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An Improved T/2 Fractionally Spaced Blind Equalization Algorithm with Coordinate Transform

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Abstract

To address the slow convergence and large residual error problems in the T/2 fractionally spaced equalization with constant modulus algorithm (T/2-FSE-CMA) for high-order QAM signal in wireless communication, we analyze the distribution characters of 16QAM signal constellations and propose an improved T/2 fractionally spaced blind equalization with coordinate transform and constant modulus algorithm (T/2-FSE-RCTCMA). In this algorithm, we equalize the real and imaginary parts of the input signal in the fractionally spaced equalizer separately. By using the coordinate transform the output signals are mapped into the same circle and the error function of constant modulus is achieved. Using the error function to adjust the weight vector of each sub-channel in the fractionally spaced equalizer, we may avoid the miss-detection caused by the T/2 FSE-CMA algorithm in multi-modulus high-order QAM signal equalization. The simulation results in wireless channel show that compared with T/2-FSE-CTCMA and T/2- FSE-CMA, the proposed algorithm has a faster convergence speed and a smaller residual error.

***Keywords**: Blind Equalization, Underwater Acoustic Channel, Fractional Spaced, Coordinates Transform*

1. Introduction

In wireless communication, the blind equalization technique without training sequences is one of the main methods to remove Inter-Symbol Interference (ISI) [1]. The baud-spaced constant modulus equalizer [2] has a simple structure. But its convergence speed is slow, and the steady-state error is large. The Fractionally spaced Equalizer (FSE), whose tap space is the fraction of baud space, has advantages convergence speed and steady-state error in constant modulus signal equalization. H;owever, for the high order QAM signal (16QAM) distributed on several known radius circles, the signal modulus is not constant. In this case, using the FSE-CMA for equalization will cause large mean square errors and may not remove ISI thoroughly.

In this paper, based on the analysis of 16QAM signal constellation distribution characters and T/2 fractionally spaced equalizer, the real and imaginary parts of each sub-channel output signals are equalized separately. The coordinates transform is performed on each equalizer output channel [6], and the error function after coordinates transform is achieved. The cost function is defined using this error function, using the cost minimization method we can get the weight vector updating equation of the T/2 fractionally spaced equalizer. We then propose an improved T/2 fractionally spaced coordinate transform blind equalization algorithm (T/2-FSE-RCTCMA). Compared with T/2-FSE-CTCMA and T/2-FSE-CMA, the equalization ability in wireless channel has improved significantly.

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2. Fractionally Spaced Blind Equalization Algorithm

The fractionally spaced equalizer is based on the baud spaced equalizer, and it over-samples the channel output signal at a sampling rate larger than the baud rate. Studies show that fractionally spaced equalizer is equal to multi-path system model [7]. As shown in Figure 1, the system input and output have the same sampling rate.

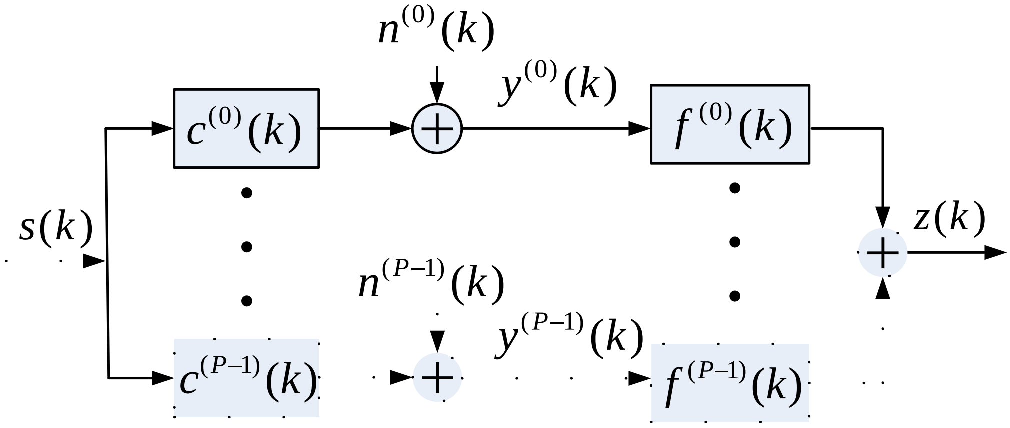


Figure 1. Structure of Fractionally Spaced Blind Equalizer

In Figure 1, $s(k)$ is the transmitted signal sequence with symbol duration T. $\boldsymbol{c}^{(i)}(k)(i=0,1\cdots P-1)$ is the impulse response of sub-channel. P is the fractionally spaced sampling factor. The impulse response of i-th channel is $\boldsymbol{c}^{(i)}(k)=c[(k+1)P-i-1]$. $\boldsymbol{n}^{(i)}(k)$ is the additional noise in the sub-channel. $\boldsymbol{y}^{(i)}(k)$ is the input signal of blind equalizer, which satisfies:

$$\boldsymbol{y}^{(i)}(k)=\sum_{j=0}^{N_c-1} s(j)\cdot c^{(i)}(j)+n^{(i)}(k) \tag{1}$$

where N_c is the length of baud spaced channel impulse response.

$\boldsymbol{f}^{(i)}(k)$ is the weight vector of equalizer, it satisfies:

$$\boldsymbol{f}^{(i)}(k+1)=\boldsymbol{f}^{(i)}(k)+\mu z^{(i)}(k)e(k)\boldsymbol{y}^{(i)*}(k)\ (i=0,\cdots P-1) \tag{2}$$

where μ is the step length, $e(n)=R_2-|z(k)|^2$ denotes the error, and the signal modulus is $R_2=\mathrm{E}\{|s(k)|^4\}/\mathrm{E}\{|s(k)|^2\}$.

The output of the equalizer is:

$$\begin{aligned} z(k)&=\sum_{i=0}^{P-1}\boldsymbol{f}^{(i)}(k)*\boldsymbol{y}^{(P-i-1)}(k)\\ &=\sum_{i=0}^{P-1}\boldsymbol{f}^{(i)}(k)*[\boldsymbol{s}(k)*\boldsymbol{c}^{(P-i-1)}(k)+\boldsymbol{n}^{(P-i-1)}(k)] \end{aligned} \tag{3}$$

The fractionally spaced blind equalization algorithm (T/2- FSE-CMA) only fits the constant modulus signal, and it may bring a large mean square error for multi-modulus signal equalization.

3. Improved T/2 Fractionally Spaced Coordinates Transform Blind Equalization Algorithm

When the transmitted signal is high-order signal, to improve the equalization result of fractionally spaced blind equalization algorithm, we make two modifications on the equalizer in Figure 1. First, equalize the real and imaginary parts of the input signal separately. It is equal to perform equalization on real signal, compared with complex signal, the calculation cost is decreased. Second, the coordinates transform is performed on the real and imaginary parts of the output signal separately, and two error functions after coordinates transform are achieved. The cost function is defined using the error functions, using the cost minimization method we can get the weight vector updating equation. The principles of the improved algorithm are shown in Figure 2. The proposed algorithm improves the equalization ability of multi-modulus signals and it also brings a faster convergence speed and a smaller steady-state error. It compensates the drawbacks of fractionally spaced constant modulus equalizer in multi-modulus signal equation.

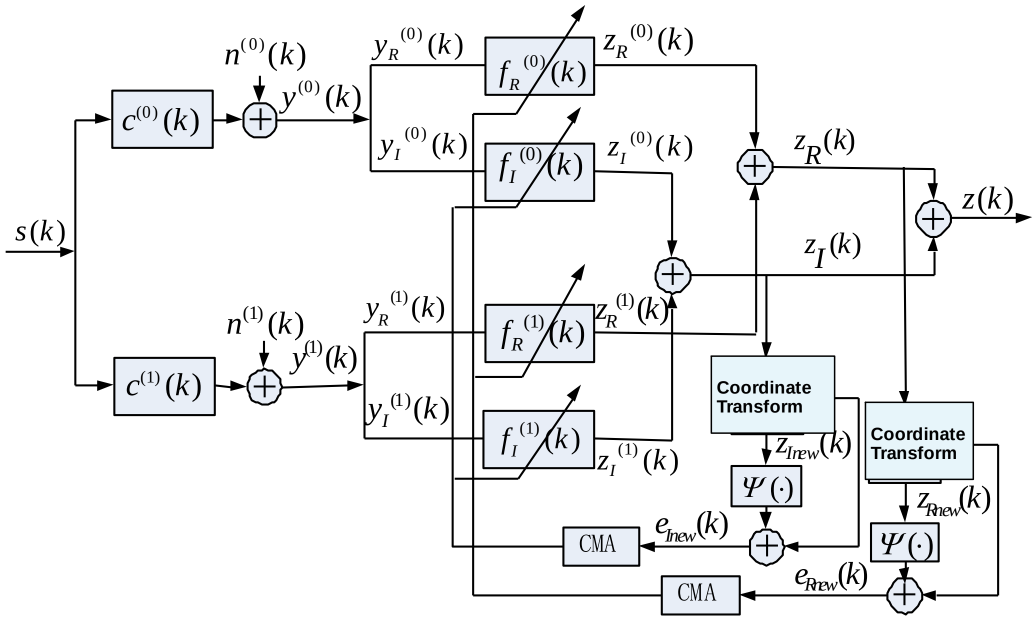


Figure 2. An Improved T/2 Fractionally Spaced Blind Equalization Algorithm with Coordinate Transform

For T/2 fractionally spaced equalizer, the oversampling factor is set to $P = 2$. Based on the channel system model in Figure 1, the channel is divided into odd sub-channel $c^{(0)}(k)$ and even sub-channel $c^{(1)}(k)$. In Figure 2, the real part and the imaginary part of the input signal $y^{(0)}(k)$ and $y^{(1)}(k)$ are separated for equalization in the improved algorithm. The input signal is represented as:

$$y^{(0)}(k) = y_R^{(0)}(k) + jy_I^{(0)}(k) \tag{4}$$

$$y^{(1)}(k) = y_R^{(1)}(k) + jy_I^{(1)}(k) \tag{5}$$

Weight vectors of each signal are $f_R^{(0)}(k)$, $f_I^{(0)}(k)$, $f_R^{(1)}(k)$ and $f_I^{(1)}(k)$. The output signals of the equalizer are: $z_R^{(0)}(k)$, $z_I^{(0)}(k)$, $z_R^{(1)}(k)$ and $z_I^{(1)}(k)$. The real part of the final output signal is represented as:

$$z_R(k) = z_R^{(0)}(k) + z_R^{(1)}(k) \tag{6}$$

The imaginary part is represented as:

$$z_I(k) = z_I^{(0)}(k) + z_I^{(1)}(k) \tag{7}$$

The final output signal can be written as:

$$z(k) = z_R(k) + jz_I(k) \tag{8}$$

We introduce the coordinate transform to the equalization process, and the principle is demonstrated in Figure 3.

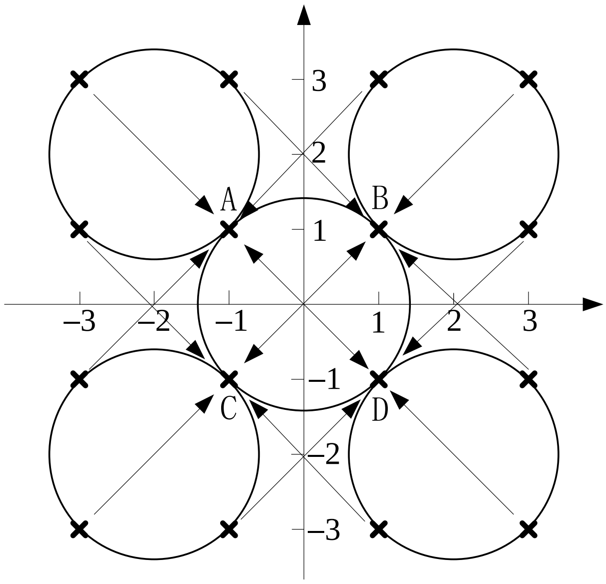


Figure 3. 16QAM Coordinate Transform Principles

In Figure 3, "×" denotes the ideal 16QAM signal points after equalization, which are distributed on four known circles. Points A, B, C and D are four points after coordinate transform, which are located on the same circle. When the constant modulus algorithm is used for equalization, the error function is $e(n) = R_2 - |z(k)|^2$. R_2 is a specific value, and when the signal is fully equalized, the error is not zero. The equalization result is not idea. Using coordinate transform, we can map the 16QAM signal points to points A, B, C and D. When the signal is fully equalized, the difference between the signal modulus and the equalized signal is zero, and the error is also zero. The equalization result is optimized.

In Figure 2, after coordinates transform, the errors of the real and imaginary parts are $e_{Rnew}(k)$ and $e_{Inew}(k)$:

$$\begin{aligned} e_{Rnew}(k) &= R_{Rnew}^2 - |z_{Rnew}|^2 \\ e_{Inew}(k) &= R_{Inew}^2 - |z_{Inew}|^2 \end{aligned} \tag{9}$$

where we have:

$$\begin{aligned} z_{Rnew}(k) &= z_R(k) - 2\text{sign}[z_R(k)] \\ z_{Inew}(k) &= z_I(k) - 2\text{sign}[z_I(k)] \end{aligned} \tag{10}$$

$$R_{Rnew}^2 = \frac{E\{|[s_R(k) - 2\text{sign}[s_R(k)]]}{E\{|[s_R(k) - 2\text{sign}[s_R(k)]]}$$

$$R_{Inew}^2 = \frac{E\{|[s_I(k) - 2\text{sign}[s_I(k)]]}{E\{|[s_I(k) - 2\text{sign}[s_I(k)]]} \tag{11}$$

The weigh vector updating equation is:

$$\boldsymbol{f}_R^{(i)}(k+1)=\boldsymbol{f}_R^{(i)}(k)+\mu z_R^{(i)}(k)e_{Rnew}(k)\boldsymbol{y}_R^{(i)*}(k)\ (i=0,1) \tag{12}$$

$$\boldsymbol{f}_I^{(i)}(k+1)=\boldsymbol{f}_I^{(i)}(k)+\mu z_I^{(i)}(k)e_{Inew}(k)\boldsymbol{y}_I^{(i)*}(k)\ (i=0,1) \tag{13}$$

The output of the equalizer is:

$$\begin{aligned} z(k) &= z_R(k)+jz_I(k) \\ &= \sum_{i=0}^{P-1} f_R^{(i)}(k)\cdot y_R^{(i)}(k)+j\sum_{i=0}^{P-1} f_I^{(i)}(k)\cdot y_P^{(i)}(k) \end{aligned} \tag{14}$$

For 16QAM signals, when the channel is fully equalized, Eq. (9) equals to zero. Eq. (4) to (14) give out the improved T/2 fractionally spaced blind equalization with coordinate transformation. In this paper, we name the algorithm that only do the coordinate transform on the output signals T/2-FSE-CTCMA. The improved algorithm T/2-FSE-RCTCMA performs equalization on real and imaginary parts separately, which is equal to the real signal. Therefore its calculation cost is smaller than that of T/2-FSE-CTCMA. Since the improved algorithm perform coordinate transform on the real and imaginary parts of the output signal separately, the convergence speed is faster and the steady-state error is smaller, compared with T/2-FSE-CTCMA and T/2-FSE-CMA.

4. Algorithm Performance Analysis

4.1. Convergence Analysis

T/2 fractionally spaced equalizer adopts a sampling rate of T/2, and it avoids the overlapping of the spectrum caused by under sampling. It also compensates the channel distortion [8]. The new algorithm performs equalization on the real and imaginary parts separately, which is equal to process real signal and the computational cost decreases. The signal is converted from multi-modulus to constant modulus by coordinate transform, therefore the steady-state error decreases and the convergence speed increases. For 16QAM, after separating the real and imaginary parts, it equals to two channels of 4PAM signals. After coordinate transform, it is converted to 2PAM, and the updating of weigh coefficients is faster. When the channel is fully equalized, error function converges to zero. Therefore, the equalization speed is significantly faster, the convergence speed is also faster, and the steady-state error is smaller. We verified the advantages of this new algorithm in wireless simulation experiments.

4.2. Analysis of Computation Load

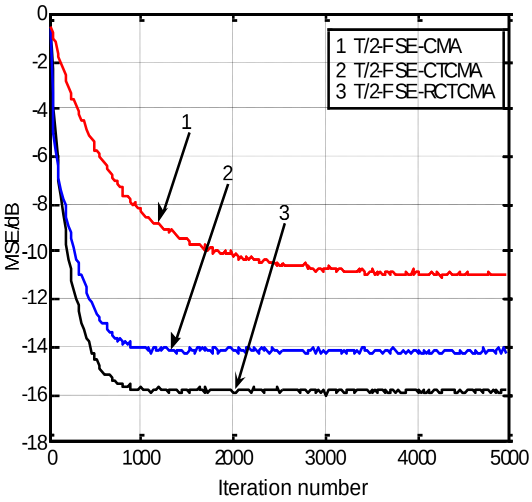
In each iteration of weight vectors in T/2-FSE-CMA, there are $(N_f/2)*4$ multiplications and $(N_f/2)*3+[(N_f/2)-1]$ additions for each channel of signal. In T/2 FSE-CTCMA, where the real and imaginary parts are separated, there are N_f-2 multiplications and $(N_f/2)-1$ additions for the real parts in each iteration. The total computation load for one channel of signal includes N_f multiplications and N_f-2 additions. Based on the above analysis, the computation load of T/2-FSE-CTCMA is almost half of the T/2-FSE-CMA. Meanwhile, the simulation results also indicate that the equalization performance of the proposed algorithm is better than that of T/2-FSE-CMA.

5. Simulation Experiments

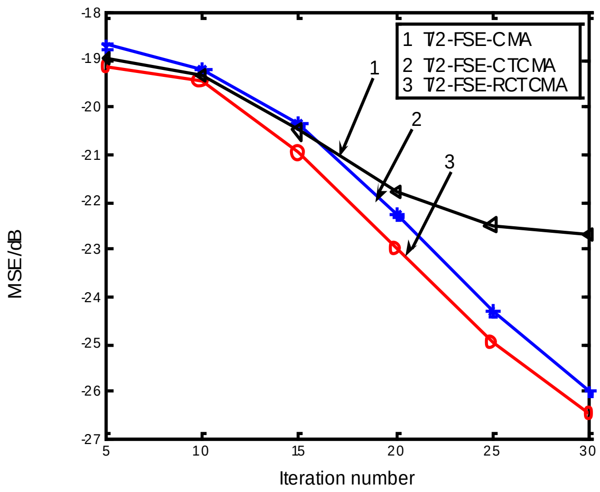
In order to verify the efficiency of T/2-FSE-RCTCMA algorithm, we compared with T/2-FSE-CTCMA and T/2-FSE-CMA in wireless channel simulation experiments.

[Experiment 1] Use the mixed phase wireless channel [9].

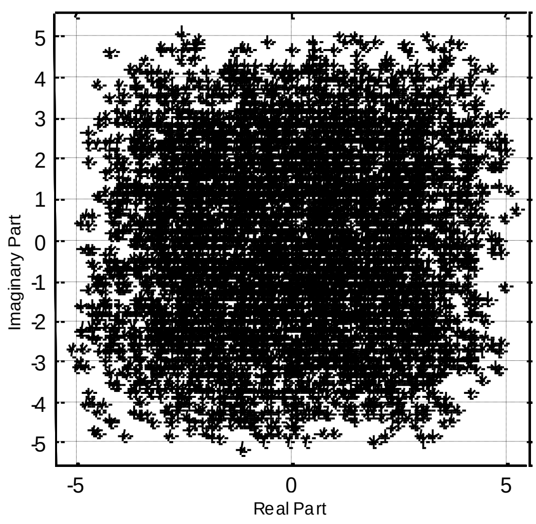
$c=[0.3132\ -0.1040\ 0.8908\ 0.3134]$, and the transmitted signal is 16QAM. The weight vector length of the equalizer is 32, SNR is 25dB. The weight vector length of each sub-channel is 16, and weight vector is initialized as central tap. The steps of three algorithms ($\mu_{\text{T/2-FSE}}$, $\mu_{\text{T/2-FSE-CTCMA}}$, $\mu_{\text{T/2-FSE-RCTCMA}}$) are set to value 0.000006 , 0.00003, and 0.0009 respectively. The 5000 Monte Carlo simulation results are shown in Figure 4.



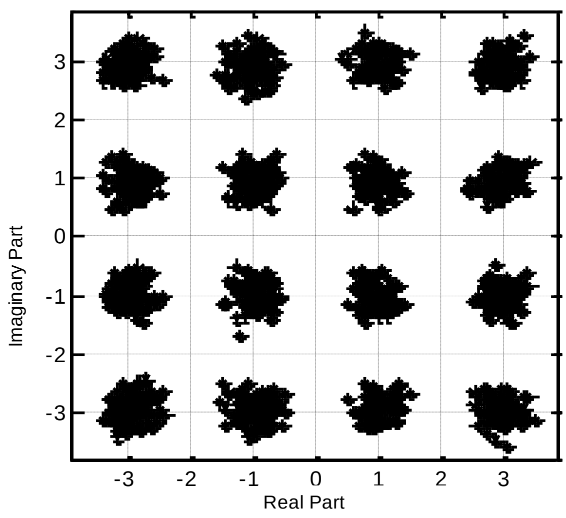
(a) Error curves



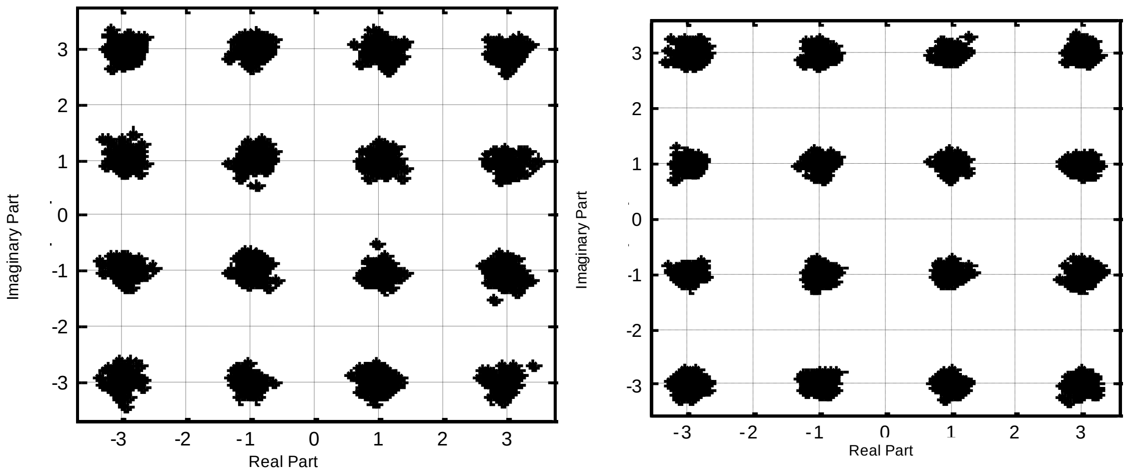
(b) Root mean square error curve



(c) Input signal of equalizer



(d) T/2-FSE-CMA constellation



(e) T/2-FSE-CTCMA constellation (f) T/2-FSE-RCTCMA constellation

Figure 4. Simulation Results

In Figure 4(a), the simulation results show that the steady-state error of T/2-FSE-RCTCMA is 2 dB smaller than that of T/2-FSE-CTCMA, and 5dB smaller than that of T/2-FSE-CMA. In convergence speed T/2-FSE-RCTCMA is the fastest one, 2000 step ahead of T/2-FSE-CMA. In Figure 4(b), under various SNRs, the root mean square error of T/2-FSE-RCTCMA is the smallest. The constellations in Figure 4 (d), (e) and (f) further proved that T/2-FSE-RCTCMA outperforms T/2 FSE-CMA. The signal constellation of T/2-FSE-RCTCMA is clearer and brings higher anti-ISI ability.

6. Conclusion

In this paper we propose the T/2 fractionally spaced blind equalization algorithm with coordinate transform (T/2-FSE-RCTCMA). Its computation load is reduced by almost fifty percent and it shows a clear advantage for 16QAM signal equalization. The wireless channel simulation results show that: compared with T/2-FSE-CTCMA and T/2-FSE-CMA, the algorithm has a faster convergence speed and smaller residual error. The effect on its constellation is quite obvious. Therefore, this algorithm may efficiently removing ISI and recover signal in real time.

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