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Research on Time Series with Garch-Copula Model in the Field of Finance

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Abstract

In modern financial issues empirical analysis shows that the volatility of financial market time series showed more variable, volatility clustering, spike and heavy tail. Garch model family in the analysis of financial time series squared error autocorrelation on widely recognized. However Garch model only analysis a single time series, correlation analysis for multiple time series on limitations. This paper introduces Copula function for studying the correlation of multiple time series. Using Garch model as marginal distribution for Copula function. At the same time, analyze correlation of t-Copula and normality Copula in the financial issus.

Keywords*: Garch model; marginal distribution; time series; Copula function*

1. Introduction

Copula theory first proposed by Sklar raised in 1959, expressed as an N-dimensional joint distribution function can be decomposed into marginal distribution of N and a Copula function. This Copula function describes the correlation between variables. Until 1999 given the strict definition of Copula function. Copula function is the joint distribution function $F(x_1,x_2,\cdots,x_N)$ of random vector $X_1,X_2,\cdots,X_N$ connection with each marginal distribution $F_{X_1}(x_1),\cdots,F_{X_N}(x_N)$ which is function $C(u_1,u_2,\cdots,u_N)$ make $F(x_1,x_2,\cdots,x_N)=C[F_{X_1}(x_1),\cdots,F_{X_N}(x_1)]$ After several years of development in many practice sequences areas have been widely used[1,3].

Copula-Garch model is together Garch model with Copula function , to analyze the correlation of multivariate financial time series. Garch model is used to describe the condition marginal distribution of the financial time series, while the Copula function is used to connect the various financial time series[4]. Different financial time series may have different conditions marginal distribution can be described by different GARCH model. The situation of marginal distribution model fit the actual data is critical for Copula function study for sequence. So there should be a method to evaluate the goodness of fit of the marginal distribution. Diehoid establish the method of sequence probability integral transformation density distribution model. it applies Copula model, means do the original sequence probability integral transformation, The new sequence obtained may be connected by a Copula function. This Copula function is used to analyze the correlation between the structure of the respective time series. So on the one hand Copula-Garch model analysis the conditional heteroskedasticity characteristic of financial time series. On the other hand reflects the non-linear correlation between sequences from a probability point of view. Very suitable for analysis of relevant financial markets.

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2. Theoretical Research of Copula-Garch

2.1. Marginal Distribution

Engle put forward Autoregressive Conditional Heteroscedasticity Model (ARCH) [5], it used to establish the conditional variance model and its prediction. However ARCH model with defect. Bollerslev proposed Generalized Autoregressive Conditional Heteroscedasticity Model (GARCH)[6]. Garch model heteroscedasticity means random error having a time-varying. Conditional forecast shows the future value depends on historical observations, describing the predicted values from back there is a link with the past observations[7]. The Garch(p, q) model is defined as.

$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \upsilon_t$$

$$\sigma_t^2 = \alpha_0(t) + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where y_t is mean equation, ε_t means random interference items, σ_t^2 expression conditional variance of ε_t , $\{\upsilon_t\}$ is a random sequence with independent identically distributed (i.i.d). α_0 α_i β_j , $i = 1, 2....p,\ j = 1, 2..., q$ determining a parameter indicative of the non-negative function.

Tgarch model [8]: Conditional variance to be redefined, $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2$ among d_{t-1} them means dummy variable: when $u_{t-1}^2 < 0$, $d_{t-1} = 1$,else $d_{t-1} = 0$.

Combine-garch model: The introduction of the conditional variance Garch (1,1): $\sigma_t^2 = \varpi + \alpha\left(u_{t-1}^2 - \varpi\right) + \beta\left(\sigma_{t-1}^2 - \varpi\right)$, ϖ means unconditional methods or long-term volatility.

2.2. Binary Copula Function

2.2.1. The Definition of Binary Copula Function

Binary copula function refers to a function $C(u,v)$ satisfies the following properties.

1) $C(u,v)$ domain is defined as $[0,1]\times[0,1]$

2) $C(u,v)$ with zero-base surface and is a two-dimensional increments.

3) For any of the $u, v \in [0,1]$, $C(u,1) = u, C(1,v) = v$ meet.

Zero-base surface means: at least one $u_0 \in [0,1]$ and one $v_0 \in [0,1]$, make $C(u_0, v) = C(u, v_0) = 0$. Two-dimensional means: for any of the $0 \le u_1 \le u_2 \le 1$, and $0 \le v_1 \le v_2 \le 1$, with $C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0$.

Assume that $F(x)$ and $G(y)$ are continuous one variable distribution function. So that $U = F(x), V = G(y)$, U, V are uniformly distributed seen on $[0,1]$. Then $C(u,v)$ is a marginal distribution are binary $[0,1]$ uniformly distributed on the joint distribution function. For any domain of definition point (u,v) , there is $0 \le C(u,v) \le 1$.

2.2.2. Sklar Theorem of Binary Distribution

Let $H(x,y)$ binary joint distribution function $F(x)$ and $G(x)$ as having marginal distribution. There is a copula function $C(u,v)$, meet.

$$H(x,y) = C[F(x), G(y)] \tag{1}$$

If $F(x)$ and $G(x)$ is a continuous function, then $C(u,v)$ is uniquely determined. Conversely, else if $F(x)$ and $G(x)$ is an element distribution function, $C(u,v)$ is a

Copula function, $H(x,y)$ determined by (1) is a binary edge distribution $F(x)$ and $G(x)$ joint distribution function.

2.2.3. Binary Copula Function of Nature

1) $C(u,v)$ respect for each variable is monotonically non-drop, that is, if a variable maintained constant, $C(u,v)$ will increase with the increase of another variable (or constant).

2) For any $u,v \in [0,1]$, $C(u,0)=C(0,v)=0$ $C(u,1)=u$, $C(1,v)=v$, that is, as long as there is a variable to 0, the corresponding value will be 0 Copula function, if a variable is 1, the Copula function value is determined by another variable.

3) For any $u_1,u_2,v_1,v_2 \in [0,1]$ with $|C(u_2,v_2)-C(u_1,v_1)| \le |u_2-u_1|+|v_2-v_1|$.

4) If U,V is independently and uniformly distributed with $[0,1]$, then $C(u,v)=uv$.

5) For any $0 \le u_1 \le u_2 \le 1$ and $0 \le v_1 \le v_2 \le 1$, with $C(u_2,v_2)-C(u_2,v_1)-C(u_1,v_2)+C(u_1,v_1) \ge 0$.

2.3. Multiple Copula Function

2.3.1. The Definition of Multiple Copula Function

Refers To a function $C(u_1,u_2,\cdots u_N)$ satisfied the following properties.

1) Domain is defined as $[0,1]^N$.

2) $C(u_1,u_2,\cdots u_N)$ with zero-base surface and is N-dimensional increments.

3) $C(u_1,u_2,\cdots u_N)$ distribution function has an edge $C_i(u_i)=$ $(i=1,2,\cdots,N)$, and satisfying $C_i(u_i)=$ $C(1,\cdots,1,u_i,1,\cdots,1)=u_i$ where $u_i \in [0,1] (i=1,2,\cdots,N)$.

2.3.2. Sklar Theorem of Multivariate Distribution

Let $F(x_1,x_2,\cdots,x_N)$ having marginal distribution $F_1(x_1),\cdots,$ $F_N(x_N)$ of N multiple joint distribution function, then there exists a copula function, $C(u_1,u_2,\cdots,u_N)$ meet.

$$F(x_1,x_2,\cdots,x_N)=C(F_1(x_1),F_2(x_2),\cdots,F_N(x_N)) \tag{2}$$

If $F_1(x_1),F_2(x_2),\cdots,F_N(x_N)$ is a continuous function, then $C(u_1,u_2,\cdots,u_N)$ is uniquely determined. Conversely, else if $F_1(x_1),F_2(x_2),\cdots,F_N(x_N)$ is an element distribution function, $C(u_1,u_2,\cdots,u_N)$ is a Copula function, $F(x_1,\cdots,x_N)$ determined by (2) is a multiple edge distribution $F_1(x_1),F_2(x_2),\cdots,$ $F_N(x_N)$ joint distribution function.

2.3.3. Multiple Copula Function of Nature

1) $C(u_1,u_2,\cdots,u_N)$ respect for each variable is monotonically non-drop.

2) $C(u_1,u_2,\cdots,0,\cdots,u_N)=0, C(1,\cdots,1,u_i,1,\cdots,1)=u_i$

3) For any of $u_i,v_i \in [0,1] (i=1,2,\cdots,N)$, with $|C(u_1,u_2,\cdots,u_N)-C(v_1,v_2,\cdots,v_N)| \le \sum_{i=1}^{N} |u_i-v_i|$

4) If $U_i \sim U(0,1) (i=1,2,\cdots,N)$ mutually independent, then $C(u_1,u_2,\cdots,u_N)=\prod_{i=1}^{N} u_i$

2.4. Common Copula Function

Distribution function and density function of Multiple normality Copula.

$C\left(u_1,u_2,...,u_N;\rho\right)=\Phi_p\left(\Phi^{-1}(u_1),\Phi(u_2)^{-1},...,\Phi(u_N)^{-1}\right)$

$$c\left(u_1,u_2,...,u_N;\rho\right)=\frac{\partial^N C\left(u_1,u_2,...,u_N;\rho\right)}{\partial u_1,\partial u_2,...,\partial u_N}=\left|\rho\right|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\xi(\rho^{-1}-I)\xi\right]$$

Where ρ is n order symmetric positive definite matrix of the diagonal are all with 1, $|\rho|$ denotes the determinant of a matrix, Φ_p denotes the distribution function of the standard normal distribution of n of correlation matrix with ρ ,Its marginal distribution is standard normal distribution. Φ^{-1} is inverse function of standard normal distribution function $\xi'=\left(\Phi^{-1}(u_1),\Phi^{-1}(u_2),...,\Phi^{-1}(u_N)\right)$. I is unit matrix.

Distribution function and density function of Multiple t-Coupla:

$$C\left(u_1,u_2,...,u_N;\rho,k\right)=t_{p,k}\left[t_k^{-1}(u_1),t_k^{-1}(u_2),...,t_k^{-1}(u_N)\right]$$

$$c\left(u_1,u_2,...,u_N;\rho,k\right)=\left|\rho\right|^{-\frac{1}{2}}\frac{\Gamma\left(\frac{k+N}{2}\right)\left[\Gamma\left(\frac{k}{2}\right)\right]^{N-1}}{\Gamma\left(\frac{k+N}{2}\right)^N}\frac{\left(1+\frac{1}{k}\xi'\rho^{-1}\xi\right)^{\frac{k+N}{2}}}{\prod_{i=1}^{N}\left(1+\frac{\xi_i^2}{k}\right)^{\frac{k+1}{2}}}$$

Where ρ is n order symmetric positive definite matrix of the diagonal are all with 1, $|\rho|$ denotes the determinant of a matrix, $t_{p,k}$ devotes the distribution function of the standard t- distribution of n with correlation matrix ρ ,freedom k , t_k^{-1} is inverse function of one variable t distribution of distribution function $\xi'=\left(t_k^{-1}(u_1),t_k^{-1}(u_2),...,t_k^{-1}(u_N)\right)$.

3. Main Title

In order to examine different periods of China's stock market correlation of each module. Paper chooses China's Shanghai series power module (DL), and building module (JZ). The date From October 21, 2011 to February 9, 2015. A total of 802 valid set of data for analysis. Defined yield = (closing price - opening price) / opening. According to the theory described above Copula, Dual Copula model can be constructed in accordance with the following procedure. First, Determines the data marginal distribution. Second, Select appropriate copula function for description related structures of data. Third, The correlation coefficient analysis about copula function.

3.1. Marginal Distribution

For description marginal distribution about two kinds of time-series. Figure 1 is power module (DL), and building module (JZ) , With 823.371 and 1984.364 for the starting point run chart from October 21, 2011 to February 9, 2015. As can be seen from the chart, DL and JZ affected by the economy there is a clear upgrade in 2014. there should be a strong correlation. Analysis of specific parameters in 3.2 . Table Ⅰ shows the basic characteristics of the data and information.

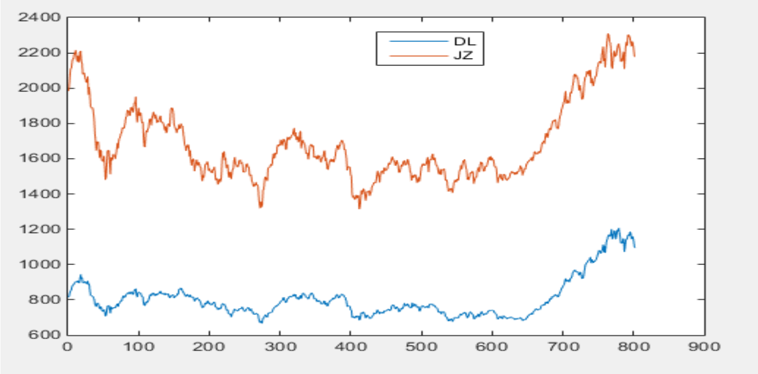

Figure 1. Run Chart of DL and JZ

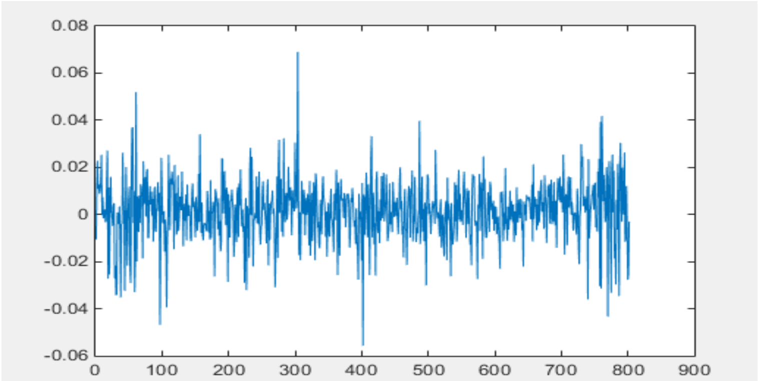

Figure 2. Yield Volatility of DL

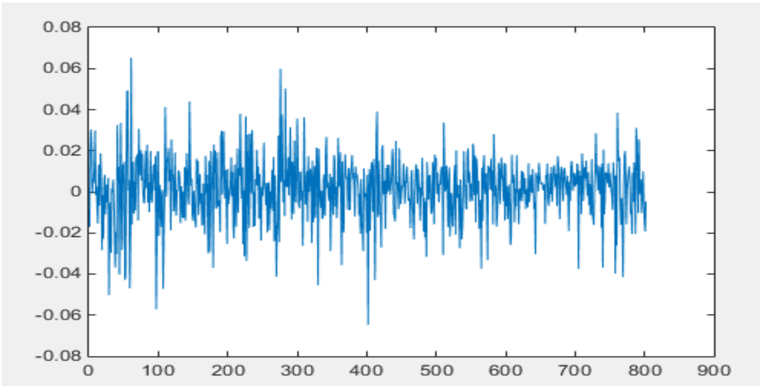

Figure 3. Yield Volatility of JZ

Since Garch model analysis time series must ask is stationary sequence. As apparent from Figure 1, the original data is not stationary sequence, so consideration should be analyzed with the yield, DL and JZ yield volatility (SDL, SJZ) is shown in Figure 2, Figure 3. ADF test for the date in Figure 2 and Figure 3. And value -27.67442, -26.67344, all significantly less than three given threshold. Therefore model analysis should be done with the yield.

Table 1. Characteristics of DL and JZ

	DL	JZ
Mean	944.7814	1947.741
Median	790.5360	1626.602
Maximum	2478.313	5071.996
Std.Dev	665.8290	1314.698
Skewness	362.2968	720.2982
Kurtosis	6.958045	2.146823

3.1.1. Establish Garch Model for SJZ

1) Garch(1,1):
Mean equation: $SJZ_t = 0.035821 SJZ_{t-1} + \hat{\mu}_t$
Garch(1,1) equation: $\sigma_t^2 = 8.48 \times 10^{-6} + 0.107478 \hat{\mu}_{t-1}^2 + 0.868229 \sigma_{t-1}^2$
2) Tgarch:
Mean equation: $SJZ_t = 0.041301 SJZ_{t-1} + \hat{\mu}_t$
Garch(1,1) equation: $\sigma_t^2 = 8.09 \times 10^{-6} + 0.06765 \hat{\mu}_{t-1}^2 + 0.06501 \mu_{t-1}^2 d_{t-1} + 0.87895 \sigma_{t-1}^2$
3) Combine-Garch:
Mean equation: $SJZ_t = 0.042175 SJZ_{t-1} + \hat{\mu}_t$
Garch(1,1) equation: $$\begin{aligned} \omega_t &= 0.000413 + 0.990391(\omega_t - \omega) + 0.65845(\mu_{t-1}^2 - \sigma_{t-1}^2) \\ \sigma_t^2 - \omega_t &= 0.57859(\mu_{t-1}^2 - \omega_{t-1}) + 0.820013(\sigma_{t-1}^2 - \omega_{t-1}) \end{aligned}$$

3.1.2. Establish Garch Model for SDZ

1) Garch(1,1):
Mean equation: $SDL_t = 0.027892 SDL_{t-1} + \hat{\mu}_t$
Garch (1,1) equation: $\sigma_t^2 = 6.16 \times 10^{-6} + 0.108583 \hat{\mu}_{t-1}^2 + 0.869547 \sigma_{t-1}^2$
2) Tgarch:
Mean equation: $SDL_t = 0.027126 SDL_{t-1} + \hat{\mu}_t$
Garch (1,1) equation: $\sigma_t^2 = 5.89 \times 10^{-6} + 0.09486 \hat{\mu}_{t-1}^2 + 0.02493 \mu_{t-1}^2 d_{t-1} + 0.87384 \sigma_{t-1}^2$
3) Combine-Garch:
Mean equation: $SDL_t = 0.034136 SDL_{t-1} + \hat{\mu}_t$
Garch (1,1) equation: $$\begin{aligned} \omega_t &= 0.000288 + 0.983308(\omega_t - \omega) + 0.087531(\mu_{t-1}^2 - \sigma_{t-1}^2) \\ \sigma_t^2 - \omega_t &= 0.050710(\mu_{t-1}^2 - \omega_{t-1}) + 0.625103(\sigma_{t-1}^2 - \omega_{t-1}) \end{aligned}$$

Tgarch model is to analyze the impact of information curve symmetry. And SJZ and SDL's $\mu_{t-1}^2 d_{t-1}$ items are 0.06501,0.02493 no significant, It is not suitable for the establishment Tgarch model. In combination Garch model: $\omega_t - \omega$ coefficients were 0.990391, 0.983308 indicating long-term parameters will slowly converge to stable.

Comparative analysis of the data obtained Garch (1,1) model better fit. So we choose Garch (1,1) model as the marginal distribution function for Garch-Copula Model.

3.2. Copula Function Correlation Coefficient Analysis

Pearson linear correlation parameter ρ: Means the linear correlation between the two series of reactions. $|\rho|$ closer to 1, the stronger the correlation between the two series. When $\rho = 0$, Description two serially uncorrelated, that there is no linear correlation.

Kendall correlation coefficient τ : When τ unchanged, description two sequences strictly monotonic and monotonic same transformation.

Spearman correlation coefficient ρ_s : Also a validation of the sequence monotone.

Table 2. Two Kinds of Copula Function Correlation Parameters

	t-copula	Normality copula
Pearson linear correlation ρ	0.8551	0.8303
Degree of free	3.6960	-
Kendall correlation coefficient τ	0.6236	0.6236
Spearman correlation coefficient ρ_s	0.6538	0.8176

After calculated Kendall=0.6538， Spearman=0.8344, t-Copula and normality Copula correlation parameter as shown in Table Ⅱ.

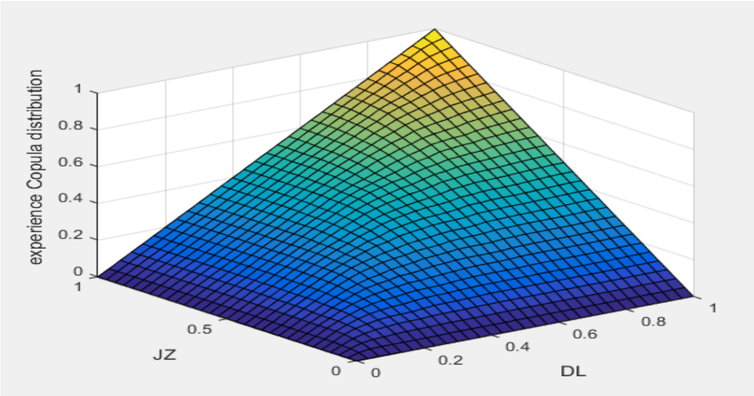

Figure 4. Experience Copula Distribution

The Kendall and spearman coefficient in Table Ⅱ compared with the Kendall and spearman coefficient calculated. The results of the t-copula more similar to Kendall correlation coefficient. However the Normal Copula in Spearman correlation coefficients better performance. To further more accurate analysis of two fit copula functions. We employ more scientific computing squared Euclidean distance method. First create experience Copula distribution graph shown in Figure 4.

3. Conclusion

This article discusses the use of Copula technical issues related to financial markets. Building a bivariate copula-garch model financial time series, and empirical study in power and building module of shanghai stock series. Because the volatility of financial market time series showed more variable, volatility clustering, spike and heavy tail. This paper analyzes the multiple Garch model family for financial realization sequence fitting situation, determines Garch (1,1) applies to the establishment of financial series model, And can be used as marginal distribution of copula function. In the empirical analysis, we introduce two kinds of Copula function(normality copula and t-Coupla) calculate the correlation coefficient of two types of functions. And analyzed to give t-Copula function use Garch(1,1) as marginal distribution fitting better.

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