

Research on Behavior of Capital Flight based on Evolutionary Game Theory

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Abstract

Through the establishment of the capital flight model of asymmetric evolutionary game, this paper focuses on the study of game relationship among perpetrators, perpetrator assistants and government, and analyzes the evolutionary stable process under different conditions by three-dimensional figures. This paper also analyzes and compares features between the two parties' model and the three parties' model, connecting the two kinds of evolutionary games and providing a theoretic basis for a better understanding of the mechanism of capital flight and motivation of all groups. The result shows that by making use of expectation, government can control the ESS of capital flight groups by adjusting relative benefits and punishment in different regulation modes. It can also expand the strategy space that is in favor of capital control through the adjustment. In the end, this paper will propose policy suggestions for the control of capital flight according to the research findings above.

Keywords: *Capital flight, three asymmetric parties, Evolutionary game theory, Dynamic replication, Evolutionarily stable strategy*

1. Introduction

Bounded rationality repeated game is a process of learning and selection. Through continuous learning and imitating strategy adjustment, game players will improve its own interests and make the final structure reach a dynamic equilibrium, so as to amend the “entirely rational” hypothesis. And meanwhile, “Evolutionarily Stable Strategy” also requires such equilibrium could “expel the invaders”, and the “variant players” will make their decisions return to the original strategy equilibrium through continuous learning and adjustment [1]. The strategy under aforesaid equilibrium is called Evolutionarily Stable Strategy (hereinafter referred to as ESS). Therefore, as a more rational research method, Evolutionary Game has already had a lot of theory creation and practical applications in the economic field [2-3]. On the perspective of theories, the model of evolutionary game has expanded from the non-symmetric game and symmetric game as finite, state dependent quasi-birth-and death processes[4], to the evolutionary game under three asymmetric parties [5]. The stability conditions and behaviors of dynamic in the game system evolutionary equilibrium can also find corresponding analysis conclusions [6-7]. Although in recent years evolutionary game theory has made a lot of new attempts and breakthroughs in economic applications, the applied researches in terms of capital flight still remain in the framework of traditional game theory and only a few literatures have introduced evolutionary game (limited in evolutionary game under two parties). However, the evolutionary game parties in the real-life capital flight game consist of three parties or more, and there is very few researches focus on this aspect. Secondly, a number of current reports only cover the game theory discussion and the analysis of stable equilibrium for two parties or three parties. There is no literature analyzes and compares features between the two parties' model and the three parties' model, connecting two kinds of evolutionary games. In addition, according to incomplete statistics, there are over \$ 20 trillion

international hot money. Most of emerging economies are facing the potential risks of malignant capital flight. However, there are few of studies for capital flight based on evolutionary game, especially from the aspect of government. Therefore, based on “bounded rational” hypothesis and Replicated Dynamic method, this paper established an evolutionary game model under three asymmetric parties, so as to analyze the capital flight. This paper focuses on the evolutionary stable process among perpetrators, perpetrator assistants and government under different conditions. And it also makes a comparison between the two parties’ model and the three parties’ model, provides a theoretic basis for a better understanding of the internal mechanism of capital flight and motivation of all groups. Corresponding policy suggestions will be proposed on these grounds.

2. A Model under Three Asymmetric Parties ($2 \times 2 \times 2$ Games)

2.1. Notation and Definition of the Game

We assume that there are three groups in the game of capital flight: the perpetrator group in the capital flight (X), and perpetrator assistant group (Y), and government (Z). Based on the reality, government regulation is discussed in two modes: the first one is active regulation (with a higher cost). The second one is passive regulation (with a lower cost). Below is a table (Table 1) of parameters and definitions.

Table 1. Main Indexes and Parameter Definitions

Definition	Parameter	Definition	Parameter	Definition	Parameter
Perpetrators’ original incomes	A	Perpetrator assistants’ original incomes	B	Cost under active regulation for capital flight groups	C
The success rate of active regulation	α	The success rate of passive regulation	β	Cost of active regulation for government	C_G
Perpetrators’ cooperative incomes	θ_1	Perpetrator assistants’ cooperative incomes	θ_2	Cost of searching	C_F
Active regulation punishment	L_A	Passive regulation punishment	L_N		

When government adopts active regulation mode, if perpetrators and perpetrator assistants do not act in concert, their initial incomes are $(A - C, B - C, -C_G)$ (the order of incomes space is (X, Y, Z)). If perpetrators and perpetrator assistants act in concert, then their incomes space turns into $(A + (1 - \alpha)\theta_1 - \alpha L_A - C, B + (1 - \alpha)\theta_2 - \alpha L_A - C, 2\beta L_A - C_G)$. If perpetrators turn to perpetrator assistants but the perpetrator assistants are unwilling to engage in capital flight, their incomes space is $(A - C_F - C, B - C, -C_G)$. On the contrary, their incomes space is $(A - C, B - C_F - C, -C_G)$. When there is no capital flight, the incomes space is $(A - C, B - C, -C_G)$. Similarly, when the government adopts passive supervision model, the income space of aforementioned four

conditions is $(A + (1 - \beta)\theta_1 - \beta L_N, B + (1 - \beta)\theta_2 - \beta L_N, 2\beta L_N)$, $(A - C_F, B, 0)$, $(A, B - C_F, 0)$, $(A, B, 0)$. Table 2 represents the incomes results under the eight kinds of game strategies combinations.

Table 2. Three Parties' Incomes under Eight Kinds of Game Strategies Combinations

Mode	Strategy	Perpetrators' incomes	Perpetrator assistants' incomes	Government's incomes
Active regulation	Acting in concert	$A + (1 - \alpha)\theta_1 - \alpha L_A - C$	$B + (1 - \alpha)\theta_2 - \alpha L_A - C$	$2\alpha L_A - C_G$
	Y not acting in concert	$A - C_F - C$	$B - C$	$-C_G$
	X not acting in concert	$A - C$	$B - C_F - C$	$-C_G$
	Both not acting in concert	$A - C$	$B - C$	$-C_G$
Passive regulation	Acting in concert	$A + (1 - \beta)\theta_1 - \beta L_N$	$B + (1 - \beta)\theta_2 - \beta L_N$	$2\beta L_N$
	Y not acting in concert	$A - C_F$	B	0
	X not acting in concert	A	$B - C_F$	0
	Both not acting in concert	A	B	0

2.2. Replication Dynamics and Evolutionary Stability

Suppose: the ratio of perpetrators choose to “act in concert” is x , and to “not act in concert” is $1 - x$; perpetrator assistants choose to “act in concert” is y , and to “not act in concert” is $1 - y$; government chooses to “active regulation” is z , and to “passive regulation” is $1 - z$. Following WEI[5], we can get the expected incomes and the average incomes of players in the evolutionary game under three asymmetric parties. Therefore, if the perpetrators choose to “act in concert”, their expected incomes and average group incomes are U_{x_1} and \bar{U}_x respectively, then:

$$\begin{aligned}
 U_{x_1} = & yz[A + (1 - \alpha)\theta_1 - \alpha L_A - C] + y(1 - z)[A + (1 - \beta)\theta_1 - \beta L_N] \\
 & + (1 - y)z(A - C_F - C) + (1 - y)(1 - z)(A - C_F)
 \end{aligned}
 \tag{1}$$

$$\begin{aligned} \bar{U}_x = & xyz[A + (1 - \alpha)\theta_1 - \alpha L_A - C] + xy(1 - z)[A + (1 - \beta)\theta_1 - \beta L_N] \\ & + x(1 - y)z(A - C_F - C) + x(1 - y)(1 - z)(A - C_F) + (1 - x)yz(A - C) \\ & + (1 - x)y(1 - z)A + (1 - x)(1 - y)z(A - C) + (1 - x)(1 - y)(1 - z)A \end{aligned} \quad (2)$$

If $F(x) / dx < 0$, the stable state is provided with stability[8]. According to these two important definitions, we can draw and prove the following conclusions.

Theorem 1 Perpetrators' ESS in the capital flight game satisfy:

(i) When $z > \frac{y(-1 - \beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N)}$, if $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N > 0$,

$x = 1$ is the balance point, and "acting in concert" is ESS;

if $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N < 0$, $x = 0$ is the balance point, and "not acting in concert" is ESS;

(ii) When $z < \frac{y(-1 - \beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N)}$, if $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N > 0$,

$x = 0$ is the balance point, and "not acting in concert" is ESS;

if $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N < 0$, $x = 1$ is the balance point, and "acting in concert" is ESS.

Proof.

The replicated dynamic equation of the ratio of perpetrators' "acting in concert" is

$$\begin{aligned} F(x) = \frac{dx}{dt} = & x(U_{x_1} - \bar{U}_x) \\ = & x(1 - x) \{ z[y((\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-1 - \beta)\theta_1 + \beta L_N - C_F - C_F \} \end{aligned} \quad (3)$$

(a) When $z = \frac{y(-1 - \beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N)}$, then $F(x) \equiv 0$, which means all

levels are stable. In this case, no matter the ratio of perpetrators' "acting in concert" strategy X_1 and "not acting in concert" strategy X_2 , their strategies will not change over time. As shown in Figure 1.

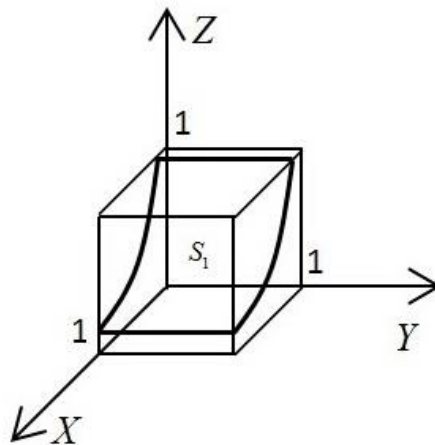


Figure 1. Perpetrators' ESS Stable Process Diagram under the Condition of
 $z = \frac{y(-1 - \beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N)}$ (Curved Surface S_1)

(b) When $z \neq \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$, let $F(x) = 0$, then $x = 0$, $x = 1$ will

be two balance points of x .

Take the derivative of $F(x)$:

$$\frac{dF(x)}{dx} = (1-2x)\{z[y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-(1-\beta)\theta_1 + \beta L_N - C_F) - C_F\}$$

,

Through the discussion of various parameters change, let $\frac{dF(x)}{dx} < 0$, so as to analyze the potential ESS:

(i) When $z > \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$, we have the following conditions:

if (1) $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N > 0$, we have

$$z[y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-(1-\beta)\theta_1 + \beta L_N - C_F) - C_F > 0,$$

when $\left. \frac{dF(x)}{dx} \right|_{x=1} < 0$, $\left. \frac{dF(x)}{dx} \right|_{x=0} > 0$, $x = 1$ is the balance point, and “acting in

concert” is ESS;

If (2) $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N < 0$, we have

$$z[y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-(1-\beta)\theta_1 + \beta L_N - C_F) - C_F < 0.$$

when $\left. \frac{dF(x)}{dx} \right|_{x=1} > 0$, $\left. \frac{dF(x)}{dx} \right|_{x=0} < 0$, $x = 0$ is the balance point, and “not acting in

concert” is ESS;

(ii) When $z < \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$, we have the following conditions:

if (1) $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N > 0$, we have

$$z[y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-(1-\beta)\theta_1 + \beta L_N - C_F) - C_F < 0,$$

when $\left. \frac{dF(x)}{dx} \right|_{x=1} > 0$, $\left. \frac{dF(x)}{dx} \right|_{x=0} < 0$, $x = 0$ is the balance point, and “not acting in

concert” is ESS;

if (2) $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N + C < 0$, we have

$$z[y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)] - y(-(1-\beta)\theta_1 + \beta L_N - C_F) - C_F > 0,$$

when $\left. \frac{dF(x)}{dx} \right|_{x=1} < 0$, $\left. \frac{dF(x)}{dx} \right|_{x=0} > 0$, so $x = 1$ is the balance point, “acting in concert” is

ESS;

Perpetrators' dynamic tendency and stability under different parameters are as shown in Figure 2 and Figure 3.

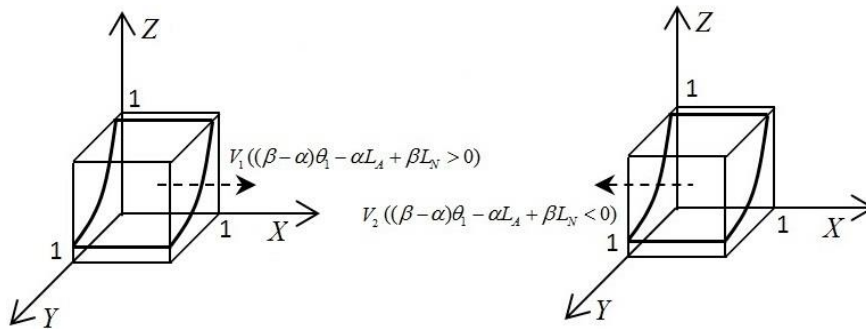


Figure 2. Perpetrators' ESS Stable Process Diagram under the Condition of
 $z > \frac{y(-1-\beta)\theta_1 + \beta L_N - C_F + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)} (V_1 \text{ or } V_2)$

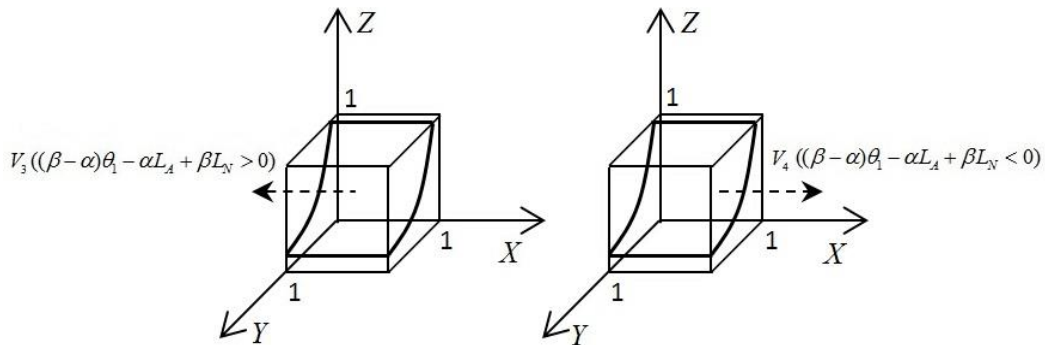


Figure 3. Perpetrators' ESS Stable Process Diagram under the Condition of
 $z < \frac{y(-1-\beta)\theta_1 + \beta L_N - C_F + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)} (V_3 \text{ or } V_4)$

Theorem 2 Perpetrator assistants' ESS in the capital flight game satisfy:

- (i) When $z > \frac{x(-1-\beta)\theta_2 + \beta L_N - C_F + C_F}{x((\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N)}$, if $(\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N > 0$, $y = 1$ is the balance point, and "acting in concert" is ESS; if $(\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N < 0$, $y = 0$ is the balance point, and "not acting in concert" is ESS;
- (ii) When $z < \frac{x(-1-\beta)\theta_2 + \beta L_N - C_F + C_F}{x((\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N)}$, if $(\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N > 0$, $y = 0$ is the balance point, and "not acting in concert" is ESS; if $(\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N < 0$, $y = 1$ is the balance point, and "acting in concert" is ESS.

As shown in Figure 4, Figure 5 and Figure 6.

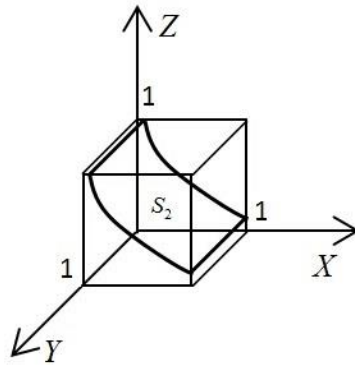


Figure 4. Perpetrator Assistants' ESS Stable Process Diagram under the Condition of $z = \frac{x(-1-\beta)\theta_2 + \beta L_N - C_F + C_F}{x((\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N)}$ (Curved Surface S_2)

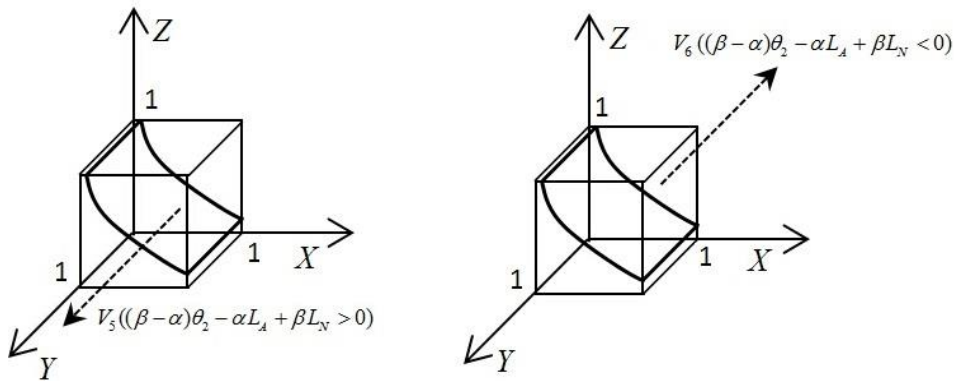


Figure 5. Perpetrator Assistants' ESS Stable Process Diagram under the Condition of $z > \frac{x(-1-\beta)\theta_2 + \beta L_N - C_F + C_F}{x((\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N)}$ (V_5 or V_6)

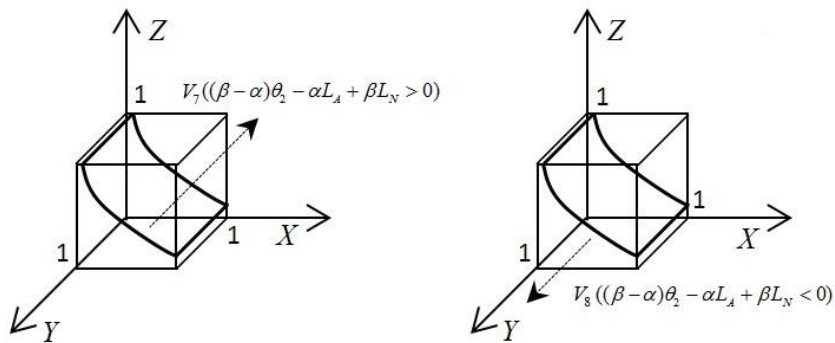


Figure 6. Perpetrator Assistants' ESS Stable Process Diagram under the Condition of $z < \frac{x(-1-\beta)\theta_2 + \beta L_N - C_F + C_F}{x((\beta-\alpha)\theta_2 - \alpha L_A + \beta L_N)}$ (V_7 or V_8)

Theorem 3 Government's ESS in the capital flight game satisfy:

- (i) When $y > \frac{C_G}{x(2\alpha L_A - 2\beta L_N)}$, if $2\alpha L_A - 2\beta L_N > 0$, $z = 1$ is the balance point, and “active regulation” is ESS; If $2\alpha L_A - 2\beta L_N < 0$, $z = 0$ is the balance point, and “passive regulation” is ESS;
- (ii) When $y < \frac{C_G}{x(2\alpha L_A - 2\beta L_N)}$, if $2\alpha L_A - 2\beta L_N > 0$, $z = 0$ is the balance point, and “passive regulation” is ESS; if $2\alpha L_A - 2\beta L_N < 0$, $z = 1$ is the balance point, and “active regulation” is ESS.
- As shown in Figure 7, Figure 8 and Figure 9.

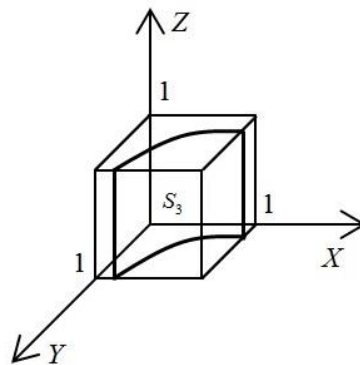


Figure 7. Government’s ESS Stable Process Diagram under the Condition of
 $y = \frac{C_G}{x(2\alpha L_A - 2\beta L_N)}$ **(Curved Surface s_3)**

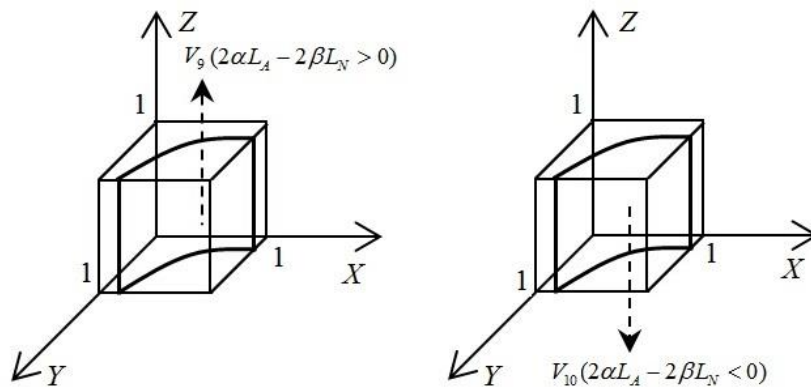


Figure 8. Government’s ESS Stable Process Diagram under the Condition of
 $y > \frac{C_G}{x(2\alpha L_A - 2\beta L_N)}$ **(V_9 or V_{10})**

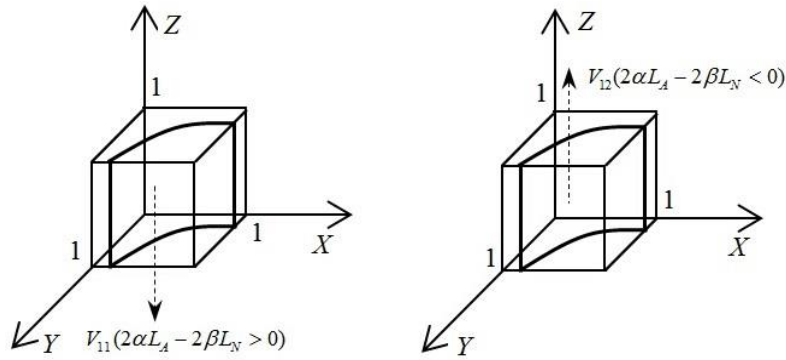


Figure 9. Government's ESS Stable Process Diagram under the Condition of

$$y < \frac{C_G}{x(2\alpha L_A - 2\beta L_N)} \quad (V_{11} \text{ or } V_{12})$$

2.3. Analysis

From the dynamic replication equation, when perpetrators adopt “acting in concert”, we can see that $z = \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$ is the critical plane S_1 . The different space of the front and back side in critical plane will decide the strategy when perpetrators reach to the balance. Considering $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N$, which can be taken as $((1-\alpha)\theta_1 - \alpha L_A) - ((1-\beta)\theta_1 - \beta L_N)$, it means the difference of incomes of perpetrators under two government regulation modes (defined as relative benefits). In the Figure 2, when $z > \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$, government's percentage of possibility for “active regulation” is higher than the ratio of critical plane (defined as strict regulatory space). If $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N > 0$ (positive relative benefits), it shows perpetrators' actual incomes are higher than those under passive regulation. As perpetrators have no better choice under that situation, whatever percentage of “acting in concert” they initially choose, “acting in concert” is the only choice to be a ESS and enables it to reach a stable balance. If $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N < 0$ (negative relative benefits), it shows, perpetrators' incomes under active regulation are lower than those under passive regulation. As perpetrators can get more incomes when government takes passive regulation, “not acting in concert” is the only ESS, and finally this group will reach a stable balance. In the Figure 3, when $z < \frac{y(-(1-\beta)\theta_1 + \beta L_N - C_F) + C_F}{y((\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N)}$, government's percentage of possibility for “active regulation” is lower than the ratio of critical plane (defined as loose regulatory space). If $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N > 0$, it shows, perpetrators' actual incomes are higher under active regulation than those under passive regulation. Because perpetrators can get more incomes when government takes active regulation, “not acting in concert” is the only ESS, and finally the group will reach a stable balance; If $(\beta-\alpha)\theta_1 - \alpha L_A + \beta L_N < 0$, it shows perpetrators' incomes are lower under active regulation than those under passive regulation. Because perpetrators have no more optimal choices than “acting in concert”, it is the only ESS, and will enable it to reach a stable balance in the end.

Under the situation of $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N$, by moving S_1 through the change of C_F , so as to change the volume of the upper and lower space, reaching the aim of change the strategy space. When $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N > 0$, C_F goes up, the difficulty of “acting in concert” also increases, and government will decrease the demand of regulation accordingly. Thus a higher percentage of assistants is needed to enable the capital flight groups to enter the strict regulatory space. In the Figure 2&3, S_1 goes up, V_1 decreases and V_3 increases, so loose regulatory space is enlarged, which raises the possibility for “not acting in concert”. When $(\beta - \alpha)\theta_1 - \alpha L_A + \beta L_N < 0$, C_F goes down, the difficulty of “acting in concert” decreases, government will improve the demand of regulation accordingly. Thus a lower percentage of perpetrator assistants is needed to enable the capital flight groups to enter the strict regulatory space. In the Figure 2&3, S_1 goes down, V_2 increases and V_4 decreases, so strict regulatory space is enlarged, which helps to increase the possibility of perpetrators entering in to “not acting in concert”.

As we hope perpetrators/perpetrator assistants' ESS to be “not acting in concert” and considering the symmetry, we conclude that: (1) By making use of capital flight groups' expectation to different policies, government can choose the proper strategy. When perpetrators/ perpetrator assistants enter into strict regulatory space, government can improve the intensity of punishment under active regulation, or decreasing the intensity of punishment under passive regulation, so as to make perpetrators/ perpetrator assistants' relative benefits negative; When perpetrators/ perpetrator assistants enter into loose regulatory space, government can improve the intensity of punishment under passive regulation, or decreasing the intensity of punishment under active regulation, so as to make perpetrators/ perpetrator assistants' relative benefits positive. It then reaches the aim of government's regulation, which is to make perpetrators/perpetrator assistants' ESS “not acting in concert”, and stay stable in a long period. (2) Enlarge the strategy space effectively so as to strengthen the stability of expected strategy. When the relative benefits are positive, government can enlarge loose regulation space by improving C_F ; When the relative benefits are negative, government can enlarge strict regulation space by decreasing C_F . It helps to increase the possibility of perpetrators/perpetrator assistants to be “not acting in concert”, and decrease the possibility of “acting in concert”.

It is expected that government's final ESS will be “passive regulation”, all three parties do not have additional cost. We also concluded based on the expected and strategy space: (1) when the perpetrators (/perpetrator assistants) enter into high risk space, the government can improve the intensity of punishment under passive regulation, so as to make relative punishment negative; when the perpetrators (/perpetrator assistants) fall into low risk space, the government can improve the intensity of punishment under active regulation, so as to make relative punishment positive. Then government can finally achieve its exclusive ESS, *i.e.* passive regulation, and keep a long-term equilibrium. (2) When the relative punishment is positive, the government can increase C_G , so as to shrink the high risk space; when the relative punishment is negative, the government can decrease C_G , so as to shrink the low risk space. It will be helpful to increase the possibility for the government to be “passive regulation” and decrease the possibility to be “active regulation”.

3. Two Simplified Models under Two Asymmetric Parties (2×2 Games)

The above-mentioned model under three asymmetric parties with respect to the capital flight has provided theoretic basis for the behavioral process of individual group, but cannot conduct evolutionary game analysis and quantitative analysis for the strategies combinations among perpetrators, perpetrator assistants and government. Therefore, in order to figure out the evolution of capital flight game, we convert the evolutionary games under three asymmetric parties model to two evolutionary game models under two parties, and then analyze the features of these three models as well as their differences.

3.1. Perpetrators and Government Model

We convert the model under three asymmetric parties to perpetrators and government model. This evolutionary game model under two parties has combined the perpetrators and perpetrator assistants into one group. From the perspective of corporate management, it can be interpreted as perpetrators make perpetrator assistants incorporated into the company. Then it can get rid of the cost for searching C_F . Set both parties' cooperative incomes as θ , perpetrators' original incomes A as the original incomes after the merger. Other parameters stay the same. Table 3 represents the incomes results under the combination of four kinds of game strategies.

Table 3. Incomes Results of Perpetrators and Government Model

		Government	
		Active regulation	Passive regulation
Perpetrators	Execution	$A + (1 - \alpha)\theta - \alpha L_A - C, \alpha L_A - C_G$	$A + (1 - \beta)\theta - \beta L_N, \beta L_N$
	Non-execution	$A - C, -C_G$	$A, 0$

Theorem 4 ESS in perpetrators and government model satisfy:

- (1) When $\beta L_N > (1 - \beta)\theta > \alpha L_A - C_G$, and $(\beta - \alpha)\theta - \alpha L_A + \beta L_N > 0$, (0,0) is the balance point, "non-execution" and "passive regulation" are ESS under this condition;
- (2) When $(1 - \beta)\theta > \beta L_N > \alpha L_A - C_G$, (1,0) is the balance point, "execution" and "passive regulation" are ESS under this condition;
- (3) When $(1 - \alpha)\theta - C_G > \alpha L_A - C_G > \beta L_N$, (1,1) is the balance point, "execution" and "active regulation" are ESS under this condition.

By calculation, we can have the equilibrium results under different parameters as shown in the Table 4.

Table 4. Local Stability Analysis for the Perpetrators and Government Model

No.	Conditions	Balance points
1	$\beta L_N > (1 - \beta)\theta > \alpha L_A - C_G$	(0,0)
2	$(1 - \beta)\theta > \beta L_N > \alpha L_A - C_G$	(1,0)
3	$\alpha L_A - C_G > (1 - \alpha)\theta - C_G$	None
4	$(1 - \alpha)\theta - C_G > \alpha L_A - C_G > \beta L_N$	(1,1)

We only consider the game within the range of positive saddle point $(\frac{C_G}{\alpha L_A - \beta L_N}, \frac{-(1 - \beta)\theta + \beta L_N}{(\beta - \alpha)\theta - \alpha L_A + \beta L_N})$ (if the saddle point is negative, then part of equilibrium point may be lost). So first $\alpha L_A - \beta L_N > 0$. As shown in the Figure 10, (1) when $\beta L_N > (1 - \beta)\theta > \alpha L_A - C_G$, and $(\beta - \alpha)\theta - \alpha L_A + \beta L_N > 0$, perpetrators' incomes under passive regulation are negative and their relative incomes are positive, so they will not engage in capital flight, and government's incomes under passive regulation are greater than the incomes under active regulation. At this time, the balance point is (0,0). "Non-execution" and "passive regulation" are ESS under this condition. (2) With the decrease intensity of the punishment under passive regulation or the increase of the incomes of capital flight, when $(1 - \beta)\theta > \beta L_N > \alpha L_A - C_G$, perpetrators' incomes under passive regulation is greater than the punishment, and government's incomes under passive regulation is greater than the incomes under active regulation. At this time, the balance point is (1,0), the perpetrators begin to engage in capital flight and the government still gets into a passive regulation. "Execution" and "passive regulation" are ESS under this condition. (3) When $(1 - \alpha)\theta - C_G > \alpha L_A - C_G$, government's incomes βL_N under passive regulation is an unlimited and variable parameter. Therefore, there are mixed strategies in this transition area that we will not discuss in this paper. (4) When $(1 - \alpha)\theta - C_G > \alpha L_A - C_G > \beta L_N$, government's intensity of punishment under passive regulation reaches the lowest point and perpetrators' incomes under active regulation is positive. At this time, the balance point is (1,1). The perpetrators will engage in capital flight and the government will get into an active regulation. "Execution" and "active regulation" are ESS under this condition, which is a very inefficient social status. Therefore, in the above four conditions, only one of these is consistent with our ultimate expectation, *i.e.* the perpetrators do not engage in capital flight, and the government's final ESS is "passive regulation". Both parties have no additional costs under passive regulation. The feasible strategy is to increase the intensity of punishment under active regulation and enhance the success rate of passive regulation, so as to let perpetrators have no incomes under passive regulation, and meanwhile, government's incomes under passive regulation is greater than the incomes under active regulation. This condition requires government to provide an efficiently passive regulation, so as to let perpetrators achieve their exclusive ESS, *i.e.* "non-execution", keep a long-term equilibrium and eventually restrain the capital flight.

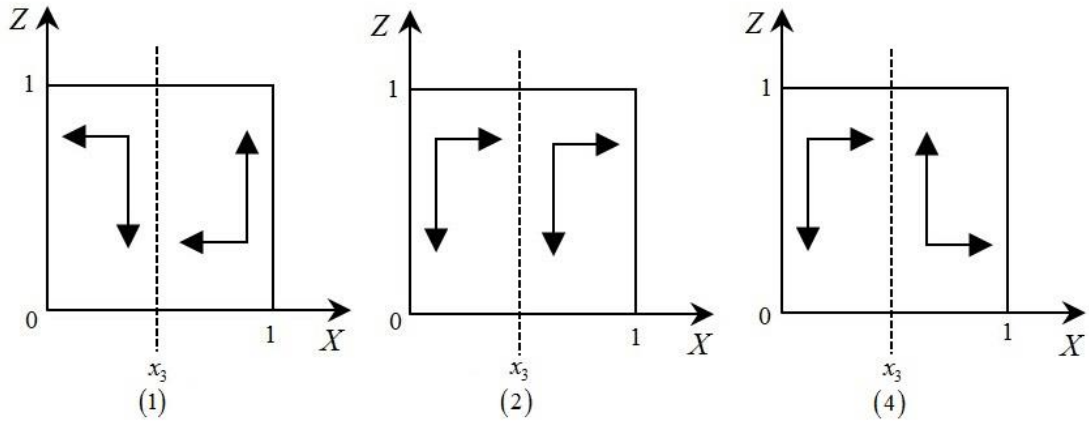


Figure 10. Perpetrators and Government’s ESS Stable Process Diagram under the Condition of (1), (2) and (4)

3.2. Perpetrators and Perpetrator Assistants Model

We convert the model under three asymmetric parties to perpetrators and perpetrator assistants model. As there is no government in the model, we combined two regulation modes into one. Then set the success rate of regulation as α , both parties’ punishment as L , and other parameters stay the same. This is a classic coordination evolutionary game so the conclusion is provided directly below.

Theorem 5 ESS in perpetrators and accomplices model satisfy:

- (1) When $(1 - \alpha)\theta_1 - \alpha L > 0$, and $(1 - \alpha)\theta_2 - \alpha L > 0$, $(0, 0)$ and $(1, 1)$ is the balance point, “acting in concert” and “acting in concert” as well as “not acting in concert” and “not acting in concert” are both ESS under this condition. See Figure 11;
- (2) When $-C_F < (1 - \alpha)\theta_1 - \alpha L < 0$, and $-C_F < (1 - \alpha)\theta_2 - \alpha L < 0$, only $(0, 0)$ is the balance point, “not acting in concert” and “not acting in concert” are ESS.

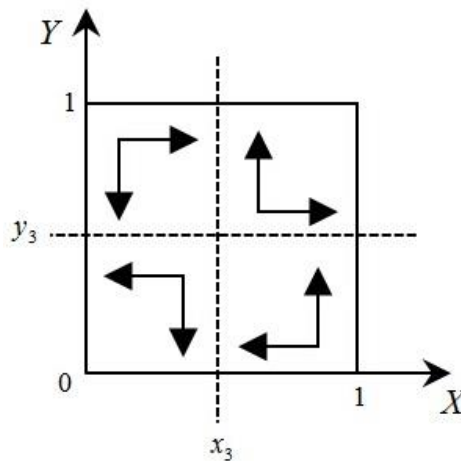


Figure 11. Perpetrators and Perpetrator Assistants’ ESS Stable Process Diagram under the Condition of $(1 - \alpha)\theta_1 - \alpha L > 0, (1 - \alpha)\theta_2 - \alpha L > 0$

3.3. Analysis and Example

In this section we will present comparisons and results of the two kinds of evolutionary game, and illustrate with two examples. At first, we discussed the relationship of perpetrators and government model, perpetrators and perpetrator assistants model and model under three asymmetric parties (only 3 separate parts of it). With no perpetrator assistants in the perpetrators and government model, compared with model under three parties (see Figure 8, Figure 9 and Figure 10), the dynamic evolutionary tendency of perpetrators and government is the same, so this model can be a form of extreme under three parties' model. It can almost explain the evolutionary conditions and process of the model under three parties. However, lacking of some conditions, room for discussion is limited. Therefore, adding the perpetrator assistants in the model is for the purpose of expansion and integrity. Without government, the perpetrators and perpetrator assistants model is not enough to explain evolutionary game. Compared with model under three parties, the difference is evident. First of all, under three parties' model, government has two regulation modes, applying relative incomes and punishment to take control of the final evolutionary process. Second, two regulatory spaces (strict regulation / loose regulation) and two risk spaces (high risk /low risk) enrich the game space. Finally, in terms of the final ESS, perpetrators and perpetrator assistants in model under three parties have little influence on each other. While in the perpetrators and perpetrator assistants model, both of them are dependent on each other, and influence each other. The reasons of such difference lie in: firstly, the function of perpetrators and perpetrator assistants in the whole game is same. Adding or removing a kind of groups will not influence game's essence. But wiping off the part of government plays will change the connotation of the game. Secondly, the perpetrators (/perpetrator assistants) of model under three parties are discussed separately, influenced little by perpetrator assistants (/perpetrators). If taking the intersection space of perpetrators and perpetrator assistants into consideration, it might reflect deeper.

Through the perpetrators and government model and the perpetrators and perpetrator assistants model, we analyze the ESS under different conditions. With the background of "not acting in concert", "passive regulation" and "not acting in concert", "not acting in concert", we analyze the influence of main factors by applying the MATLAB (R2014a) software. Finally, we get relevant results which can be theoretical reference for government to control capital flight.

Example 1 In perpetrators and government model, we can see perpetrators' descending speed of "execution" ratio (define as evolutionary speed) is mainly influenced by incomes and punishment. Therefore, we mainly observe the change of parameters θ and L_A . According to the conditions in the model, the setting of each parameter is as follows: $\alpha = 0.2, \beta = 0.8, \theta = 0.5, L_A = 2.5, L_N = 0.5, x = 0.5, z = 0.5$. As Figure 12 shows, through the comparison of decreasing 20% of incomes and increasing 20% of punishment, it is found that when the punishment is increased, perpetrators' evolutionary speed is faster than that of the speed when incomes are decreased. So, according to the numerical simulation, when government uses the same policy intensity to control capital flight, the effect of increasing punishment should be greater than the effect of decreasing the incomes.

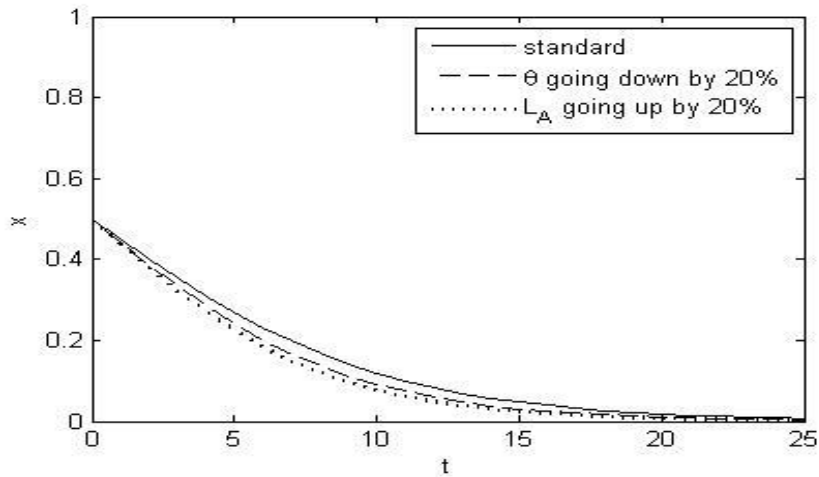


Figure 12. The Influence of the Two Policies on Evolutionary Speed in the Perpetrators and Government Model

Example 2 In the perpetrators and perpetrator assistants model, when perpetrators adopt “not acting in concert”, the evolutionary speed is mainly influenced by incomes, punishment and cost of searching. So, we mainly observe the change of parameters θ_1 , L and C_F . According to the conditions in the model, each parameter is set as follows: $\alpha = 0.3$, $\theta = 0.5$, $L = 0.6$, $C_F = 0.2$, $x = 0.5$, $y = 0.5$. As Figure 13 shows, when decrease incomes by 20% and increase punishment by 20%, as well as increase cost of searching by 20%, it is demonstrated that, the evolutionary speed is the fastest when punishment is increased. Next comes the condition of raising the cost of searching. The evolutionary speed when applying the strategy of decreasing the incomes is the slowest. So, according to the numerical simulation results, when government uses the same policy intensity to control capital flight, increasing the punishment may be the optimal choice. Next is setting up a barrier to improve the cost of searching for both. The least favored choice is to decrease the incomes of perpetrators. The influence of three policies on evolutionary speed reduces in order.

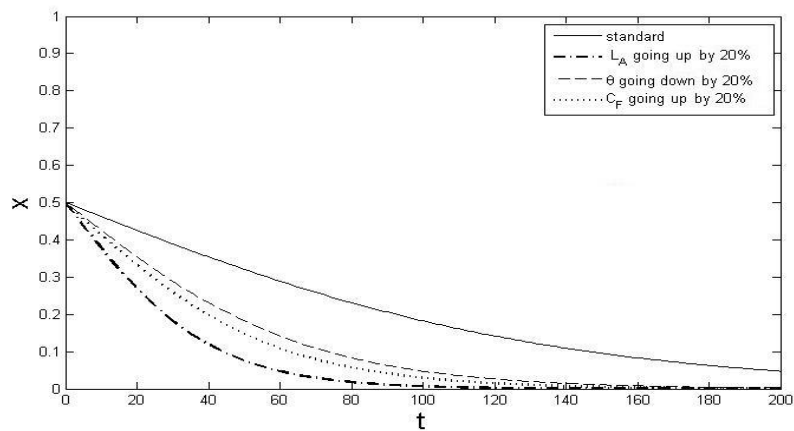


Figure 13. The Influence of Three Policies on Evolutionary Speed in the Perpetrators and Perpetrator Assistants Model

3.4. Discussion and Summary

The result of Evolutionary Game under three parties shows that when the capital flight groups are in regulatory space (strict space/loose space), government could adjust relative benefits (decrease as negative/increase as positive), so that “not acting in concert” would be ESS. Or through the modifying of C_r to increase the possibilities of perpetrators/perpetrator assistants entering into “not acting in concert”. When the capital flight groups are in risk space (high risk/low risk), government could adjust relative punishment (decrease as negative/increase as positive), so that “passive regulation” would be ESS. Or through modifying C_g to increase the possibilities of government entering into “passive regulation”. Perpetrators and government model shows the perpetrators’ incomes are negative under passive regulation, and their relative benefits is positive, “not acting in concert” and “passive regulation” are ESS. Perpetrators and perpetrator assistants model shows, as long as the punishment reached to the threshold value, incomes are zero. The ESS will be stable at “not acting in concert” and “not acting in concert”. As numerical simulation results under the same policy intensity show, punishment policy has the greatest influence on the evolutionary speed of perpetrators who adopt “not acting in concert”.

According to research conclusions, we move forward to give the following policy suggestions: First, in the process of dynamic evolution, it is hard to control capital flight. However, by making use of expectation, and adjusting relative punishment, it is possible for government to take control of the ESS of capital flight groups. Since there are two regulation modes, for the purpose to the capital flight groups giving up the current capital flight plan, government can use “deceiving measure” to modify the relative punishment of its regulation mode within a short time, so that capital flight groups might not enter a certain regulation mode or to wait to enter a certain regulation mode. The temporary balance not only controls the capital flight, but also gives participants more time and space to make proper adjustment. Second, government can change conditions such as C_F and C_G through various administrative means to enlarge its own strategy space which are beneficial to, and at the same time, enlarging the strategy space which is not beneficial for capital flight so as to reach the aim of controlling capital flight. Third, evolutionary speed under different policies varies. Through the theoretical calculation and practices, we can find the most practical and the most effective policy to control the speed of capital flight and decrease the reaction time for capital flight.

Acknowledgments

This research was supported by National Natural Science Foundation of China (71171128) and Research Fund of Program Foundation of Education Ministry of China (10YJA790233).

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