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Research on Adaptive Sliding Mode Control of Simplified Supersonic Missile System with Three Kinds of Uncertain Parameters

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Abstract

A new kind of adaptive sliding mode control method is proposed for simplified missile model of pitch channel with time varying and uncertain parameters. Sliding mode control is an effective method for coping with system uncertainties. Adaptive method is integrated with sliding mode method and a Lyapunov function is constructed to guarantee the whole system is stable. Through the theoretical analysis and numericala simulation, comparison between PID control and sliding mode control shows that the adaptive sliding mode control of the uncertain missile system has a better control effect. And with consideration of three kind of parameter uncertainties in simulation, sliding mode control has better robustness.

Keywords*: Sliding mode control; uncertainty; supersonic missile; PID control; robustness*

1. Introduction

Second order system has very complex dynamics and many complex engineering objects such as missile, airplane and rocket, can be viewed as a second order system for designers. So research on second order system is meaningful and enough[1-5]. Model uncertainties are always exist and they are caused by environment changes or random reasons [6-8]. For example, air dynamic coefficients changes as its speed and height and air density changes. Especially for supersonic missiles or hypersonic missiles, model parameters will change in a very big range. Also, the weight change as flue consume, that will also affect the parameters of simplified missile model. Common PID control or optimal control or feedback control can not provide satisfied performance as parameters change or uncertainties are very big. There are many papers tried adaptive method to solve those uncertainties [9-18]. But adaptive method is always integrated with other method, such as adaptive robust control or adaptive sliding mode control and else.

Sliding mode control is also named variable structure control which is famous for its strong robustness. And sliding mode control also has advantages that it is very simple and it is not require the system model to be accurate. So it is very convenient to be applied in missile system. In this paper, adaptive control is integrated with sliding mode method to solve system uncertainties. Also, PID control is designed and it is compared with adaptive sliding mode control. Detailed numerical simulations were done with PID and adaptive sliding mode control. Three kinds of parameter uncertainties are considered to do the simulation. And simulation result shows that the adaptive sliding mode control has a better performance and a strong robustness than PID control.

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2. Problem Description

The simplified linear model of supersonic missile pitch channel can be written as following second order system:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \tag{1}$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z \tag{2}$$

Where a_{ij} is air dynamic coefficient of missile, α is attack angle of missile, ω_z is the rotate speed of pitch angle and.

The control objective is to design a control law such that the attack angle α can track the desired angle α^d . Without loss of generality, assume $\alpha^d = 1$.

3. The Design of PID Control Law

The structure figure of PID control system is shown in Figure 2.1. The system is composed of PID controller and the controlled object. The control signals that PID controller produces control the controlled object. The PID controller is composed of proportion (P), integral (I) and differential (D).

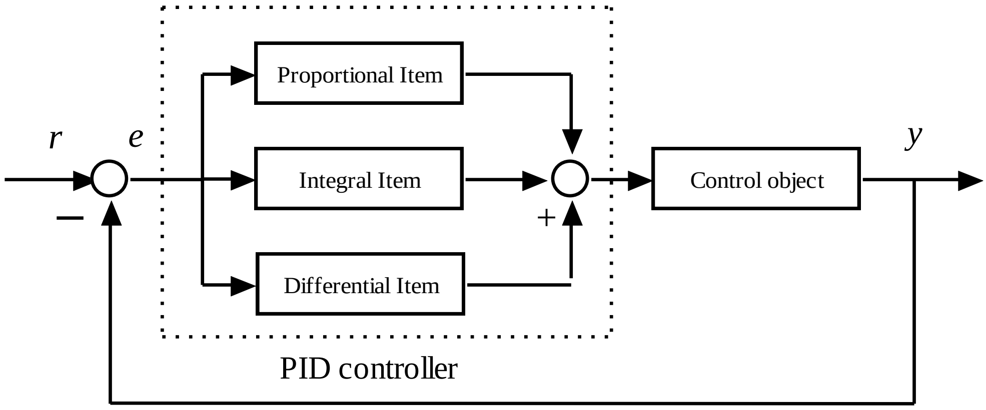

Figure 2.1. PID Control System Structure Diagram

PID controller is a linear controller. It constitutes control deviation according to the expectation α^d and actual output values α .

$$e(t) = \alpha - \alpha^d \tag{3}$$

The PID control law is:

$$u(t) = k_p\left(e(t) + \frac{1}{T_I}\int_0^t e(t)dt + \frac{T_D e(t)}{dt}\right) \tag{4}$$

The form of transfer function is written.

$$G(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{5}$$

In type: k_p is the proportional coefficient; T_I is the integral time constant; T_D is the differential time constant.

4. The Design of Sliding Mode Control Law

Hypothesis 1: a_{12} is not zero, the direction is known, and $a_{12} > 0$。

Hypothesis 2: The expectations x_1^d is constant, its derivative is zero.

The design of sliding mode control system is usually considered as a comprehensive method. Its characteristic is simple and flexible. The basic steps of designing sliding mode control system include two relatively independent parts:

1. Designing the sliding mode function $s(x)$, make the sliding mode gradually stable and have good quality.

2. Calculating the sliding mode control, make the system meet the conditions of reaching sliding mode, and form a sliding mode.

In this way, Sliding mode control not only ensures approaching movement reach the sliding surface in the limited time. But also ensures the sliding mode plane in the sliding mode area. Once the sliding mode function $s(x)$ and the sliding mode control $u(x)$ have been gotten. Sliding mode control system is completely established[10-12].

Defining error variables $e = \alpha - \alpha^d$, There are:

$$\dot{e} = \dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \tag{6}$$

The second derivative of error is:

$$\begin{aligned} \ddot{e} &= \dot{\omega}_z - a_{34}\dot{\alpha} - a_{35}\dot{\delta}_z \\ &= a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z - a_{34}(\omega_z - a_{34}\alpha - a_{35}\delta_z) - a_{35}\dot{\delta}_z \\ &= (a_{24} + a_{34}^2)\alpha + (a_{22} - a_{34})\omega_z + (a_{25} + a_{34}a_{35})\delta_z - a_{35}\dot{\delta}_z \end{aligned} \tag{7}$$

How to eliminate error is the key for error model. Using the sliding mode controller can achieve the effect of eliminating error. First defining the sliding surface is as follows.

$$s = \dot{e} + c_1 e + c_2 \int e dt \tag{8}$$

In order to ensure the sliding mode surface meaningful, when the sliding mode surface is equal to zero, differential equation is steady. There are $c_1 > 0$, $c_2 > 0$.

Then calculate the derivative of sliding mode surface:

$$\begin{aligned} \dot{s} &= \ddot{e} + c_1\dot{e} + c_2 e \\ &= (a_{24} + a_{34}^2 - c_1 a_{34})\alpha + (a_{22} - a_{34} + c_1)\omega_z \\ &\quad + (a_{25} + a_{34}a_{35} - c_1 a_{35})\delta_z - a_{35}\dot{\delta}_z + c_2 e \end{aligned} \tag{9}$$

Define:

$$l_1 = a_{24} + a_{34}^2 - c_1 \tag{10}$$

$$l_2 = a_{22} - a_{34} \tag{11}$$

$$l_3 = c_2 \tag{12}$$

Define:

$$T = (a_{25} + a_{34}a_{35} - c_1 a_{35})\delta_z - a_{35}\dot{\delta}_z \tag{13}$$

Design

$$T = -c_1\omega_z - \hat{l}_1\alpha - \hat{l}_2\omega_z - \hat{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s) - k_3 \int s dt \tag{14}$$

Define

$$\tilde{l}_1 = l_1 - \hat{l}_1 \tag{15}$$

$$\tilde{l}_2 = l_2 - \hat{l}_2 \tag{16}$$

$$\tilde{l}_3 = l_3 - \hat{l}_3 \tag{17}$$

Then

$$\dot{s} = \tilde{l}_1\alpha + \tilde{l}_2\omega_z + \tilde{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s) - k_3\int sdt \tag{18}$$

$$\dot{s} + k_3\int sdt = \tilde{l}_1\alpha + \tilde{l}_2\omega_z + \tilde{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s) \tag{19}$$

$$\dot{s}s + k_3 s\int sdt = \tilde{l}_1\alpha s + \tilde{l}_2\omega_z s + \tilde{l}_3 es - k_1 s^2 - k_2 \operatorname{sgn}(s)s \tag{20}$$

Define adjustment rule of weights is as follows.

$$\dot{\hat{l}}_1 = \Gamma_1 \alpha s \tag{21}$$

$$\dot{\hat{l}}_2 = \Gamma_2 \omega_z s \tag{22}$$

$$\dot{\hat{l}}_3 = \Gamma_3 es \tag{23}$$

Selection Lyapunov function as

$$V_1 = \sum_{i=1}^{3} \frac{1}{2\Gamma_i}\tilde{l}_i^{\,2} \tag{24}$$

Solve its derivative

$$\dot{V}_1 = -\tilde{l}_1\alpha s - \tilde{l}_2\omega_z s - \tilde{l}_3 s \tag{25}$$

Choose Lyapunov function

$$V_2 = \frac{1}{2a_{12}}s^2 + \frac{k_3}{2}\left(\int sdt\right)^2 \tag{26}$$

Then

$$\dot{V}_2 = \frac{1}{a_{12}}\dot{s}s + k_3 s\int sdt = \tilde{l}_1\alpha s + \tilde{l}_2\omega_z s + \tilde{l}_3 es - k_1 s^2 - k_2 \operatorname{sgn}(s)s \tag{27}$$

Selecting total Lyapunov function of system

$$V = V_1 + V_2 \tag{28}$$

Calculating the derivative

$$\dot{V} = -k_1 s^2 - k_2 \operatorname{sgn}(s)s \le 0 \tag{29}$$

Visible that system is stable under the action of sliding mode control. and it does not require that the gain of system is larger values.

5. The Simulation Analysis

The aerodynamic parameters of supersonic missile in a feature point are used to simulate in this paper. The selection of pneumatic parameter is as follows:

$$a_{25} = -167.87; a_{35} = 0.243; a_{22} = -2.876; a_{24} = -193.65; a_{34} = 1.584$$

Since aerodynamic data that the missiles use in the digital simulation is obtained by wind tunnel data interpolation. So aerodynamic data of missile in actual flight may differ simulation data. The robustness of the designed controller is verified by all aerodynamic data perturbation 50% or even 500% up and down in the vicinity of feature points.

5.1. Simulation Results of PID Control

The feature points of supersonic missile are selected as parameters of controlled system. At last, the control parameters are selected as follows after several simulation and modification.

$$k_p = 2, k_i = 5, k_d = 5$$

First, simulation results can be obtained under the situation of no parameter perturbation.

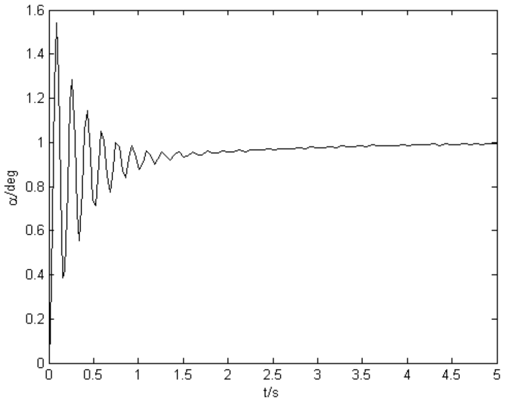

Figure 1. Simulation Results with No Parameters Perturbation

Then situation of having parameter perturbation is considered. Setting the fixed perturbation k, and the perturbation ballistic parameters are as follows.

$$A_{22} = a_{22} \cdot (1+k);\ A_{24} = a_{24} \cdot (1+k);\ A_{25} = a_{25} \cdot (1+k);$$

$$A_{34} = a_{34} \cdot (1+k);\ A_{35} = a_{35} \cdot (1+k)$$

The expectation of signal is 1, the simulation results are as follows.

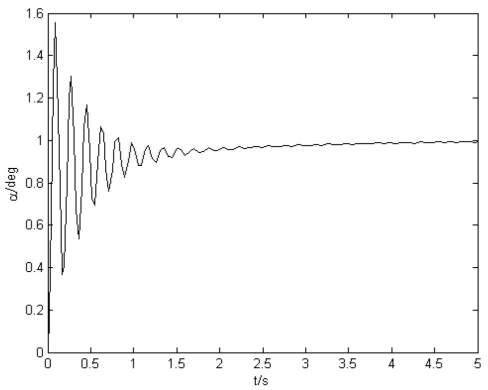

Figure 2. The Simulation Results of $k = -10\%$

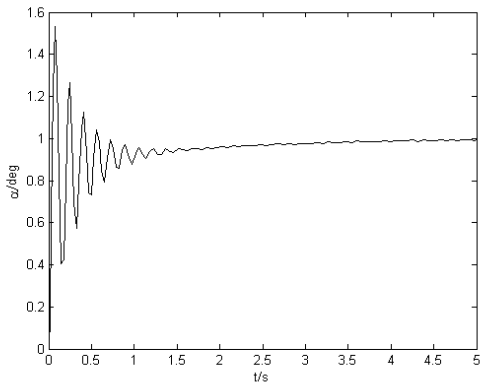

Figure 3. The Simulation Results of $k = 10\%$

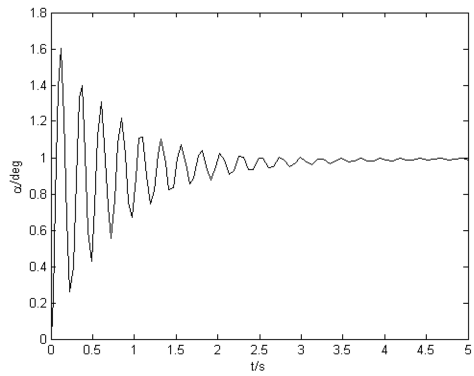

Figure 4. The Simulation Results of $k = -50\%$

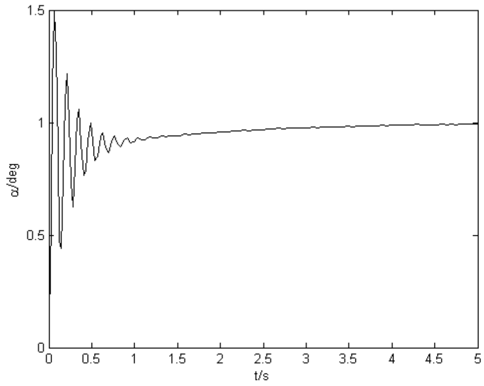

Figure 5. The Simulation Results of $k = 50\%$

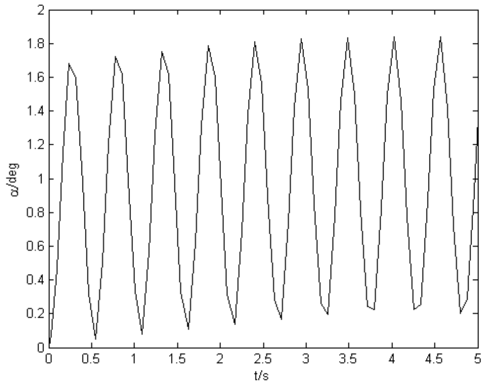

Figure 6. The Simulation Results of $k = -90\%$

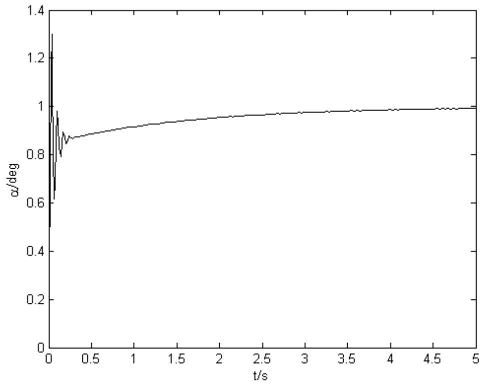

Figure 7. The Simulation Results of $k = 500\%$

Setting the random perturbation k_s, because the perturbation quantity is random. The rand function instruction in MATLAB are used to select random numerical in this paper. The expressions of random perturbation are as follow.

$$k_s = 2K * \left(\mathrm{rand}\left(5,1\right) - 0.5\right)$$

Among them, variable rand function expresses random value in the scope (0, 1). K is the absolute value of the random perturbation range. The five random number in corresponding perturbation scope can be generated by every performing this function instructions in MATLAB software, that k_{s1}、k_{s2}、k_{s3}、k_{s4}、k_{s5}.

In this case, trajectory parameters after perturbation are:

$$A_{22} = a_{22} \cdot (1 + k_{s1})；A_{24} = a_{24} \cdot (1 + k_{s2})；A_{25} = a_{25} \cdot (1 + k_{s3})；$$
$$A_{34} = a_{34} \cdot (1 + k_{s4})；A_{35} = a_{35} \cdot (1 + k_{s5})$$

The simulation results can be obtained as follows.

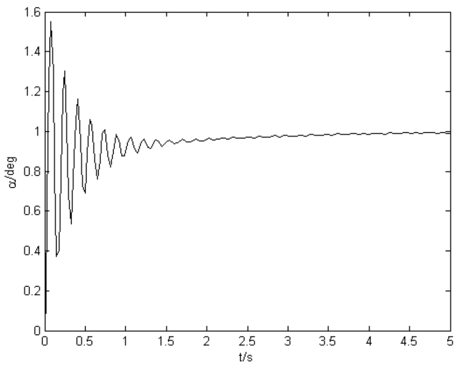

Figure 8. The Simulation Results of $k_s \in (-10\%, 10\%)$

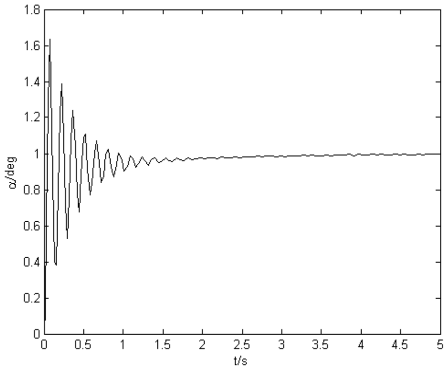

Figure 9. The Simulation Results of $k_s \in (-50\%, 50\%)$

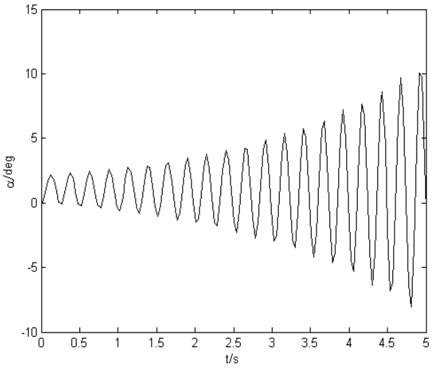

Figure 10.The Simulation Results of $k_s \in (-90\%, 100\%)$

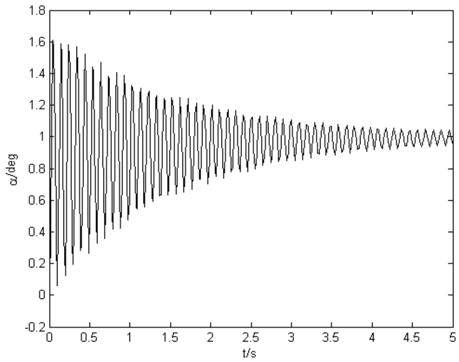

Figure 11. The Simulation Results of $k_s \in (-90\%, 500\%)$

Simulation results show that PID control can ensure system stability in the smaller disturbance range. But when the disturbance range becomes larger, especially under the condition of negative disturbance, the system will be unstable, and have greater volatility.

5.2. Simulation Results of Sliding Mode Control

Control parameters are selected as follows:

$$c_1 = 8, c_2 = 5, q_1 = 1, q_2 = 1, q_3 = 1, k_1 = 20, k_2 = 20, k_3 = 20$$

First, simulation results can be obtained under the situation of no parameter perturbation.

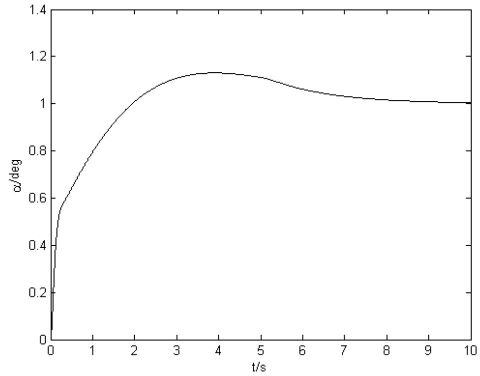

Figure 12. Simulation Results No Parameters Perturbation

Simulation results can be obtained by the PID fixed parameter perturbation method.

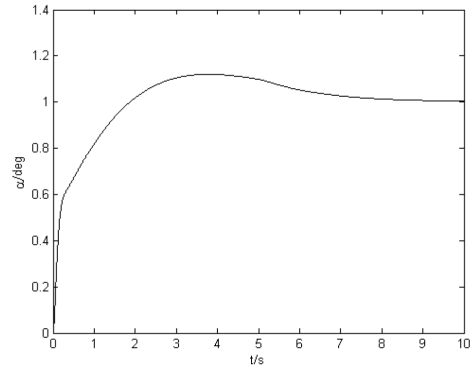

Figure 13. The Simulation Results of $k = -10\%$

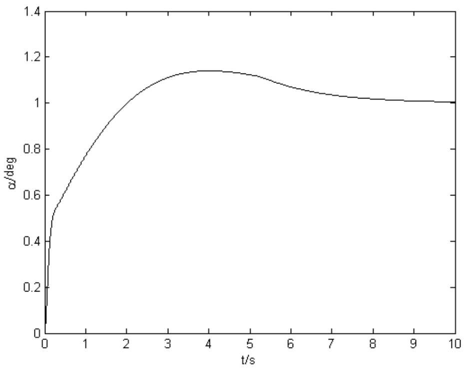

Figure 14. The Simulation Results of $k = 10\%$

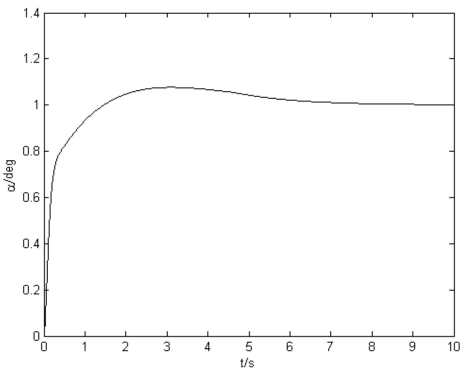

Figure 15. The Simulation Results of $k = -50\%$

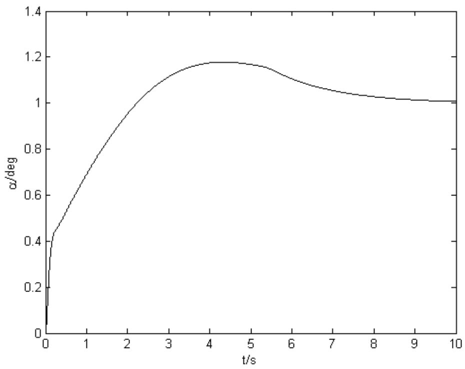

Figure 16. The Simulation Results of $k = 50\%$

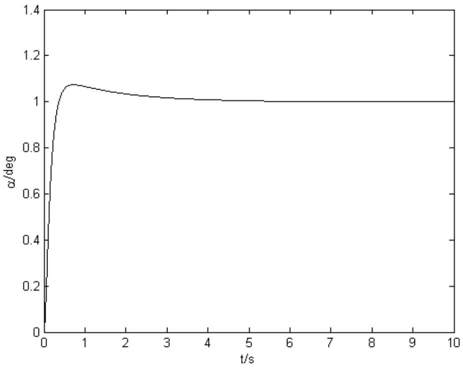

Figure 17. The Simulation Results of $k = -90\%$

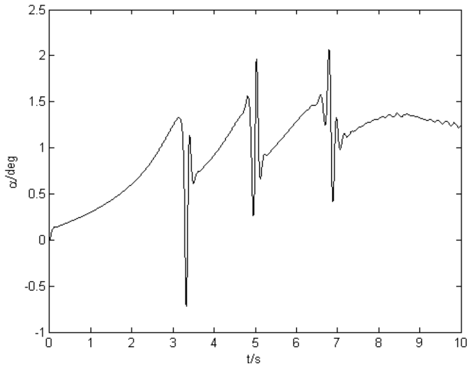

Figure 18. The Simulation Results of $k = 500\%$

Finally, the following results can be gotten by simulation in accordance with PID random parameters perturbation method.

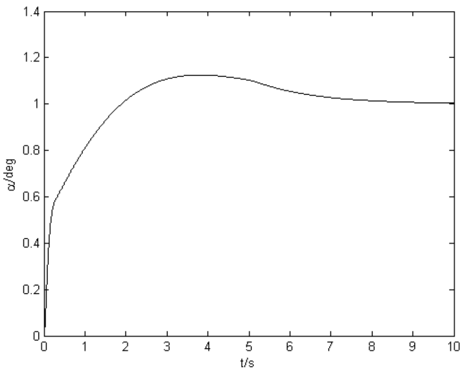

Figure 19. The Simulation Results of $k_s \in (-10\%, 10\%)$

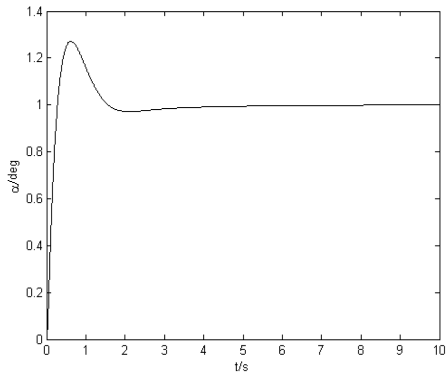

Figure 20. The Simulation Results of $k_s \in (-50\%, 50\%)$

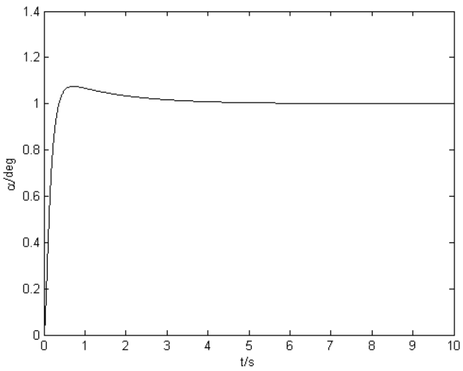

Figure 21. The Simulation Results of $k_s \in (-90\%, 100\%)$

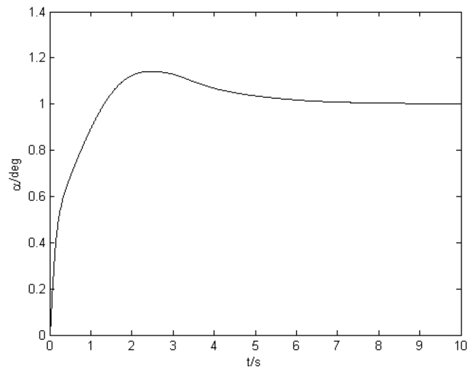

Figure 22. The Simulation Results of $k_s \in (-90\%, 500\%)$

The simulation results are stable under the condition of different perturbation. It shows that the control method has strong robustness.

5.3. Comparison and Analysis

The above simulation results shows that both PID control and adaptive sliding mode control methods can make the missile system stable in some characteristic height with consideration of small uncertainties. But if the uncertainties increase, the PID control is not effective and especially if the uncertainties are random, the control effect of PID control is not as good as adaptive sliding mode control. And both sliding mode control and PID control are stable but the adaptive sliding mode control method has better robustness and it is not necessary to know the system parameter in advance.

6. Conclusion

A new kind of adaptive sliding mode control law was proposed for pitch channel control of supersonic missile. And three kinds of parameter uncertainties are considered to testify the robustness of proposed method. Also, the numerical simulation was done with PID control method. And simulation result shows that if the parameters change in a small range, the PID method is as effective as adaptive sliding mode method. But if the simplified missile parameters change in a big range, the proposed adaptive sliding mode control will have a better performance and it is more robust compared with PID control.

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References

[1] S. S. Ge, C. Wang and T. H. Lee, "Adaptive backstepping control of a class of chaotic systems', Int J Bifurcation and chaos, vol. 10, no. 5, **(2000)**, pp. 1140-1156.
[2] X. Ye and J. Jiang, "Adaptive nonlinear design without a priori knowledge of control directions", Automatic Control, IEEE Transactions on, vol. 43, no. 11, **(1998)**, pp. 1617-1621.
[3] X. Ye, "Asymptotic regulation of time-varying uncertain nonlinear systems with unknown control Directions", Automatica, vol. 35, no. 5, **(1999)**, pp. 929-935.
[4] X. Ye, "Semiglobal output feedback control of uncertain nonlinear systems with unknown high frequency gain sign", Automatic Control,IEEE Transcations on, vol. 45, no. 12, **(2000)**, pp. 2402-2405.
[5] X. Ye, "Adaptive nonlinear output—feedback control with unknown high-frequency gain sign", Automatic, Control, IEEE Transactions on, vol. 46, no. 1, **(2001)**, pp. 112-115.
[6] P. Kokotovic, "Constructive nonlinear control", Progress in the 90'S in Proceedings of IFAC 14nd World Congress. Beijing, **(1999)**.
[7] A. Saberi, EK. and S. Hussmann, "Global Stabilization of Partially Linear Composite", Systems, Journal of Control Optimization, vol. 28, **(1990)**, pp:1491-1503.
[8] A. Richard, D.S. Hull and Z. Qu, "Esign and Evaluation of Robust Nonlinear Missile Autopilots from a Performance Perspective", In Proceeding of the American Control Conferences, Washington, **(1995)**.
[9] M. Krstic, I. anellakopoulos and P. Kokotovic, "Nonlinear and Adaptive Control Design", New York: Wiley-Interscience Publication, **(1995)**.
[10] D. Swaroop, J.C. Gerdes and J.K. Hedrick, "Dynamic surface control of nonlinear systems", Proceedings of the American Control Conference. Albuquerque, New Mexico, **(1997)**.
[11] J. Li and D. Xu, "Adaptive backstepping sliding mode control of nonlinear system with Unmatched uncertainties", Control and Decision, vol. 14, no. 1, **(1999)**, pp. 46-50.
[12] C. Qian and W. Lin, "Output feedback control of a class of nonlinear systems: A nonseparation principle paradigm", IEEE Transactions on Automatic Control, **(2002)**, vol. 47, pp.1710–1715.
[13] H. Lei and W. Lin, "Universal adaptive control of nonlinear systems with unknown growth rate by output feedback", Automatica, vol. 42, **(2006)**, pp. 1783-1789.
[14] J. P. Gauthier, H. Hammouri and S. Othman, "A simple observer for nonlinear systems, applications to bioreactors", IEEE Transactions on Automatic Control, vol. 37, **(1992)**, pp. 875–880.

[15] H. K. Khalil and A. Saberi, "Adaptive stabilization of a class of nonlinear systems using high-gain feedback", IEEE Transactions on Automatic Control, vol. 32, **(1987)**, pp. 1031-1035.
[16] C. Qian and W. Lin, "Nonsmooth output feedback stabilization and tracking of a class of nonlinear systems", Proceedings of the 42th IEEE CDC, **(2003)**, pp. 43-48.
[17] G. Chen and X. Dong, "On Feedback Control of Chaotic Continuous-Time Systems", JEEE Trans.on CAS, part I, vol. 40, no. 10, **(1993)**, pp. 591-601.
[18] J. Leitner, "Helicopter Nonlinear Control Using Adaptive Feedback Linearization", Ph.D. Thesis, Georgia Institute of Technology, Atlanta, CA, **(1995)**.

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