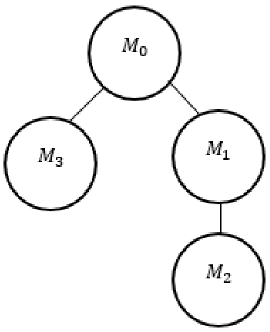

Figure 4. Fixed Communication Architecture

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \xi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Xi = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, for the first case, we take up the motors without considering the delay effects in the network as well as without introducing any sort of load to the motors. As given by Figure 5, the speed trajectories of following 3 agents under fixed topology efficiently track the speed trajectory of leading agent after minor dislocation. So, in such condition, it is said that the motors speed become synchronize and a consensus is reached in the multi-motor system using proposed methodology.

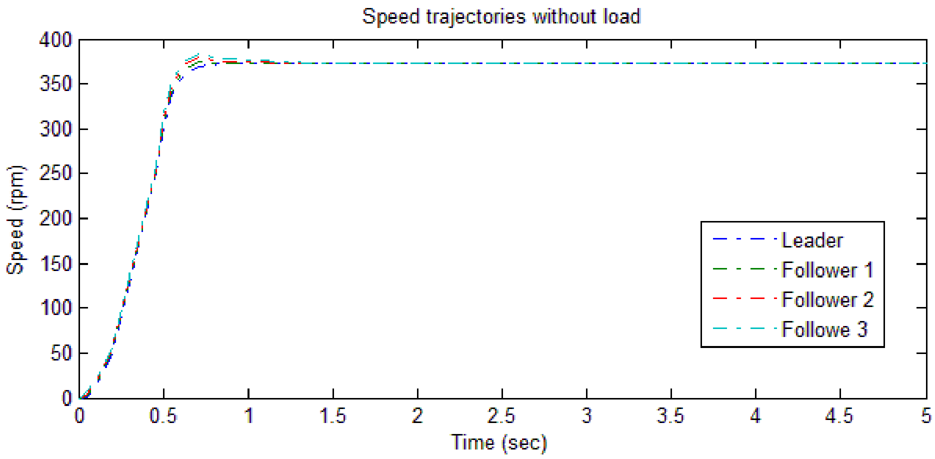

Figure 5. Leader & Followers Speed Trajectory Response without Applying Load to Motors

Moreover, for the second case, we again use the same scenario as that of an earlier case but with the introduction of load to the motors ($\mathbb{T}_L = 60\ Nm$). Once again, it is shown in Figure 6 that, the system reaches the consensus and the following agents effectively track the leader trajectory with minor dislocation before and after application of load.

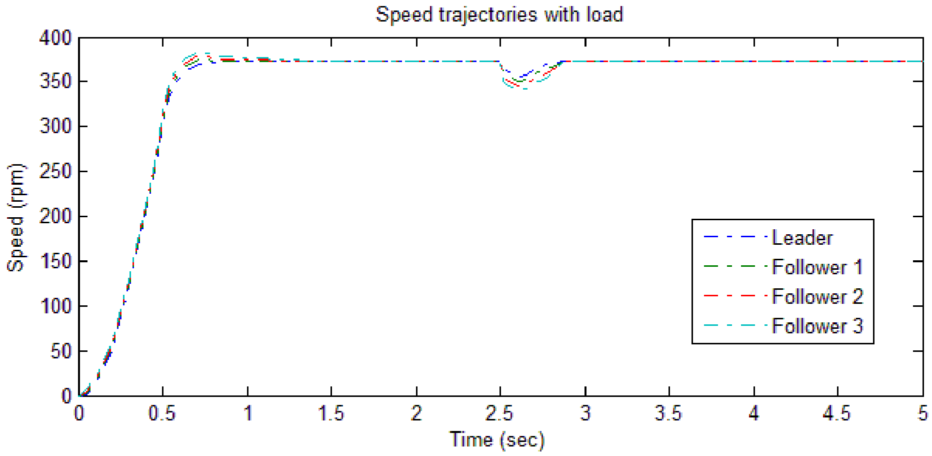

Figure 6. Leader & Followers Speed Trajectory Response with the Application of Load to Motors

Now for the third case, we simulated the proposed system under fixed topology, given above in Figure 4, by introducing a constant delay i-e $\tau = 0.01\ sec,$ in the system but didn't apply any load to the motors. On observing Figure 7, one can notice that the three following agents track the leader's trajectory effectively but start out with the slight delay. Once more, the proposed system design and controller gives an exceptionally sound performance and a consensus is reached successfully.

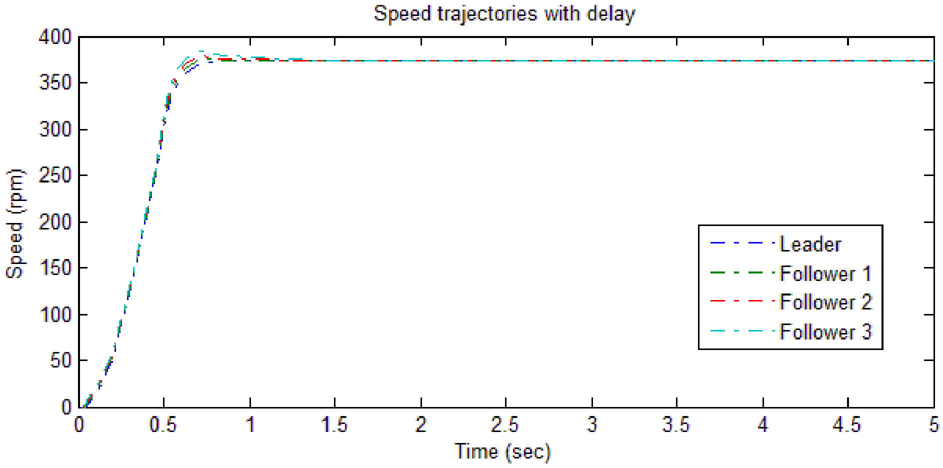

Figure 7. Leader & Followers Speed Trajectory Response with Delay

Finally, for the last case, we consider the system with same constant delay as well as with the same load as that of earlier cases. The system performance under such situation is given by Figure 8, and one can observe that system is still maintain its stability and reach consensus, obviously after minor distortion due to sudden application of the load to the motors.

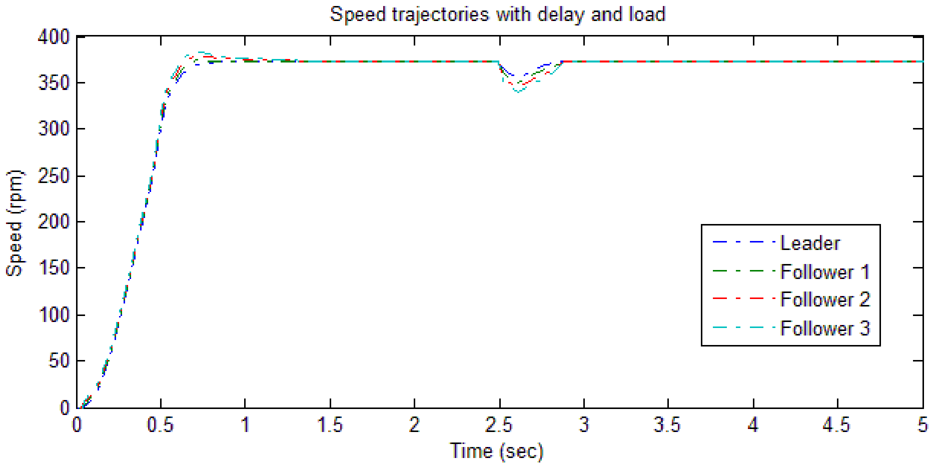

Figure 8. Leader & Followers Speed Trajectory Response with Delay & Load

Remark: The complete system responses are given by Figures 5-8 in various conditions that can probably encounter in synchronizing speed multi-motor system. The proposed work provides a novel approach of using the consensus protocol of leader following MAS along with VS-MRAC for speed tracking of BLDC motors.

7. Conclusion

This paper addresses the problem of synchronize speed in the multi-motor system with BLDC motors using leader following consensus protocol of MAS with advised hybrid controller. The complete system is modeled as a leader following MAS having predefined fixed communication pattern. The connectivity between leader and following agents (to ensure that exchange of information exists) are provided by using graph theoretical concepts. Moreover, system stability is ensured by lyapunov function. The paper not only addresses the aforementioned problem by considering the effects of delays in the network but also without delay (ideal case), besides with and without applying the load to the motors too. In any situation, it is proved that the proposed integration of hybrid controller with consensus algorithm of MAS along with specified communication pattern ensure synchronize BLDC motors speed in the multi-motor system. The proposed method is more flexible and robust for industrial applications using networks for motors connectivity. For future work, it is worthy to explore the system performance and stability in case of applying load randomly to selected motors rather applying to the entire system simultaneously.

Acknowledgement

This work is supported by National Natural Science Foundation of China under grant 61273114, the Innovation Program of Shanghai Municipal Education Commission under grant 14ZZ087, the Pujiang Talent Plan of Shanghai City China under grant 14PJ1403800, the International Corporation Project of Shanghai Science and Technology Commission under grants 14510722500, 15220710400.

References

[1] R .Nirali and S. K. Shah, “Multi Agent Implementation for Optimal Speed Control of Three Phase Induction Motor”, In Advances in Computing and Information Technology, Springer Berlin Heidelberg, **(2011)**, pp. 488-496.

[2] Y. Ke, Q. Chen, Y. Li, D. Huang and Y. Ge, “Leader-following control of multi-motor systems based on the consistency theory under the network environment”, In 2016 Chinese Control and Decision Conference (CCDC), IEEE, **(2016)**, pp. 2312-2318.

[3] M. Guinaldo, J. Sánchez and S. Dormido, “Distributed adaptive control of linear multi-agent systems with event-triggered communications”, Applied Mathematics and Computation, vol. 274, **(2016)**, pp. 195-207.

[4] F. Abdollahi and K. Khorasani, “A decentralized Markovian jump control routing strategy for mobile multi-agent networked systems”, Control Systems Technology, IEEE Transactions on, vol. 19, no. 2, **(2011)**, pp. 269-283.

[5] L. Fucai, X. Zhang, and L. Liu, “Synchronous control theory and practical study of multi-motor synchronous driving system”, Control Engineering of China, vol. 9, no. 4, **(2002)**, pp. 87-90.

[6] H.-H. Lee and U.-H. Jeong, “A study on speed synchronization for multi-motors using controller area network”, In Science and Technology, 2000. KORUS 2000, Proceedings. The 4th Korea-Russia International Symposium on, IEEE, vol. 2, **(2000)**, pp. 234-239.

[7] F. J. Pérez-Pinal, C. Nunez, and R. Alvarez, “Multi-motor synchronization technique applied in traction devices”, In IEEE International Conference on Electric Machines and Drives, vol. 2005, **(2005)**.

[8] K. Shahid and K. D. KumarSate “llite formation flying using variable structure model reference adaptive control”, Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, vol. 223, no. 3, **(2009)**,pp. 271-283.

[9] C.-J. Chien and L.-C. Fu, “Adaptive variable structure tracking control using neural network design”, In Control Conference (ECC), 1999 European, IEEE, **(1999)**, pp. 4359-4364.

[10] C.-W. Chang, C.-L. Chen, Y.-H. Chang and C. W. Tao, “Adaptive fuzzy sliding-mode formation control for second-order multi-agent systems”, In 2010 International Conference on System Science and Engineering, IEEE, **(2010)**, pp. 310-314.

[11] R. R. Nair and L. Behera, “Robust adaptive gain nonsingular fast terminal sliding mode control for spacecraft formation flying’, In 2015 54th IEEE Conference on Decision and Control (CDC), IEEE, **(2015)**, pp. 5314-5319.

[12] C. Godsil and G. F. Royle. “Algebraic graph theory”, Springer Science & Business Media, vol. 207, **(2013)**.

[13] Z. Li and Z. Duan, “Cooperative Control of Multi-Agent Systems: A Consensus Region Approach”, Automation and Control Engineering, CRC Press, **(2014)**.

[14] M. Mesbahi and M. Egerstedt, “Graph theoretic methods in multiagent networks”, Princeton University Press, **(2010)**.

[15] Y. Hong, J. Hu and L. Gao, “Tracking control for multi-agent consensus with an active leader and variable topology”, Automatica, vol. 42, **(2006)**, pp. 1177 – 1182.

[16] K. Peng and Y. Yang, “Leader-following consensus problem with a varying-velocity leader and time-varying delays”, Physica A: Statistical Mechanics and its Applications, vol. 388, **(2009)**, pp. 193 – 208.

[17] E. Anitha and P. Vairaprakash, “Design of dynamic states and digital speed control of BLDC motor”, In 2016 International Conference on Circuit, Power and Computing Technologies (ICCPCT), IEEE, **(2016)**, pp. 1-6,.

[18] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays”, Automatic Control, IEEE Transactions on, vol. 49, no. 9, **(2004)**, pp. 1520-1533.

[19] J. Gong and R. Xie, “A Novel Sensorless Control for Small Wind Turbines via a Speed Estimator Based on Model Reference Adaptive Controller”, International Journal of Control and Automation, vol. 9, no. 7, **(2016)**, pp. 159-168.

[20] Z. A. Ali, D.-B. Wang, R. Javed and A. Akbar, “Modeling & Controlling the Dynamics of Tri-rotor UAV Using Robust RST Controller with MRAC Adaptive Algorithm”, International Journal of Control and Automation, vol. 9, no. 3, **(2016)**, pp. 61-76.

[21] Kumar, R. Senthil and V. Ganapathy, “Implementation of a Sliding Mode Controller Trained ANN for Energy Conservation in Induction Motor”, International Journal of Control and Automation, vol. 9, no. 7 **(2016)**, pp. 221-234.