

Peristaltic Hemodynamic Jeffery Fluid through a Tapered Channel with Heat and Mass Transfer under the Influence of Radiation –Blood Flow Model

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Abstract

In this paper, impact of heat and mass transfer on peristaltic hemodynamic Jeffery fluid model through a tapered vertical channel under the influence of radiation is studied under long wavelength and low Reynolds number assumptions are investigated. Closed form expressions for the axial velocity pressure gradient, temperature, heat transfer coefficient and Concentration distribution (Mass transfer distribution) on the channel walls have been computed numerically. In addition, it has been illustrated graphically for significant various parameters such as, Radiation parameter, Prandtl number, heat source/sink parameter, Soret number, Schmidt number and Jeffery fluid parameter.

Keywords: Heat transfer, Mass transfer, MHD, Hemodynamic fluid, tapered channel

1. Introduction

The peristaltic flow of a fluid is very significant for medical analysis and it has many clinical applications. These applications are swallowing of food bolus through the esophagus, the urine transport from a kidney to the bladder, the movement of chime in the tract, the transport of lymph in the lymphatic vessels and the vasomotion of small blood vessel. The flow of non-Newtonian fluids also has advantage to the field environmental engineering, chemical and biomedical. Furthermore, the peristaltic pump is found in many applications of medicine, engineering and water waste. In addition, peristaltic pumping occurs in numerous practical applications involving biomechanical systems. Jaffrin and Shapiro [1] explained the basic principles of peristaltic pumping in a two dimensional channel and brought out clearly the significance of the various parameters governing the flow. M. J. Manton [2] studied on peristaltic transport with long wavelength at low Reynolds number. Influence of induced magnetic field and heat transfer on the peristaltic motion of a Jeffrey fluid in an asymmetric channel: closed form solutions by S Safia Akramet al. [3]. Peristaltic flow of a Jeffrey fluid under the effect of radially varying magnetic field in a tube with an endoscope examined by A. M. Abd-Alla *et al.* [4]. Peristaltic Flow of Phan-Thien-Tanner Fluid in an Asymmetric Channel with Porous Medium discussed by Vajravelu *et al.* [5].

Influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls studied by Srinivas *et al.* [6]. Srinivas *et al.* [7] investigated the peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. In another attempt, K. Vajravelu *et al.* [8] discussed on Peristaltic transport of a conducting Jeffrey fluid in an inclined asymmetric channel.

Hayat, T. *et al.* [9] investigated the Newtonian and Joule Heating Effects in Two-Dimensional Flow of Williamson Fluid. Partial Slip Consequences on Peristaltic Transport of Williamson Fluid in an Asymmetric Channel discussed by Safia AKRAM *et al.* [10]. Heat Transfer in MHD Squeezing Flow using Brinkman Model by Satish Chandra RAJVANSHI and Sargam WASU [11]. Some more works on this topic are cited in (S. Nadeem and Noreen Sher Akbar [12], T. Hayat *et al.*[13], K. Vajravelu *et al.*[14], S. Srinivas and M. Kothandapani [15], T. Hayat *et al.*[16], O. U. Mehmood *et al.*[17], Musharafa Saleem and Aun Haider [18], G. Radhakrishnamacharya, Ch. Srinivasulu [19], K. Vajravelu *et al.*[20], K. Ramesh, M. Devakar [21], M Kothandapani *et al.*[22] and Sk Abzal *et al.* [23]).

2. Formulation of the Problem

The model simulates the peristaltic transport of a viscous fluid through an infinite two-dimensional asymmetric vertical tapered channel through the porous medium. Asymmetry in the flow is due to the propagation of peristaltic waves of different amplitudes and phase on the channel walls. We assume that the fluid is subject to a constant transverse magnetic field B_0 . The flow is generated by sinusoidal wave trains propagating with steady speed c along the tapered asymmetric channel walls.

The geometry of the wall surface is defined as

$$Y = \overline{H}_2 = b + m' \overline{X} + d \sin \left[\frac{2\pi}{\lambda} (\overline{X} - ct) \right] \quad (2.1)$$

$$Y = \overline{H}_1 = -b - m' \overline{X} - d \sin \left[\frac{2\pi}{\lambda} (\overline{X} - ct) + \phi \right] \quad (2.2)$$

Where b is the half-width of the channel, d is the wave amplitude, c is the phase speed of the wave and m' ($m' \ll 1$) is the non-uniform parameter, λ is the wavelength, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and further b , d and ϕ satisfy the following conditions for the divergent channel at the inlet $d \cos \left(\frac{\phi}{2} \right) \leq b$.

It is assumed that the left wall of the channel is maintained at temperature T_0 while the right wall has temperature T_1 .

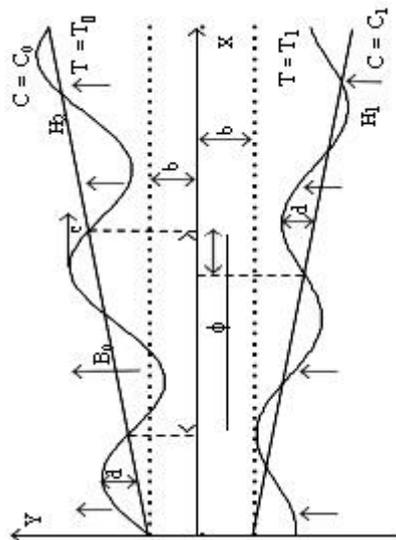


Figure 1. Schematic Diagram of the Physical Model

The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\bar{I} + \bar{S} \quad (2.3)$$

$$\bar{S} = \frac{\mu}{1 + \lambda_1(\bar{r} + \lambda_2\dot{\bar{r}})} \quad (2.4)$$

where \bar{T} and \bar{S} are Cauchy stress tensor and extra stress tensor, respectively, \bar{p} is the pressure, \bar{I} is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time $\dot{\bar{r}}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.

In laboratory frame, the equations of continuity, momentum, energy and concentration are described as follows

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad (2.5)$$

$$\rho \left[\bar{U} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{Y}} \right] = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{X}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} \quad (2.6)$$

$$\rho \left[\bar{U} \frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{V}}{\partial \bar{Y}} \right] = -\frac{\partial \bar{P}}{\partial \bar{Y}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} \quad (2.7)$$

$$\rho C_p \left[\bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{T} = k \left[\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2} \right] \bar{T} + Q_0 - \frac{\partial q}{\partial y} \quad (2.8)$$

$$\left[\bar{U} \frac{\partial \bar{C}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{C}}{\partial \bar{Y}} \right] = D_m \left[\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{Y}^2} \right] + \frac{D_m k_T}{T_m} \left[\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} \right] \quad (2.9)$$

Where

$$\bar{S}_{\bar{X}\bar{X}} = \frac{2\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(\bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right) \frac{\partial \bar{U}}{\partial \bar{X}}$$

$$\bar{S}_{\bar{X}\bar{Y}} = \frac{\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(\bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right) \left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}} \right)$$

$$\bar{S}_{\bar{Y}\bar{Y}} = \frac{2\mu}{1 + \lambda_1} \left(1 + \lambda_2 \left(\bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right) \frac{\partial \bar{V}}{\partial \bar{Y}}$$

\bar{U} and \bar{V} are the velocity components in the laboratory frame (\bar{X}, \bar{Y}) , k_1 is the permeability of the porous medium, ρ is the density of the fluid, p is the fluid pressure, k is the thermal conductivity, μ is the coefficient of the viscosity, Q_0 is the constant heat addition/absorption, C_p is the specific heat at constant pressure, σ is the electrical conductivity, g is the acceleration due to gravity \bar{T} is the temperature of the fluid, \bar{C} is the concentration of the fluid, T_m is the mean temperature, D_m is the coefficient of mass diffusivity, and K_T is the thermal diffusion ratio.

The relative boundary conditions are

$$\bar{U} = 0, \bar{T} = T_0, \bar{C} = C_0 \text{ at } \bar{Y} = \bar{H}_1$$

$$\bar{U} = 0, \bar{T} = T_1, \bar{C} = C_1 \text{ at } \bar{Y} = \bar{H}_2$$

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = \bar{X} - ct, y = \bar{Y}, \bar{u} = \bar{U} - c, \bar{v} = \bar{V}, \bar{p}(x) = \bar{P}(\bar{X}, t) \quad (2.10)$$

Where \bar{u} , \bar{v} are the velocity components in the wave frame (\bar{x}, \bar{y}) , \bar{p} is pressures and \bar{P} fixed frame of references. We introduce the following non-dimensional variables and

parameters for the flow:

$$\begin{aligned} x &= \frac{\bar{x}}{\lambda} & y &= \frac{\bar{y}}{b} & \bar{t} &= \frac{ct}{\lambda} & u &= \frac{\bar{u}}{c} & v &= \frac{\bar{v}}{c\delta} & S &= \frac{b\bar{S}}{\mu c} & h_1 &= \frac{\bar{H}_1}{b} & h_2 &= \frac{\bar{H}_2}{b} & p &= \frac{b^2\bar{p}}{c\lambda\mu} \\ \theta &= \frac{\bar{T}-T_0}{T_1-T_0} & \Theta &= \frac{\bar{C}-C_0}{C_1-C_0} & \delta &= \frac{b}{\lambda} & \text{Re} &= \frac{\rho cb}{\mu} & S_c &= \frac{\mu}{D_m\rho} & S_r &= \frac{D_m\rho k_T(T_1-T_0)}{\mu T_m(C_1-C_0)} \\ M &= B_0 b \sqrt{\frac{\sigma}{\mu}} & \text{Pr} &= \frac{\mu C_p}{k} & E_c &= \frac{c^2}{C_p(T_1-T_0)} & \beta &= \frac{Q_0 b^2}{\mu C_p(T_1-T_0)} & \varepsilon &= \frac{d}{b} \end{aligned} \quad (2.11)$$

where $\varepsilon = \frac{d}{b}$ is the non-dimensional amplitude of channel, $\delta = \frac{b}{\lambda}$ is the wave number,

$k_1 = \frac{\lambda m'}{b}$ is the non-uniform parameter, Re is the Reynolds number, M is the

Hartmann number, $K = \frac{k}{b^2}$ Permeability parameter, Pr is the Prandtl number, E_c is the

Eckert number, β is the heat source/sink parameter, $B_r (= E_c Pr)$ is the Brinkman number, S_c Schmidt number and S_r Soret number.

3. Solution of the Problem

In view of the above transformations (2.10) and non-dimensional variables (2.11), equations

(2.5-2.9) are reduced to the following forms.

$$\delta \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = 0 \quad (3.1)$$

$$\text{Re} \delta \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \left[-\frac{\partial p}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right] \quad (3.2)$$

$$\text{Re} \delta^3 \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \left[-\frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} \right] \quad (3.3)$$

$$\text{Re} \left[\delta u \frac{\partial \theta}{\partial x} + v \delta \frac{\partial \theta}{\partial y} \right] = \frac{1}{\text{Pr}} \left[\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] + \beta + \frac{N^2 \theta}{\text{Pr}} \quad (3.4)$$

$$\text{Re} \delta \left[u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} \right] = \frac{1}{S_c} \left[\delta^2 \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right] + S_r \left[\delta^2 \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right] \quad (3.5)$$

Where

$$S_{xx} = \frac{2\delta}{1+\lambda_1} \left(1 + \frac{\lambda_2 \delta c}{d} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right) \frac{\partial u}{\partial x}$$

$$S_{xy} = \frac{1}{1+\lambda_1} \left(1 + \frac{\lambda_2 \delta c}{d} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right) \left(\frac{\partial u}{\partial x} + \delta^2 \frac{\partial v}{\partial x} \right)$$

$$S_{yy} = \frac{2}{1+\lambda_1} \left(1 + \frac{\lambda_2 \delta c}{d} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right) \frac{\partial v}{\partial y}$$

Applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers. Equations (3.1-3.5) become

$$\frac{\partial^2 u}{\partial y^2} = (1 + \lambda_1) \frac{\partial p}{\partial x} \quad (3.6)$$

$$\frac{\partial p}{\partial y} = 0 \quad (3.7)$$

$$\frac{\partial^2 \theta}{\partial y^2} + N^2 \theta = -\beta \text{Pr} \quad (3.8)$$

$$\left[\frac{\partial^2 \Theta}{\partial y^2} \right] + S_c S_r \frac{\partial^2 \theta}{\partial y^2} = 0 \quad (3.9)$$

The relative boundary conditions in dimensionless form are given by

$$u = -1, \theta = 0, \Theta = 0 \text{ at } y = h_1 = -1 - k_1 x - \varepsilon \sin[2\pi(x-t) + \phi] \quad (3.10)$$

$$u = -1, \theta = 1, \Theta = 1 \text{ at } y = h_2 = 1 + k_1 x + \varepsilon \sin[2\pi(x-t)] \quad (3.11)$$

The solutions of velocity and temperature with subject to boundary conditions (3.10) and (3.11) are given by

$$u = A_1 + A_2 y + A y^2 \quad (3.12)$$

Where

$$A = \frac{(1 + \lambda_1)p}{2}$$

$$A_1 = - \left(1 + \left(\frac{A(h_2^2 - h_1^2)}{(h_1 - h_2)} \right) h_1 + A h_1^2 \right)$$

$$A_2 = \frac{A(h_2^2 - h_1^2)}{(h_1 - h_2)}$$

$$\theta = A_3 \text{Cos}[N y] + A_4 \text{Sin}[N y] - A_5 \quad (3.13)$$

Where

$$A_3 = \left(\frac{1}{\cos N h_1} \right) \left(A_5 + \left(\frac{(\cos[N h_1] + A_5(\cos[N h_1] - \cos[N h_2])) \text{Sin}[N h_1]}{\text{Sin}[N h_1] \cos[N h_2] - \text{Sin}[N h_2] \cos[N h_1]} \right) \right)$$

$$A_4 = \left(\frac{-\cos[N h_1] - A_5(\cos[N h_1] - \cos[N h_2])}{\text{Sin}[N h_1] \cos[N h_2] - \text{Sin}[N h_2] \cos[N h_1]} \right) \quad A_5 = \frac{\beta \text{Pr}}{N^2}$$

$$\Theta = A_{10} + A_{11} y + A_8 \text{Cos}[N y] + A_9 \text{Sin}[N y] \quad (3.14)$$

Where

$$A_6 = A_3 N^2 \quad A_7 = A_4 N^2 \quad A_8 = - \frac{S_c S_r A_6}{N^2} \quad A_9 = - \frac{S_c S_r A_7}{N^2}$$

$$A_{10} = - \left(\frac{h_1}{(h_1 - h_2)} \right) \left(-1 - A_8 [\text{Cos}[N h_1] - \text{Cos}[N h_2]] - A_9 \text{Sin}[N h_1] - \text{Sin}[N h_2] \right) - A_8 \text{Cos}[N h_1] - A_9 \text{Sin}[N h_1]$$

$$A_{11} = \left(\frac{1}{(h_1 - h_2)} \right) \left(-1 - A_8 [\text{Cos}[N h_1] - \text{Cos}[N h_2]] - A_9 \text{Sin}[N h_1] - \text{Sin}[N h_2] \right)$$

The coefficients of the heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ respectively, are given by

$$Zh_1 = \theta_y h_{1,x} \quad (3.15)$$

$$Zh_2 = \theta_y h_{2,x} \quad (3.16)$$

The solutions of the coefficient of heat transfer at $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \theta_y h_{1,x} = \left[\frac{-A_3 \sin[Ny]}{N} + \frac{A_4 \cos[Ny]}{N} \right] * [-2\pi \varepsilon \cos[2\pi(x-t) + \phi] - k_1] \quad (3.17)$$

$$Zh_2 = \theta_y h_{2,x} = \left[\frac{-A_3 \sin[Ny]}{N} + \frac{A_4 \cos[Ny]}{N} \right] * [2\pi \varepsilon \cos[2\pi(-t+x)] + k_1] \quad (3.18)$$

4. Numerical Results and Discussion

The main object of this investigation has been to study impact of heat and mass transfer on peristaltic Jeffery fluid model through a tapered asymmetric channel under the influence of radiation. The analytical expressions for velocity distribution, and temperature and heat transfer coefficient have been derived in the previous section. The numerical and computational results are discussed through the graphical illustration. Mathematica software is used to find out numerical results.

Figure 2 shows the variation of the axial velocity for different values of Jeffery fluid with being fixed $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = 0.5$. We observe from this figure that the axial velocity diminished when the increasing the values of Jeffery fluid parameter ($\lambda_1 = 0.1, 0.5, 1$). It can be observed from Figure 3 that when the non-uniform parameter increases ($k_1 = 0.1, 0.2, 0.3$) the velocity of the fluid decreases in the entire vertical tapered channel with fixed $\lambda_1 = 1$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = 0.5$. Figure 4 is drawn to study the effect of non-dimensional amplitude of channel (ε) on axial velocity distribution (u) with $\lambda_1 = 1$, $\phi = \pi/6$, $x = 0.6$, $t = 0.4$, $dp/dx = 0.5$. We notice that the velocity of the fluid diminished when increasing the values of non-dimensional amplitude of channel.

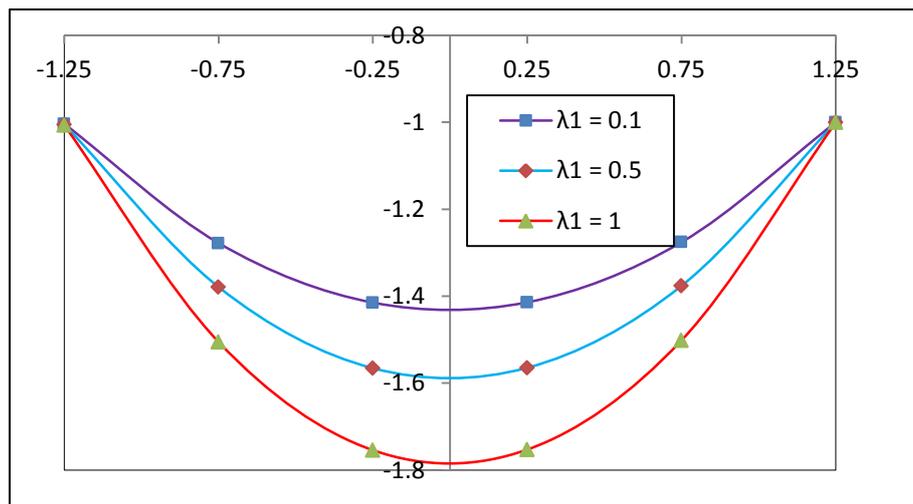


Figure 2. The Variation the Velocity Distribution (U) With Different Values of Λ_1 When $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = 0.5$

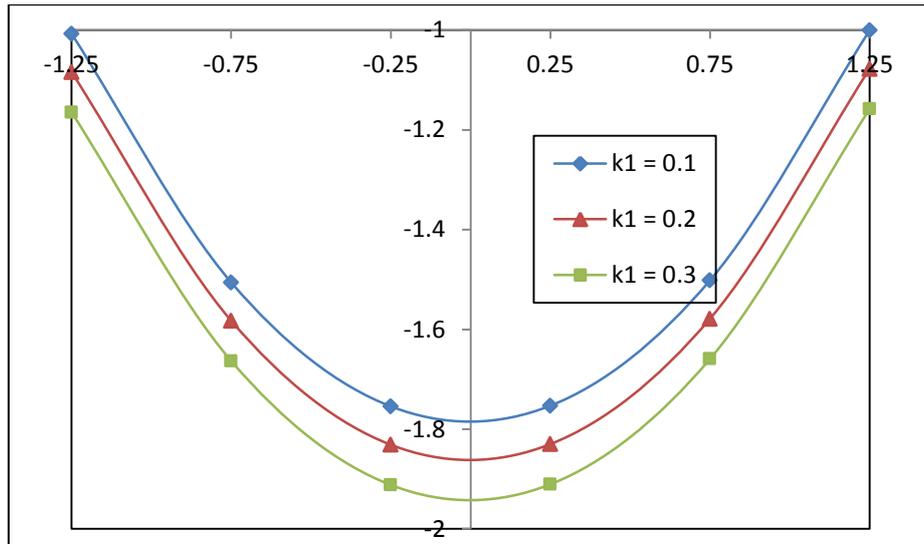


Figure 3. The Variation the Velocity Distribution (U) With Different Values Of Λ_1 When $\Lambda_1 = 1, \Phi = \pi/6, E = 0.2, K_1 = 0.1, X = 0.6, T = 0.4, Dp/Dx = 0.5$

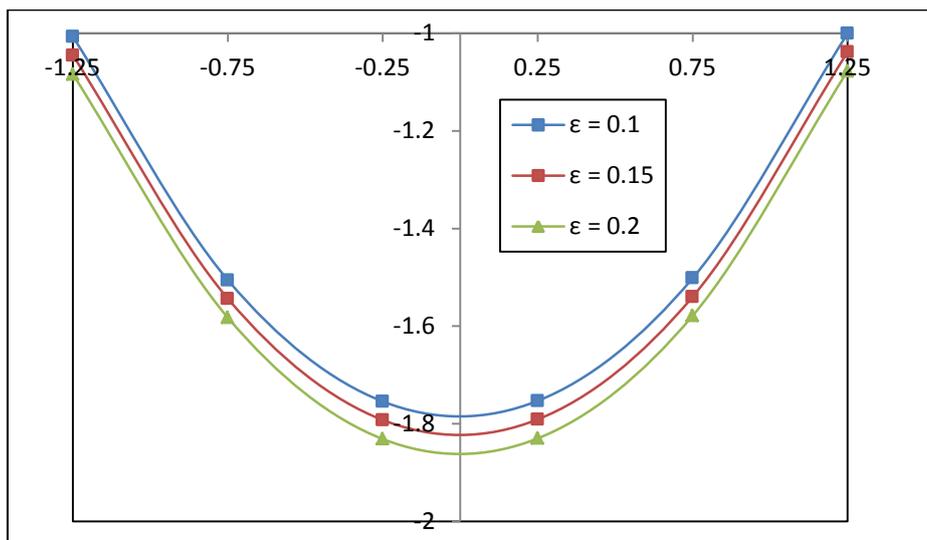


Figure 4. The Variation the Velocity Distribution (U) With Different Values Of Λ_1 When $\Lambda_1 = 1, \Phi = \pi/6, X = 0.6, T = 0.4, Dp/Dx = 0.5$.

Figure 5 is made to see the variation of temperature distribution (θ) for various values of radiation parameter (N) with fixed $\beta = 0.1, Pr = 2, \phi = \pi/6, \epsilon = 0.2, k_1 = 0.1, x = 0.6, t = 0.4$. We notice that the temperature of the fluid increases with increasing the values of radiation parameter. Figure 6 displays the influence of Prandtl number (Pr) on temperature profile. We examine that the temperature of the fluid enhances with increase in Prandtl number with fixed other parameters. The effect of heat source/sink parameter (β) on the temperature of the fluid (θ) is depicted in Figure 7. This figure shows that the results in temperature of the fluid enhances with increase in heat source/sink parameter ($\beta = 0.1, 0.2, 0.3$). Figure 8 depicts to examine the effect phase angle $\phi (\phi = \pi, \frac{\pi}{2}, \frac{\pi}{6})$ on the temperature of the fluid (θ) with $N = 0.3, Pr = 2, \beta = 0.1, \epsilon = 0.2, k_1 = 0.1, x = 0.6, t = 0.4$. This figure indicates that the temperature of the fluid enhances with decrease in phase

angle. Figure 9 presents the flow structure of temperature (θ) for different values of non-uniform parameter k_1 ($k_1=0.1, 0.2, 0.3$) with $N = 0.3$, $Pr = 2$, $\beta = 0.1$, $\varepsilon = 0.2$, $\phi = \pi/6$, $x = 0.6$, $t = 0.4$. We notice that the temperature of the fluid increases with increase in non-uniform parameter (k_1). Hence we conclude that the temperature of the fluid increases with an increase in radiation parameter, Prandtl number, heat generation parameter, non-uniform parameter and phase angle and also we notice that the temperature profile is found almost parabolic in nature.

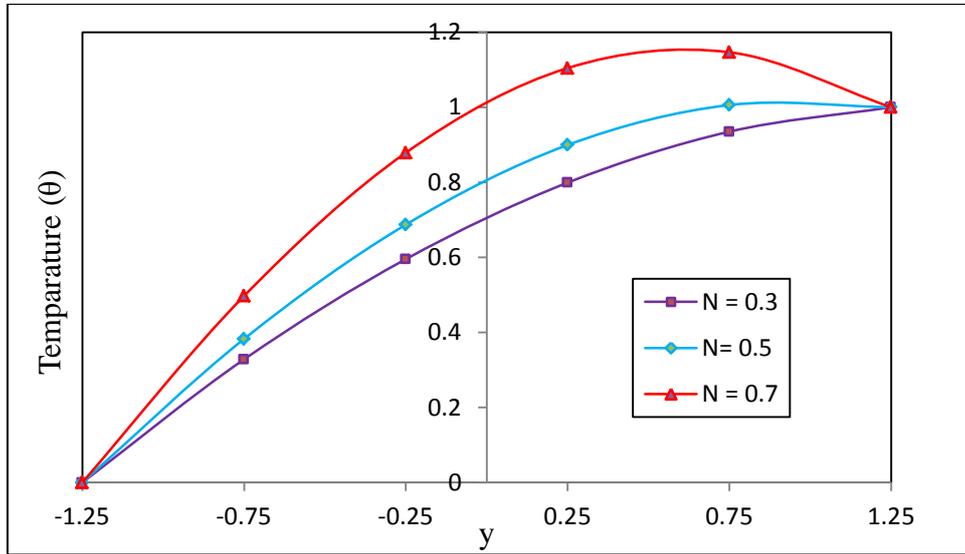


Figure 5. Temperature Distribution (θ) For Different Values of N with $B=0.1$, $Pr = 2$, $\Phi = \pi/6$, $E = 0.2$, $K_1= 0.1$, $X= 0.6$, $T = 0.4$

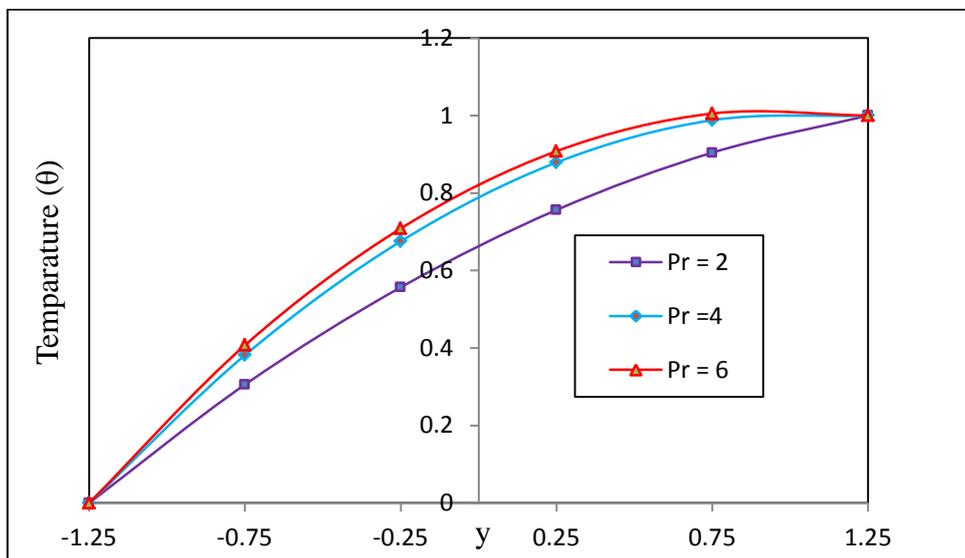


Figure 6. Temperature Distribution (θ) for Different Values of Pr with $\beta = 0.1$, $N = 0.3$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1= 0.1$, $x= 0.6$, $t = 0.4$

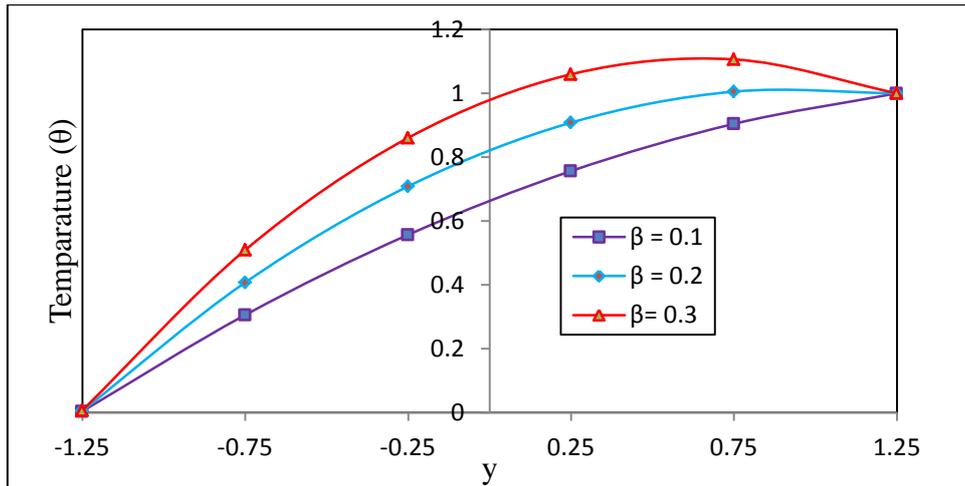


Figure 7. Temperature Distribution (θ) for Different Values of β with $N = 0.3$, $Pr = 2$, $\phi = \pi/6$, $\epsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

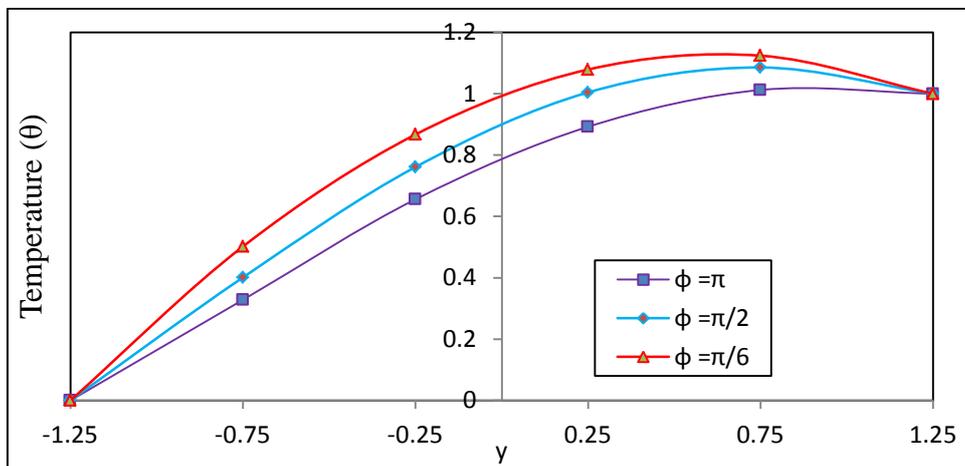


Figure 8. Temperature Distribution (θ) for Different Values of ϕ with $N = 0.3$, $Pr = 2$, $B = 0.1$, $E = 0.2$, $K_1 = 0.1$, $X = 0.6$, $T = 0.4$

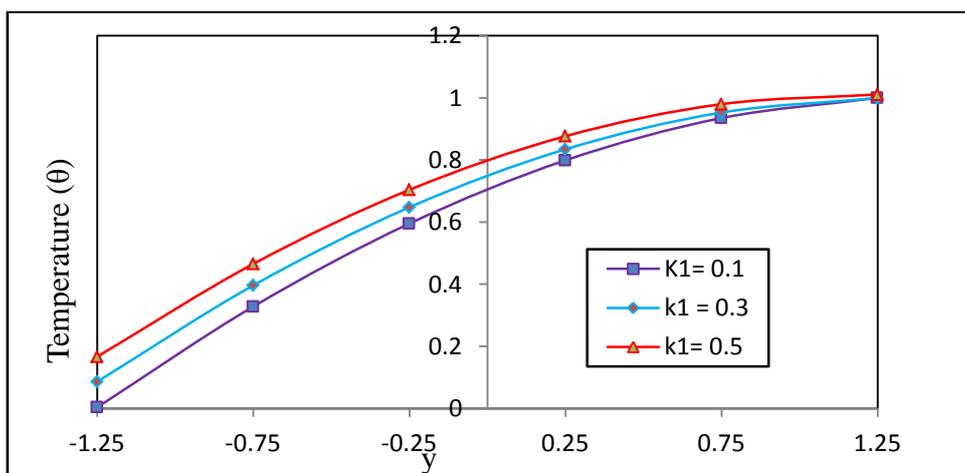


Figure 9. Temperature Distribution (θ) for Different Values of k_1 with $N = 0.3$, $Pr = 2$, $\beta = 0.1$, $\epsilon = 0.2$, $\phi = \pi/6$, $x = 0.6$, $t = 0.4$

Radiation parameter N on heat transfer coefficient at the wall $y = h_1$ is depicted in Figure 10 with fixed $\beta = 0.1$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. This figure shows that the heat transfer coefficient increases in the portion of the tapered channel $x \in [0, 0.05] \cup [0.55, 1]$ and decreases in the other portion of the tapered channel $x \in [0.05, 0.55]$ with increase in N ($N = 0.5, 0.7, 0.9$). Figure 11 shows the influence of Prandtl number (Pr) on heat transfer coefficient at the wall $y = h_1$ with $N = 0.5$, $\beta = 0.1$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. This figure reveals that the fluid in heat transfer coefficient increases in the portion of the channel $x \in [0, 0.05] \cup [0.55, 1]$ and diminishes in the other portion of the tapered channel $x \in [0.05, 0.55]$ with increase in Pr ($Pr = 2, 4, 6$). Effect of heat source/sink parameter β on heat transfer coefficient at the wall $y = h_1$ is presented in Figure 12 with $N = 0.5$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. It can be seen that the results in heat transfer coefficient enhances in the portion of the channel $x \in [0, 0.05] \cup [0.55, 1]$ and diminishes in the other portion of the tapered channel $x \in [0.05, 0.55]$ with increase in increase in heat source/sink parameter β . Figure 13 reveals the variation in heat transfer coefficient at the wall $y = h_1$ with non- uniform parameter k_1 with fixed $N = 0.5$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $\beta = 0.1$, $x = 0.6$, $t = 0.4$. It is interested to note the results in heat transfer coefficient decreases in the entire tapered channel $x \in [0, 1]$ with increase in non- uniform parameter k_1 ($k_1 = 0.1, 0.2, 0.3$).

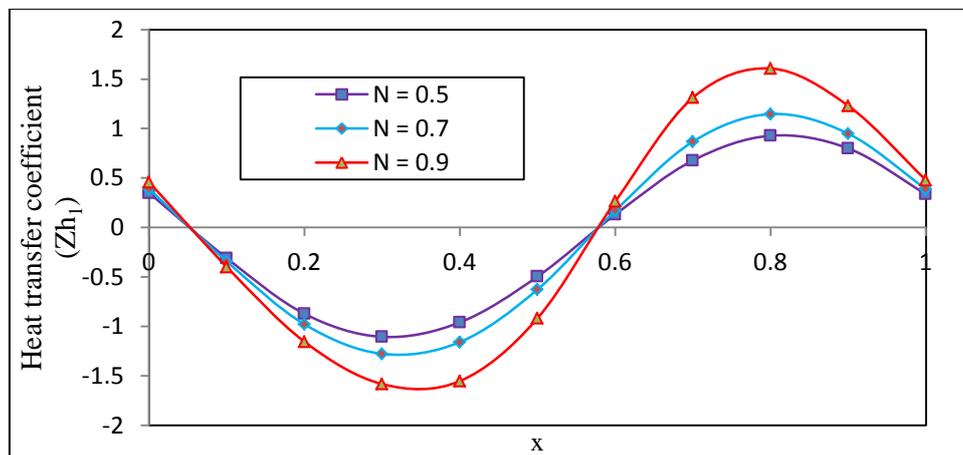


Figure 10. Heat Transfer Coefficient ($y = h_1$) for Different Values of N with fixed $\beta = 0.1$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

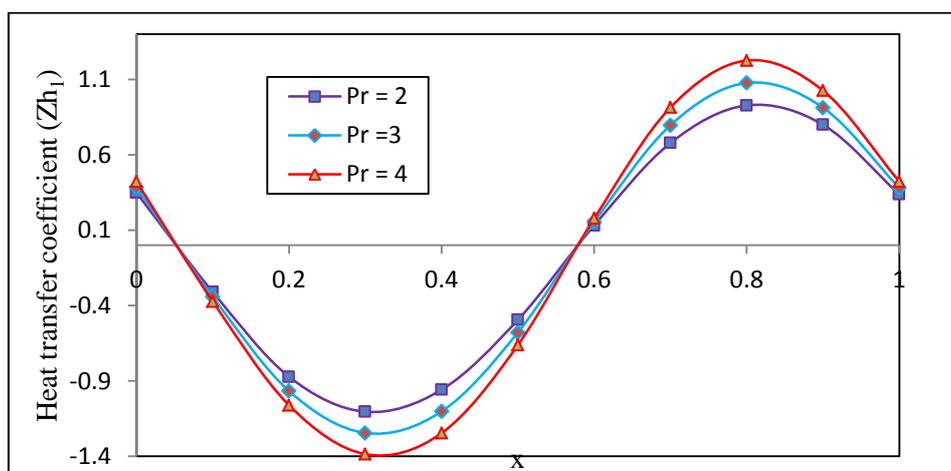


Figure 11. Heat Transfer Coefficient ($y = h_1$) for Different Values of Pr with fixed $N = 0.5$, $\beta = 0.1$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

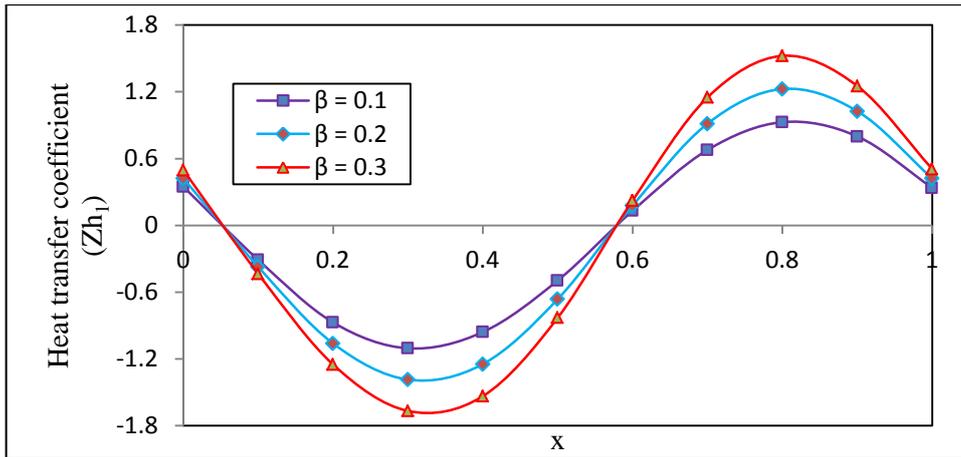


Figure 12. Heat Transfer Coefficient ($y = h_1$) for Different Values of β with Fixed $N= 0.5$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1= 0.1$, $x= 0.6$, $t = 0.4$

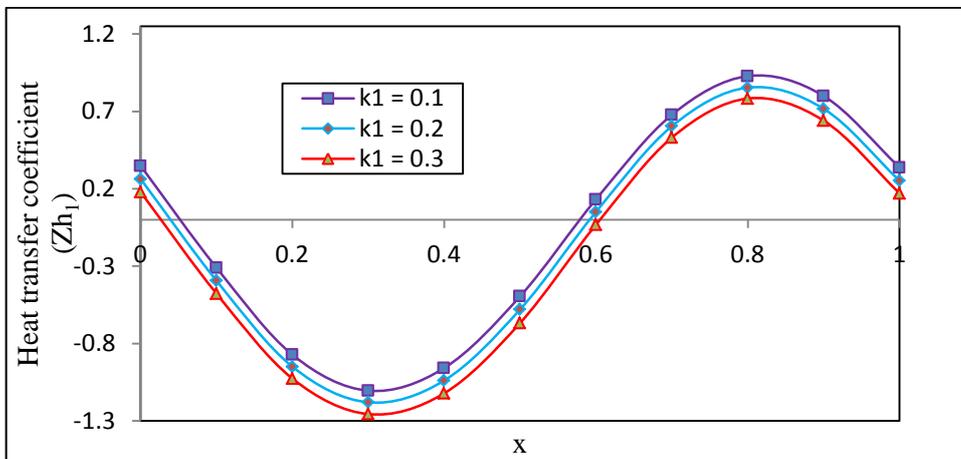


Figure 13. Heat Transfer Coefficient ($y = h_1$) for Different Values of k_1 with Fixed $N= 0.5$ $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $\beta = 0.1$, $x= 0.6$, $t = 0.4$

Figures (14) - (18) are plotted to study the effects of β , Pr , N , Sc and Sr on the concentration distribution. Figure 14 shows that the influence of Schmidt number (Sc) on concentration distribution (Φ) with $Sr = 2$, $Pr = 2$, $N = 0.5$, $\beta = 0.1$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. It can be seen from this figure that the results in concentration field decreases with increase in Schmidt number ($Sc = 0.3, 0.5, 0.7$). Figure 15 presents to examine the effect of Soret number (Sr) on concentration distribution (Φ) with $Sc = 0.3$, $Pr = 2$, $N = 0.5$, $\beta = 0.1$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. This figure reveals that the results in concentration field diminished with increase in Soret number ($Sr = 2, 4, 6$). Figure 16 reveals to examine the effect of source/sink parameter β on concentration distribution (Φ) with $Sr = 2$, $Sc = 0.3$, $Pr = 2$, $N = 0.5$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$. We observe from this figure that the results in concentration field decreases with increase in source/sink parameter β . Figures 17–18 show the variations of concentration distribution (Φ) for different values of Prandtl number (Pr) and Radiation parameter (N) being fixed other parameters. In both figures, it is clear that the results in concentration distribution decreases with increase in Prandtl number (Pr) and Radiation parameter (N). Finally, we conclude that the results concentration distribution decreases with increase in β , Pr , Sc , N and Sr in an entire channel.

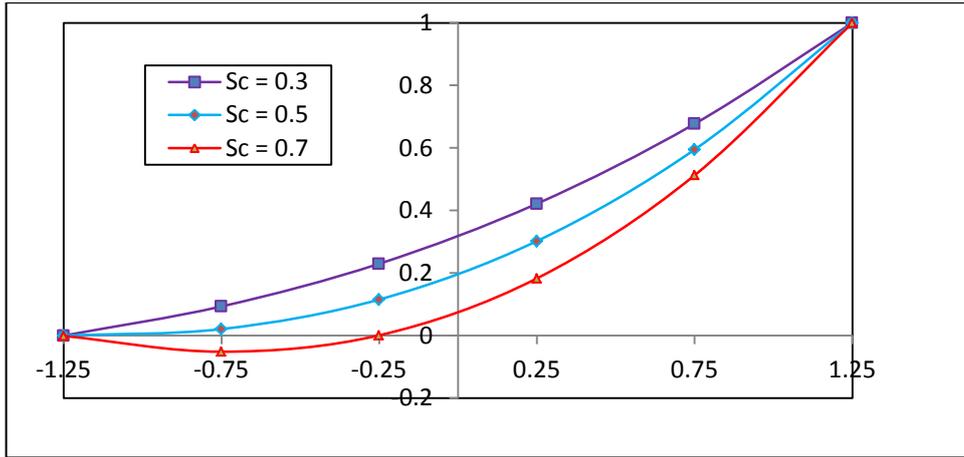


Figure 14. Concentration Distribution (Φ) for Different Values of Sc with $Sr = 2$, $Pr = 2$, $N = 0.5$, $\beta = 0.1$, $\phi = \pi/6$, $\epsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

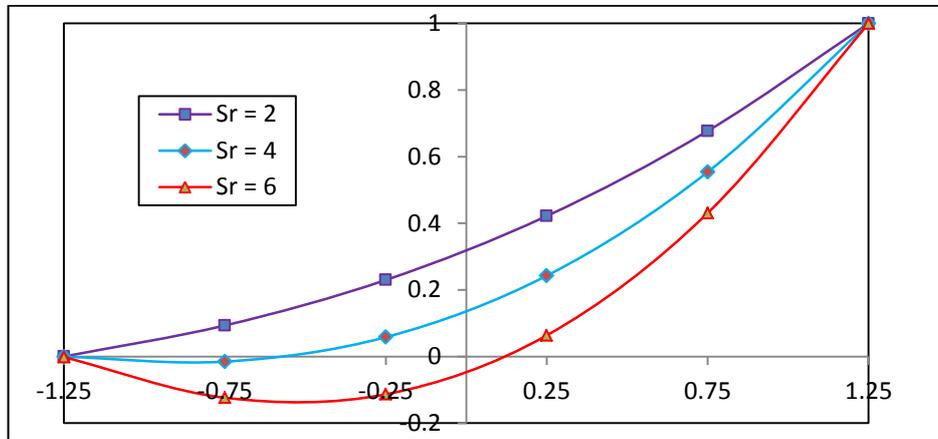


Figure 15. Concentration Distribution (Φ) For Different Values of Sr with $Sc = 0.3$, $Pr = 2$, $N = 0.5$, $B = 0.1$, $\Phi = \pi/6$, $E = 0.2$, $K_1 = 0.1$, $x = 0.6$, $t = 0.4$

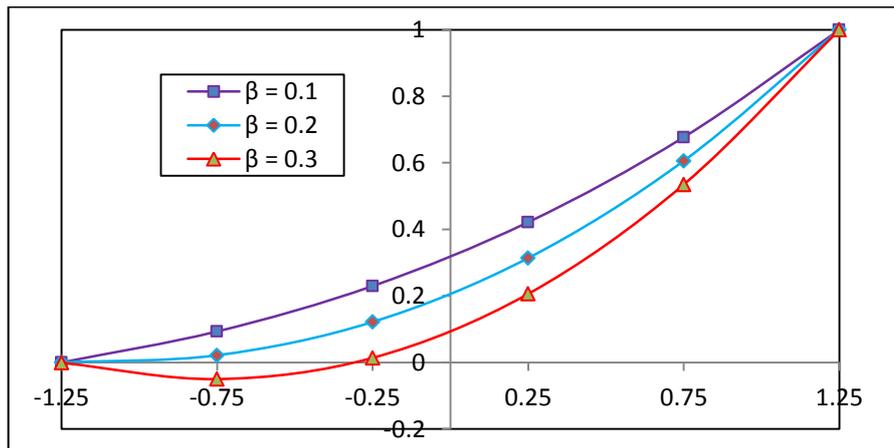


Figure 16. Concentration Distribution (Φ) For Different Values of β with $Sr = 2$, $Sc = 0.3$, $Pr = 2$, $N = 0.5$, $\phi = \pi/6$, $\epsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

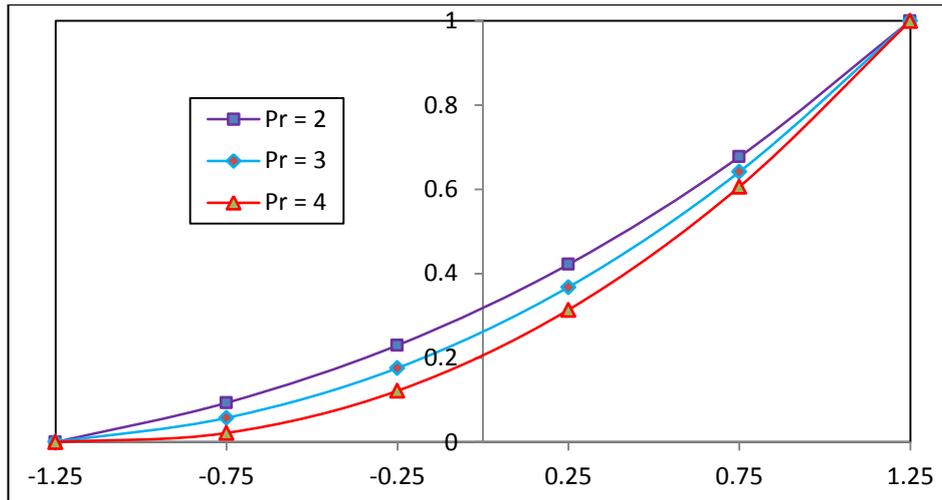


Figure 17. Concentration Distribution (Φ) for Different Values of Pr with $Sr = 2$, $Sc = 0.3$, $\beta = 0.1$, $N = 0.5$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

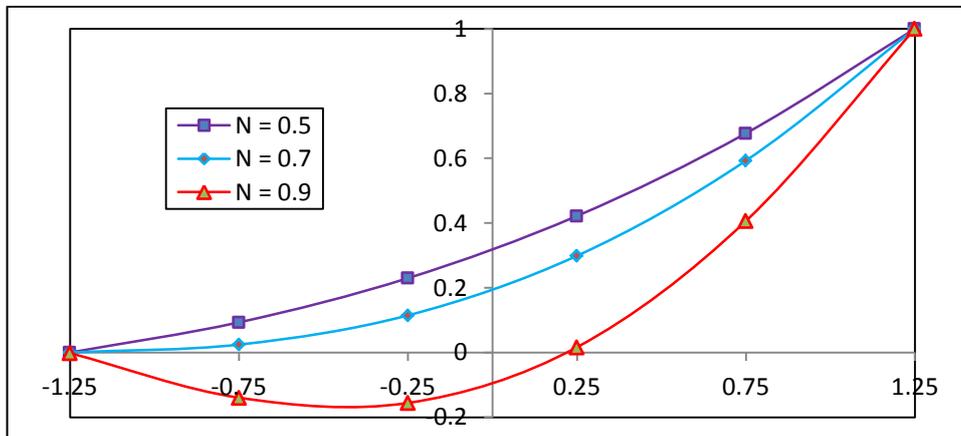


Figure 18. Concentration Distribution (Φ) For Different Values of N with $Sr = 2$, $Sc = 0.3$, $\beta = 0.1$, $Pr = 2$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$

5. Conclusions

In this paper, impact of heat and mass transfer on peristaltic hemodynamic Jeffery fluid model through a tapered asymmetric vertical channel under the influence of radiation is studied under long wavelength and low Reynolds number assumptions are investigated. The major outcomes of the present analysis are as follows:

- 1 Velocity of the fluid depicts in the entire tapered channel with increase in Jeffery fluid parameter, non – uniform parameter and non-dimensional amplitude of the channel.
- 2 Temperature of the fluid enhances in the entire tapered channel with increase in N, Pr, β and k_1
- 3 Temperature of the increases with decrease in phase angle ϕ .
- 4 Heat transfer coefficient increases in the portion of the channel $x \in [0, 0.05] \cup [0.55, 1]$ and diminishes in the other portion of the tapered channel $x \in [0.05, 0.55]$ with increase in N, Pr and β
- 5 Heat transfer coefficient decreases in the entire tapered channel $x \in [0, 1]$ with increase in non- uniform parameter k_1 .

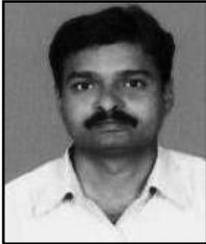
6 Concentration distribution decreases with increase in β , Pr, Sc, N and Sr in an entire channel.

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