International Journal of Hybrid Information Technology
Vol.9, No.1 (2016), pp. 339-352
http://dx.doi.org/10.14257/ijhit.2016.9.1.29

Incremental Optimization of Hub and Spoke Network under Changes of Spokes' Number and Flow

Yanfeng Wang[1,2] and Youfang Huang[1]
[1]*Logistics Research Center, Shanghai Maritime University, China*
[2]*School of Economics and Management, Shanghai Maritime University, China*
guiyan_1234@163.com

Abstract

Hub and spoke network has been widely used in air transport, road freight, shipping network and urban distribution areas as part of the strategic decisions, which may have a profound effect on the future of enterprises. In view of the existing network structure, as time goes on, the number of spokes and the flow change because of different sources of uncertainty. Hence, the incremental optimization of the capacitated single-allocation hub and spoke network problem is considered in this paper, and the decisions to be made to cope with change comprise (i) whether new hubs need to be setup and their selection, (ii) whether the capacity levels of original hubs need to be adjusted, and their new capacity levels, (iii) whether the original hubs need to be closed, (iv) the new allocation of the spoke nodes to the hubs, (v) the flow distribution through the sub network defined by all the hubs. The objective is to minimize the total cost, which includes the setup costs for the new hubs, the closure costs and the adjustment costs for the original hubs as well as the flow routing costs. Two mixed-integer linear programming formulations are proposed and analyzed for this problem. A set of computational tests are performed, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on the network.

Keywords: *hub and spoke network, incremental optimization, hub location, capacity choice, single-allocation*

1. Introduction

Hub and spoke network consists of a set of nodes and some flow shipped between them. Some nodes chosen to be the hubs will consolidate, process and redistribute the flow, others not chosen are called spokes. One important reason for considering hub and spoke network is the possibility of taking advantage of economies of scale. Hub and spoke network is widely used in areas such as public transportation, telecommunications and logistics distribution systems [1].

Hub and spoke network is first put forward by Goldman in 1969, then O'Kelly proposed the mathematic model for hub and spoke network [2], which draw worldwide concerns and deepen research from numerous scholars. Hub and spoke network can be classified according to different standards. One important perspective concerns the allocation pattern of the spokes to the hubs. Based on this standard, it is often considered: single allocation and multiple allocation. Regarding the single allocation, each spoke is connected to a single hub. This is the case, for instance in literatures [3-9]. As far as the multiple allocation, a spoke node can send and receive flow from more than one hub. Some works considering this allocation pattern include [10-16]. Some works address both allocation patterns as it is the case in the papers [1-2, 17]. When considering some type of constraint exists in the flow through the network, capacity constraints may indicate the nodes or to the edges. Some works regard to nodes capacity [3-4, 16]. Besides, capacity

ISSN: 1738-9968 IJHIT

constraints both in the hubs and in the links have been studied together in research [18]. Most of the above literatures are about the static hub and spoke network problems using a variety of models and algorithms. They ignore the number of spokes and flow change with the times and circumstances, and there is little research on the incremental optimization of hub and spoke network driven by the number of spokes and the flow.

This paper studies the incremental optimization problem of the single-allocation hub and spoke network with a limited capacity, which means that each spoke is allocated to one and only one hub and also that there are capacity constraints. In real life, most enterprises have constructed their initial hub and spoke network in public transportation, urban distribution and logistics areas. In view of the existing network structure, as time goes on, it is worth mentioning that the number of spoke nodes and the flow routing though the network change because of different sources of uncertainty. For instance, CHINA DEPPON LOGISTICS, a leading integrated service-oriented logistics provider with the national 5A qualification, is dedicated to the domestic road and air freight services. Deppon has constructed the initial hub and spoke network, which includes about 50 hub centers and 4800 service points within China by the end of July 2014. With the growth in demand, its service points are extended from the original 4800 to the present 5400 in the last months, and the corresponding goods flows have also changed.

In such cases, it makes sense to research the incremental optimization of the capacitated single-allocation hub and spoke network problem in this paper. As the spokes' number and flow change during each time period, the policy makers should adopt a series of strategies to cope with the change, and the decisions to be made comprise (i) whether new hubs need to be setup and their selection, (ii) whether the capacity levels of original hubs need to be adjusted, and their new capacity levels, (iii) whether the original hubs need to be closed, (iv) the new allocation of the spoke nodes to the hubs, (v) the flow distribution through the sub network defined by all the hubs. The construction of new hubs or closure of original hubs needs to spend sizable amounts of money on infrastructure, and consume huge manpower physical resource. The adjustment of capacity levels for original hubs also gives rise to the expenditure. Therefore, the decision makers need to make choice by comprehensive consideration, to minimize the total cost and meet the needs.

The remainder of this paper is organized as follows. In Section 2 two mixed-integer linear programming formulations are proposed and analyzed for this problem. One is the initial optimization model as the base, and the other is the incremental optimization model of hub and spoke network. In Section 3 a set of computational tests are performed, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on the network. The paper ends with some conclusions drawn from the work presented and some directions for further research.

2. Formulations and Properties

In this section, we introduce the basic setting for our analysis. Two mixed-integer linear programming formulations for the capacitated single-allocation hub and spoke network problem are proposed and discussed. We consider an initial optimization model in order to get a more focused demonstration of the incremental optimization model proposed.

Suppose there are no direct links between spoke nodes, and the connection must be realized by hubs. We assume the hub level network is a complete graph. It should be noted that, for $k \in N$, the following relation is assumed: $T_k^{q_1} < T_k^{q_2}$ for $q_1, q_2 \in Q$, such that $q_1 < q_2$. Regarding the fixed setup cost F_k^q, can also include fixed operation costs for the hubs (dependent on the capacity level) when they exist.

Hereafter, the following notation is considered.

N	set of nodes ($n=1,2,...,N$)
Q	set of capacity levels ($q=1,2,...,Q$)
F_k^q	fixed setup cost for installing a hub with capacity of level q at node k $(k\in N, q\in Q)$
T_k^q	capacity of a hub installed at node k with a level of capacity q $(k\in N, q\in Q)$
w_{ij}	flow originated at node i that is destined to node j $(i,j\in N)$
O_i	total flow originated at node i $(i\in N)$, $O_i=\sum_{j=1}^{N} w_{ij}$
D_i	total flow destined to node i $(i\in N)$, $D_i=\sum_{j=1}^{N} w_{ji}$
CC	collection cost per unit of flow and per unit of distance from a spoke to a hub
DC	distribution cost per unit of flow and per unit of distance from a hub to a spoke
TC	transfer cost per unit of flow and per unit of distance between hubs. It is assumed that TC is smaller than CC and DC
d_{ij}	distance between node i and node j . It is assumed that $d_{ii}=0$ and that the distances satisfy the triangle inequality $(i,j\in N)$
λ	the weight of setup costs
θ	the weight of flow shipment costs

2.1 The Initial Optimization Model

The basic decisions to be made in the initial hub and spoke network problem comprise the selection of the nodes that should become hubs and the way flow should be routed through the network. The objective is to minimize the overall cost, which includes setup costs for the hubs and flow shipment costs. With regard to the latter, it consists of collection costs, transfer costs, and distribution costs, which respectively for the flow sent from the spokes to the hubs, shipped between hubs, and sent from the hubs to the spokes. The decision variables are as follows.

$$x_{ik}=\begin{cases}1, & \text{if node } i \text{ is assigned to hub } k\\ 0, & \text{otherwise}\end{cases}, \quad i,k\in N$$

$$x_{kk}=\begin{cases}1, & \text{if node } k \text{ is selected to be a hub}\\ 0, & \text{otherwise}\end{cases}, \quad k\in N$$

$$z_k^q=\begin{cases}1, & \text{if node } k \text{ receives as a hub with capacity lever } q\\ 0, & \text{otherwise}\end{cases}, \quad k\in N, q\in Q$$

y_{ks}^i the amount of flow with origin at i that goes through hub k and hub s

The problem can be formulated as follows:

$$\min \quad f=\lambda\sum_{k=1}^{N}\sum_{q=1}^{Q}F_k^q z_k^q+\theta\left(\sum_{i=1}^{N}\sum_{k=1}^{N}CCd_{ik}O_i x_{ik}+\sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{s=1}^{N}TCd_{ks}y_{ks}^i+\sum_{i=1}^{N}\sum_{k=1}^{N}DCd_{ki}D_i x_{ik}\right) \tag{1}$$

s.t

$$\sum_{k=1}^{N}x_{ik}=1, \quad i\in N \tag{2}$$

$$x_{ik}\le x_{kk}, \quad i,k\in N \tag{3}$$

$$\sum_{i=1}^{N}y_{ks}^i-\sum_{i=1}^{N}y_{sk}^i=O_i x_{ik}-\sum_{j=1}^{N}w_{ij}x_{jk}, \quad i,k\in N \tag{4}$$

$$\sum_{\substack{s=1 \\ s\neq k}}^{N} y_{ks}^{i} \leq O_i x_{ik}, \quad i,k \in N \tag{5}$$

$$\sum_{i=1}^{N} O_i x_{ik} \leq \sum_{q=1}^{Q} T_k^q z_k^q, \quad k \in N \tag{6}$$

$$\sum_{q=1}^{Q} z_k^q \leq 1, \quad k \in N \tag{7}$$

$$x_{ik} \in \{0,1\}, \quad i,k \in N \tag{8}$$

$$y_{ks}^{i} \geq 0, \quad i,k,s \in N \tag{9}$$

$$z_k^q \in \{0,1\}, \quad k \in N, q \in Q \tag{10}$$

In the model, the objective function (1) represents minimizing the total cost. Constraint (2) makes sure that each node is a hub or is allocated to a single hub. Constraint (3) makes sure that a spoke node can only be allocated to the opened hubs. Constraint (4) is the flow balance constraint. Constraint (5) ensures that a y_{ks}^{i} can only be different from 0 if x_{ik} is equal to one and in this case, all the flow originated at node i is sent to hub k. Constraint (6) is the capacity constraint. Constraint (7) guarantees that for each hub at most one size is chosen. Finally, constraints (8), (9) and (10) are domain constraints.

2.2 The Incremental Optimization Model

Based on the existing network structure in Section 2.1, as time goes on, the number of spokes and the flow will change due to different sources of uncertainty. We propose the incremental optimization model for the capacitated single-allocation hub and spoke network problem. As the spokes' number and flow change during each time period, the policy makers should adopt a series of strategies to cope with the change, such as setup new hubs, adjust the capacity levels of original hubs, or close some original hubs, *etc*. Concerning the cost structure of the incremental optimization model, we consider setup costs for new hubs, adjustment costs for original hubs, closure costs for original hubs, and all flow routing costs.

Beyond using the variables introduced above, a new set of variables should be added to describe the incremental optimization problem. Hereafter, a few new variables can be considered in the following notation.

M	set of the initial hubs ($m=1,2,...,M$)
N	set of spoke nodes ($n=1,2,...,N$)
AC_m^q	adjustment cost of initial hub m for changing its original capacity level to the current level q $(m \in M, q \in Q)$
G_m^q	closure cost for closing the initial hub m with capacity level q $(m \in M, q \in Q)$

Here a new decision variable is added based on the original decision variables.

$$H_m^q = \begin{cases} 1, \text{ if the initial hub } m \text{ change its original capacity lever to the current lever } q \\ 0, \text{ otherwise} \end{cases}, \quad m \in M, q \in Q$$

The incremental optimization problem can be formulated as follows:

$$\min \begin{aligned} f = & \sum_{i=1}^{N}\sum_{m=1}^{M} CCD_{im} O_i x_{im} + \sum_{i=1}^{N}\sum_{k=1}^{N} CCD_{ik} O_i x_{ik} + \sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{l=1}^{M} TCd_{ml} y_{ml}^{i} + \sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{N} TCd_{mk} y_{mk}^{i} + \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{s=1}^{N} TCd_{ks} y_{ks}^{i} \\ & + \sum_{i=1}^{N}\sum_{m=1}^{M} DCD_{mi} O_i x_{im} + \sum_{i=1}^{N}\sum_{k=1}^{N} DCD_{ki} O_i x_{ik} + \sum_{k=1}^{N}\sum_{q=1}^{Q} F_k^q z_k^q + \sum_{m=1}^{M} H_m^q AC_m^q + \sum_{m=1}^{M}\sum_{q=1}^{Q} G_m^q * \left(1 - \sum_{i=1}^{N} x_{im}\right) \end{aligned} \tag{11}$$

s.t

$$\sum_{k=1}^{M} x_{im} + \sum_{k=1}^{N} x_{ik} = 1, \quad i \in N \tag{12}$$

$$x_{ik} \le x_{kk}, \quad i,k \in N \tag{13}$$

$$\sum_{l=1}^{M} y_{ml}^{i} - \sum_{l=1}^{M} y_{lm}^{i} = O_i x_{im} - \sum_{j=1}^{N} w_{ij} x_{jm}, \quad i \in N, m \in M \tag{14}$$

$$\sum_{m=1}^{M} y_{km}^{i} - \sum_{m=1}^{M} y_{mk}^{i} = O_i x_{ik} - \sum_{j=1}^{N} w_{ij} x_{jk}, \quad i,k \in N \tag{15}$$

$$\sum_{i=1}^{N} y_{ks}^{i} - \sum_{i=1}^{N} y_{sk}^{i} = O_i x_{ik} - \sum_{j=1}^{N} w_{ij} x_{jk}, \quad i,k \in N \tag{16}$$

$$\sum_{\substack{l=1 \\ l \ne m}}^{M} y_{ml}^{i} \le O_i x_{im}, \quad i \in N, m \in M \tag{17}$$

$$\sum_{m=1}^{M} y_{km}^{i} \le O_i x_{ik}, \quad i,k \in N \tag{18}$$

$$\sum_{\substack{s=1 \\ s \ne k}}^{N} y_{ks}^{i} \le O_i x_{ik}, \quad i,k \in N \tag{19}$$

$$\sum_{i=1}^{N} O_i x_{im} \le \sum_{q=1}^{Q} T_m^q H_m^q, \quad m \in M \tag{20}$$

$$\sum_{i=1}^{N} O_i x_{ik} \le T_k^q z_k^q, \quad k \in N \tag{21}$$

$$\sum_{q=1}^{Q} H_m^q \le 1, \quad m \in M \tag{22}$$

$$\sum_{q=1}^{Q} z_k^q \le 1, \quad k \in N \tag{23}$$

$$x_{ik}, x_{im} \in \{0,1\}, \quad i,k \in N, m \in M \tag{24}$$

$$y_{ml}^{i}, y_{mk}^{i}, y_{ks}^{i} \ge 0, \quad i,k \in N, m,l,s \in M \tag{25}$$

$$z_k^q, H_m^q \in \{0,1\}, \quad k \in N,\ m \in M, q \in Q \tag{26}$$

The objective function (11) represents the total cost of incremental optimization to be minimized. Constraint (12) assures that each node is a hub or is allocated to a hub. Constraint (13) makes sure that a spoke node is only allocated to operating hubs. Constraints (14), (15) and (16) are flow balance constraints. Constraints (17), (18) and (19) ensure that y_{ml}^{i}, y_{km}^{i}, y_{ks}^{i} can only be different from 0 if x_{im}, x_{ik} are respectively equal to one. Constraints (20) and (21) are the capacity constraints. Constraints (22) and (23) guarantee that for each hub at most one size is chosen. Finally, constraints (24), (25) and (26) are domain constraints.

3. Computational Experience

In this Section, we present computational analysis with the initial and incremental optimization models, to assess the effects of change of spokes' number and flow on the resulting solutions.

3.1 The Initial Optimization Analysis and Conclusion

For this analysis, we assume there are twenty nodes in the network. Random generated twenty nodes, and the demands between these nodes are as followed in Table 1. The collection cost and distribution cost per unit are taken equal to one, that means $CC = DC = 1$. For the value of unit transfer cost TC, assumed it is smaller than the collection cost and distribution cost per unit, so we let TC is taken equal to 0.5. The weights of setup costs and flow shipment costs are taken equal to 0.5.

Four capacity levels are available which are equal to 240000, 320000, 400000 and 480000 for each hub. The fixed setup cost and closure cost for the hub depend on the capacity level chosen, which can be seen in Table 2. Adjustment cost for the hub depends on the original capacity level and adjusted capacity level. Adjustment costs between different capacity levers are as seen in Table 3.

The decision maker needs to choose some nodes as hubs to construct distribution centers and deliver goods to meet demands. This is the initial hub and spoke network. Then, as time goes on, the number of spokes and the flow may change because of different sources of uncertainty. Then the leader should make choice to meet new demands and decrease the total cost.

Table 1. Demands between Nodes（*100）

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	16	7	42	45	45	42	49	44	46	41	46	6	24	12	34	4	25	7	29
2	12	0	10	26	49	45	48	49	43	42	45	48	32	15	7	19	15	37	6	17
3	30	9	0	10	10	41	49	44	43	40	44	49	39	38	37	11	23	16	8	20
4	45	47	14	0	42	41	47	44	49	39	38	37	11	23	16	35	31	17	24	18
5	38	46	10	1	0	27	22	37	41	34	49	43	15	42	27	9	18	23	6	32
6	23	21	1	34	8	0	17	7	26	23	50	22	9	16	6	17	5	42	19	28
7	41	3	22	42	19	17	0	40	45	40	22	34	26	7	23	19	14	26	8	12
8	22	18	47	25	43	27	48	0	10	11	25	11	19	28	36	18	17	28	10	28
9	31	41	23	35	43	36	26	15	0	29	30	42	32	16	27	18	39	24	33	10
10	40	2	41	41	30	45	44	33	38	0	3	32	25	6	12	39	19	42	14	26
11	40	21	40	33	42	34	37	32	35	14	0	35	12	7	29	34	25	14	28	12
12	7	46	48	2	47	38	20	9	2	2	41	0	19	28	25	8	10	23	14	27
13	32	11	16	30	5	48	2	6	2	9	3	33	0	26	16	24	43	16	34	9
14	19	15	46	13	13	27	44	33	37	12	34	26	8	0	21	9	5	19	12	24
15	40	24	22	30	40	26	45	25	25	44	3	48	2	16	0	29	24	8	25	18
16	27	12	10	35	2	12	40	39	24	2	4	32	17	6	12	0	15	9	15	27
17	18	42	45	11	46	24	5	36	45	25	26	40	14	23	6	25	0	14	8	16
18	47	10	49	6	36	31	13	45	30	9	5	23	17	8	26	12	34	0	9	15
19	2	14	8	37	14	25	6	17	8	35	26	9	10	14	28	35	16	27	0	10
20	18	41	3	19	4	25	19	37	19	21	32	18	36	24	5	9	17	6	12	0

Table 2. Capacity, Fixed Setup Cost and Closure Cost for Hub

Capacity level	1	2	3	4
Capacity	240000	320000	400000	480000
Fixed setup cost	280000	336000	392000	448000
Closure cost	6000	7200	8400	9600

Table 3. Adjustment Costs for Different Capacity Levels

Adjusted capacity level	Original capacity level			
	1	2	3	4
1	5000	10000	14000	16000
2	10000	5000	12000	14000
3	14000	12000	5000	13000
4	16000	14000	13000	5000

The instance of the initial optimization is solved to optimality with MATLAB version 7.0. We do not report the CPU times separately for each instance since all the instances are solved within a few seconds.

We present the results in Figure 1 for the initial optimization of hub and spoke network problem. In Figure 1, the sequence numbers of 9, 14 and 16 are chosen to be the hubs (represented by stars) that consolidate, process and redistribute the flow, and others not chosen are spokes (represented by spots). The capacity lever of each hub is the same, with the lever of 2 and the capacity of 320000. The allocation patterns between spokes and hubs are described by solid line. The links between hubs are described by dotted line. This indicates that the decision maker should setup three hubs with the capacity of 320000, and the total cost of the initial hub and spoke network is 30940000.

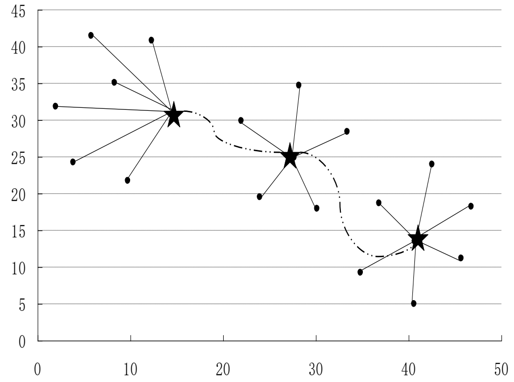


Figure 1. The Initial Hub and Spoke Network Structure

3.2 The Incremental Optimization Analysis and Conclusion

Based on the initial network structure in Section 3.1, considering in the actual operation of the process, as time goes on, the number of spokes and the flow change because of different sources of uncertainty, such as the uncertainty of the market and the adjustment of enterprise strategic layout, the number of spokes may be increased or decreased, and the flow will also change, so the decision maker needs to do corresponding measures in different periods to cope with the changing circumstances. This paper designs a series of related experiments for the incremental optimization, respectively for spoke nodes and flow change scenarios. Then we will study the influence of the variables in the model on the network.

Experiment one: Incremental optimization analysis under the change of the number of spoke nodes. Based on the initial hub and spoke network in Figure 1, two spokes are increased at each time, and the corresponding demands are generated at random. Then we solved the incremental optimization model to optimality using MATLAB version 7.0 on the same computer with the properties given in Section 2.2. The results are presented in Figure 2.

Experiment two: Incremental optimization analysis under the change of flow. As far as the demands are concerned, they may be estimated in advance, however, the time elapsed between the moment the decision is made and the moment the network starts operating may make such information completely obsolete. Based on the initial hub and spoke network in Figure 1, the flow changes in certain proportion at each time. Then we solved the incremental optimization models to optimality using MATLAB version 7.0 on the same computer with the properties given in Section 2.2. The results are presented in Figure 3.

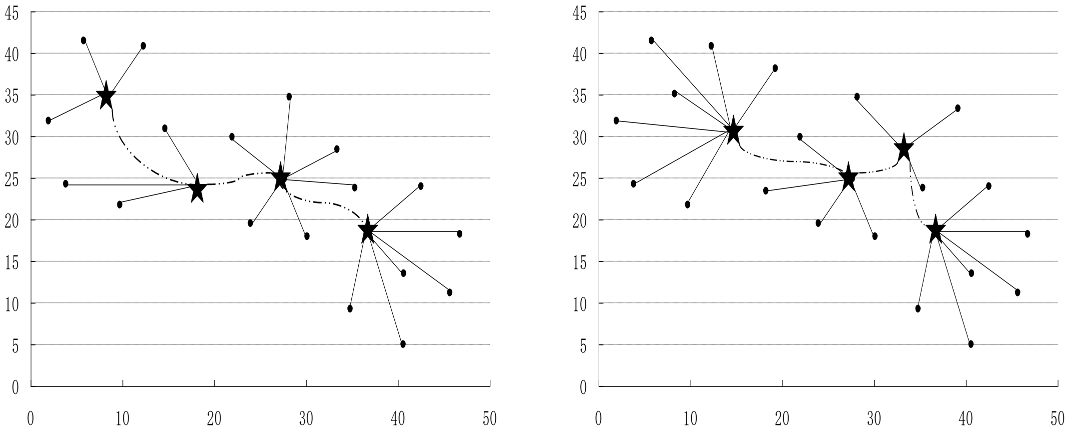
Experiment three: Pareto analysis for setup costs and flow shipment costs of the network. Based on the initial hub and spoke network in Figure 1, for the value of λ, $\lambda \in \{0.00, 0.01, ..., 1.00\}$, $\theta = 1 - \lambda$, then we solved the incremental optimization models to optimality using MATLAB software. The results are presented in Figure 4.

Experiment four: Analysis the influence of fixed setup cost on the network. In practice, the actual fixed setup cost varies due to many factors such as the price of the property or the price of the raw-materials. Considering the variation in the fixed setup cost, we make the fixed setup cost randomly from the interval $F_k^q \in \left[0.2F_k^q, 2F_k^q\right]$ for each potential hub. Then we solved the optimization models to optimality using MATLAB software on the same computer. The results presented in Figure 5.

Based on the four experiments, by analysis and calculation this paper draws the relevant conclusions as follows:

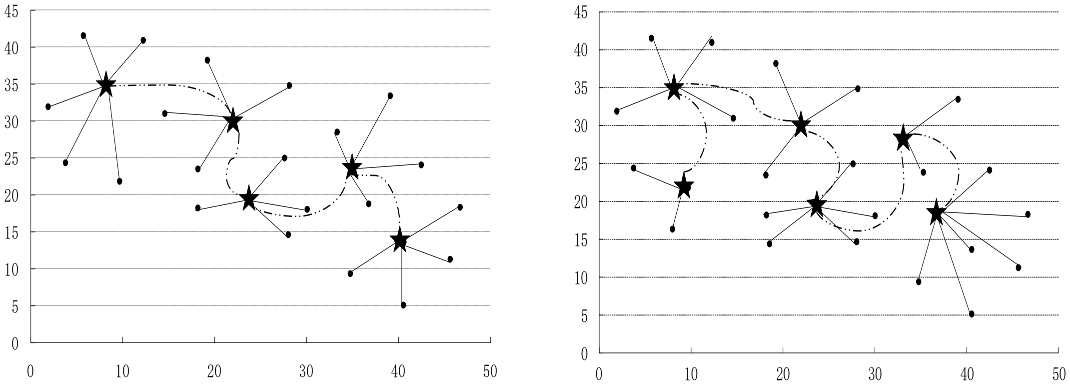
From Experiment one results can be seen, in Figure 2, (a) to (f) show the selection of hubs and the allocation between spokes and hubs under the changes of the number of spokes at each period of time. The new network structure at each time is determined by the changes of spokes for achieving the goal of the minimum total cost including the setup costs, flow shipment costs, adjustment costs and closure costs for the decision makers. Figure 2 shows when the number of spokes increases by 2, 4, 6 and 8, in turn the number of nodes chosen as hubs is 4, 4, 5 and 6. Compared with the initial network, the number, location and capacity levels of hubs are all changed, and the allocation relationship between hubs and spokes also changed. Similarly, when the number of spoke nodes decreases by 2 and 4, in turn the number of nodes chosen as hubs is 3 and 3, and the location, capacity level and allocation relationship between hubs and spokes are also changed.

From Experiment two results can be seen, in Figure 3, (a) to (f) show the selection of hubs and the allocation between spokes and hubs under the changes of the flow at each period of time. The new network structure is determined at each time by the changes of demands for achieving the goal of the minimum total cost. From Figure 3 we can see when the flow changes each time by 0.5, 0.75, 1.25, 1.5, 1.75 and 2 times of the initial flow, in accordance with the number of nodes chosen as hubs are 2, 3, 4, 5, 6 and 7. The hub location, capacity level and allocation relationship between hubs and spokes are also changed.

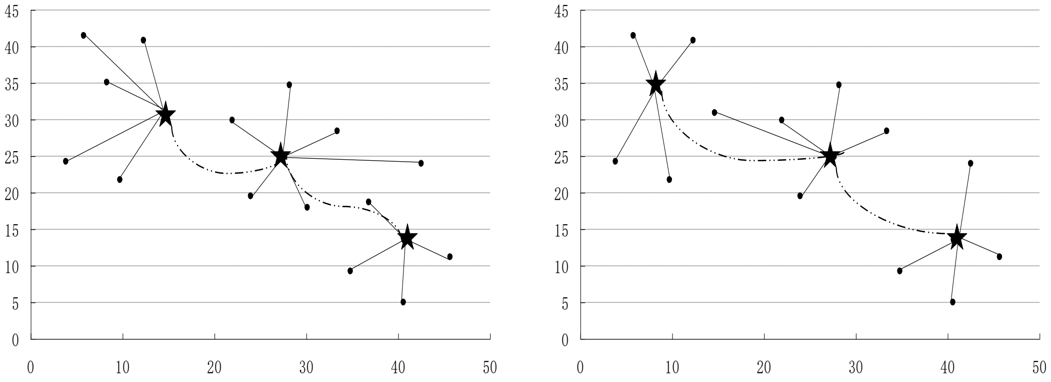


(a) Adding Two Spokes

(b) Adding Four Spokes



(c) Adding Six Spokes **(d) Adding Eight Spokes**



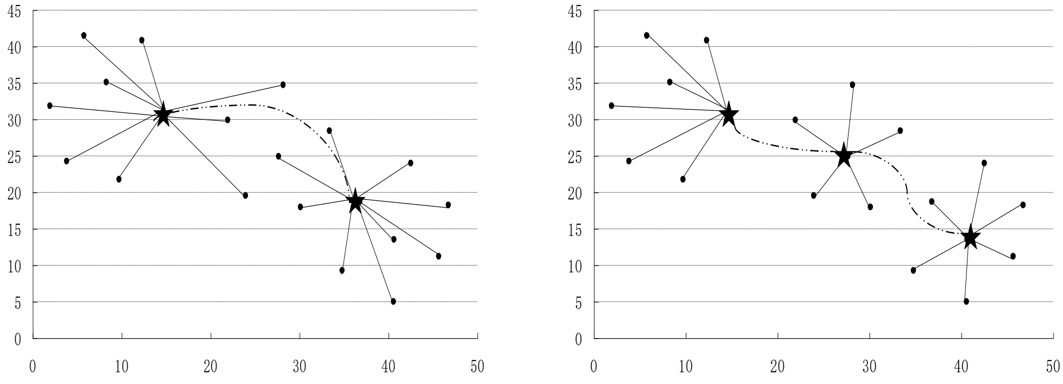
(e) Removing Two Spokes **(f) Removing Four Spokes**

Figure 2. Incremental Optimization Results under the Change of the Number of Spokes

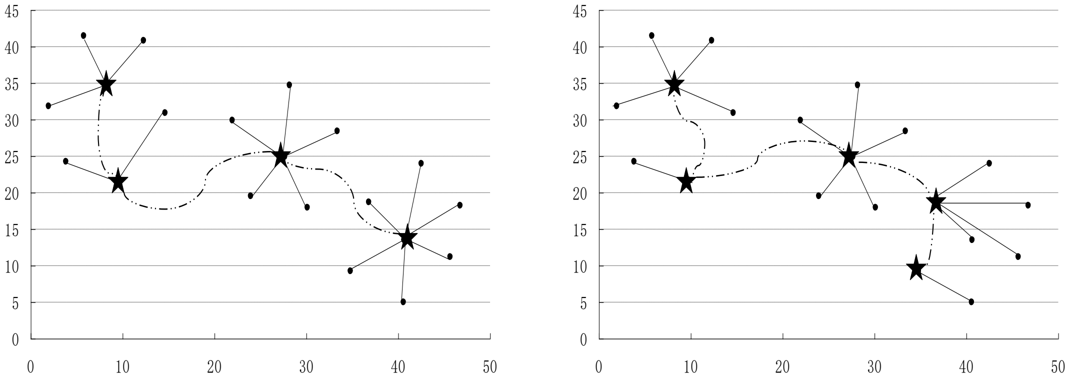
From Experiment three results can be seen, in Figure 4, there is a reciprocal relationship between setup costs and flow shipment costs. When the setup costs increase by 84%, the corresponding flow shipment costs fall by 57.3%. With the change of weight coefficients, the variation of setup costs becomes more obvious. Taking the cases under the different weights to analyze, when the weight coefficient of setup costs is 0.32, the setup costs and flow shipment costs are 14560000 and 18065600, the sequence numbers for the five hubs selected are node 2, 8, 9, 15 and 18, and the capacity levels respectively are 1, 1, 2, 1 and 1; when the weight coefficient is 0.78, the setup costs and flow shipment costs are 8960000 and 24740000, the sequence numbers for the two hubs selected are node 16 and node 18, and the capacity levels respectively are 4 and 4. The setup costs in the above two scenarios are reduced by 38.5%, while the flow shipment costs correspondingly increase by 36.9%. In addition, during the incremental optimization we can see: with the changes of the number of spokes and the flow at each time period, when the weight coefficient of setup costs is 0.32, the numbers of hubs change very quickly, and the capacity levels of initial hubs are almost unchanged; while the weight is 0.78, the capacity levels of original hubs are continuously changing from low level to high level, and the numbers of new or closed hubs are rarely.

This means that, for the incremental optimization of hub and spoke network under the changes of the number of spokes or the flow, the decision makers tend to be preferentially of choosing to setup new hubs or close initial hubs to meet the needs in the scenarios of smaller setup cost weight. But for the scenarios of larger weight of setup costs, the priority should be given to change the capacity levels of original hubs, and the number of hubs does not change obviously. For the later scenarios, the main reason is that the setup costs have a large influence on the total cost. Therefore, the larger weight is given to the setup costs, and its influence on the total cost is greater. In general, the adjustment cost of changing the capacity level of original hub is smaller than the fixed setup cost of new hub, hence, in order to seek the lowest total cost, the decision makers give priorities on changing the capacity level of original hub until the maximum capacity is exceeded, then they may choose to setup new hubs. On the contrary, for the former with smaller weight, the effect of setup costs on the total cost is small while the flow shipment costs become an important factor influencing the total cost. In this case it may cause a sharp increase in the flow shipment costs only by changing the original hub levels. It is effective on reducing the total cost of the optimization network that changing the number of hubs. Therefore, for the smaller weight scenarios, policy makers appear to be preferentially of choosing to setup new hubs or close the initial hubs to to accommodate changing requirements.

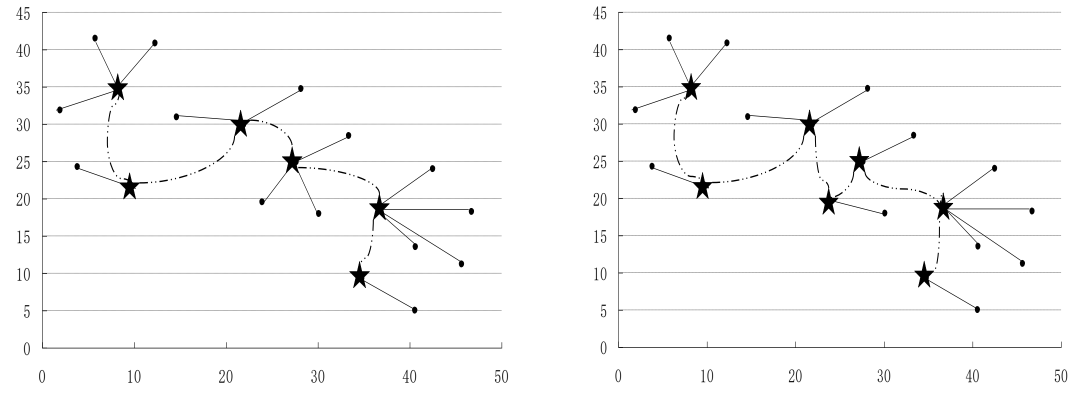
By the study on the results of Experiment four, in Figure 5, the fixed setup cost will have a certain impact on the hub and spoke network. With the fixed setup cost changes from initial 0.2 times increase to 2 times, the setup costs first get slow and then fast while the flow shipment costs first get fast and then slow to be moderation, and the total cost is approximately linear upward trend. Before reaching the 0.8 times, the fixed setup cost has an obvious influence on the three costs, especially for the flow shipment costs. When up to 0.8 times, with the increase of fixed setup cost, the setup costs and the total cost increase greatly, but the change rate of flow shipment costs remains basically unchanged. Therefore, in order to reduce the costs of the hub and spoke network, it is necessary to improve the management level and the operational efficiency, to make full use of hub capacity efficiently, and to increase the amount of transfer goods to meet the needs.



(a) 0.5 Times of the Initial Flow **(b) 0.75 Times of the Initial Flow**



(c) 1.25 Times of the Initial Flow **(d) 1.5 Times of the Initial Flow**



(e) 1.75 Times of the Initial Flow **(f) 2 Times of the Initial Flow**

Figure 3. Incremental Optimization Results under the Change of Flow

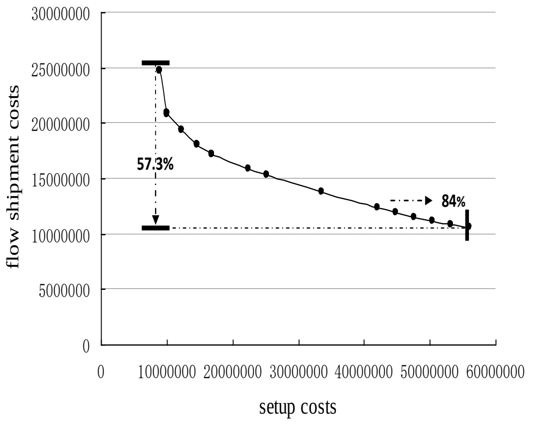


Figure 4. Pareto Result

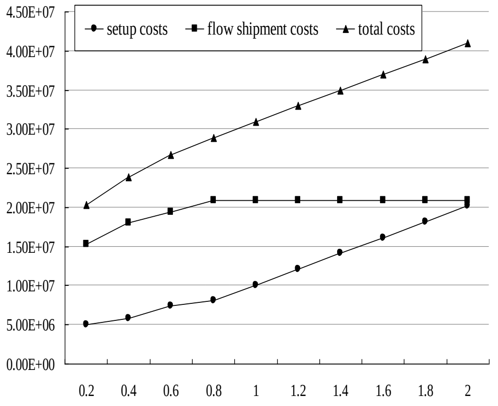


Figure 5. The Impact of Fixed Setup Cost on Network

4. Conclusion

In this paper, we study the incremental optimization problem of the capacitated single-allocation hub and spoke network. In view of the existing network structure, as time goes on, the number of spokes and the flow change because of different sources of uncertainty. The objective is to minimize the total cost, which includes the setup costs for the new hubs, the closure costs and the adjustment costs for the original hubs as well as the flow routing costs. Two mixed-integer linear programming formulations are proposed and analyzed for this problem. A set of computational tests are performed, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on the network.

The results show that when the number of spokes or the flow changes, the hub location, capacity level and allocation relationship between hubs and spokes are also changed. There is a reciprocal relationship between the setup costs and the flow shipment costs. When the setup costs increase by 84%, the corresponding flow shipment costs fall by 57.3%. For the instances of smaller weight of setup costs, the decision makers tend to be preferentially of choosing to setup new hubs or close initial hubs to meet the needs. But for the larger weight scenarios, the priority should be given to adjust the capacity levels of original hubs. The fixed setup cost has a great influence on the total cost and setup costs, and has a certain influence on the flow shipment costs for the hub and spoke network.

Two lines of research can be drawn from here. One regards more complex incremental optimization problem of hub and spoke network such as problems with dynamic network design decisions. Another line of research regards the development of heuristic procedures for giving good quality feasibly solutions for large instances of the problem.

Acknowledgements

This work is supported by the Doctoral Fund Project of Chinese Ministry of Education (No.20123121110004), Graduate Innovation Fund Project of Shanghai Maritime University (No.2013ycx061), Excellent Doctoral Dissertation Cultivation Project of Shanghai Maritime University (No.2014bxlp002) and China National Science Foundation (No.71301101).

References

[1] A. S. Alumur, S. Nickel and F. S. da-Gama, "Hub location under uncertainty", Transportation Research Part B: Methodological, vol. 46, no. 4, **(2012)**, pp. 529-543.

[2] M. O'Kelly, D. Bryan, D. S. Kapov and J. S. Kapov, "Hub network design with single and multiple allocation: a computational study", Location Science, vol. 4, no. 3, **(1996)**, pp. 125-138.

[3] I. Correia, S. Nickel and F. S. da-Gama, "The capacitated single-allocation hub location problem revisited: a note on a classical formulation", European Journal of Operational Research, vol. 207, no. 1, **(2010)**, pp. 92-96.

[4] I. Correia, S. Nickel and F. S. da-Gama, "Hub and spoke network design with single-assignment, capacity decisions and balancing requirements", Applied Mathematical Modelling, vol. 35, no. 10, **(2011)**, pp. 4841-4851.

[5] R. S. de-Camargo, G. Miranda and R. P. M. Ferreira, "A hybrid Outer-Approximation/Benders Decomposition algorithm for the single allocation hub location problem under congestion", Operations Research Letters, vol. 39, no. 5, **(2011)**, pp. 329-337.

[6] R. S. de-Camargo and G. Miranda, "Single allocation hub location problem under congestion: Network owner and user perspectives", Expert Systems with Applications, vol. 39, no. 3, **(2012)**, pp. 3385-3391.

[7] M. Labbé and H. Yaman, "Projecting flow variables for hub location problems", Networks, vol. 44, no. 2, **(2004)**, pp. 84-93.

[8] M. Labbé, H. Yaman and E. Gourdin, "A branch and cut algorithm for the hub location problems with single assignment", Mathematical Programming, vol. 102, no. 2, **(2005)**, pp. 371–405.

[9] J. Puerto, A. B. Ramos and A. M. R.Chía, "A specialized branch & bound & cut for Single-Allocation Ordered Median Hub Location problems", Discrete Applied Mathematics, vol. 161, no. 16, **(2013)**, pp. 2624-2646.

[10] N. Boland, M. Krishnamoorthy, A. T. Ernst and J. Ebery, "Preprocessing and cutting for multiple allocation hub location problems", European Journal of Operational Research, vol. 155, no. 3, **(2004)**, pp. 638–653.

[11] I. Contreras, E. Fernández and A. Marín, "Tight bounds from a path based formulation for the tree of hub location problem", Computers & Operations Research, vol. 36, no. 12, **(2009)**, pp. 3117–3127.

[12] S. García, M. Landete and A. Marín, "New formulation and a branch-and-cut algorithm for the multiple allocation p-hub median problem", European Journal of Operational Research, vol. 220, no. 1, **(2012)**, pp. 48-57.

[13] H. Hamacher, M. Labbé, S. Nickel and T. Sonneborn, "Adapting polyhedral properties from facility to hub location problems", Discrete Applied Mathematics, vol. 145, no. 1, **(2004)**, pp. 104-116.

[14] J. Kratica, "An electromagnetism-like metaheuristic for the uncapacitated multiple allocationp-hub median problem", Computers & Industrial Engineering, vol. 66, no. 4, **(2013)**, pp. 1015-1024.

[15] A. Marin, "Formulating and solving splittable capacitated multiple allocation hub location problems", Computers & Operations Research, vol. 32, no. 12, **(2005)**, pp. 3093-3109.

[16] J. Sender and U. Clausen, "Heuristics for solving a capacitated multiple allocation hub location problem with application in German wagonload traffic", Electronic Notes in Discrete Mathematics, vol. 41, no. 5, **(2013)**, pp. 13-20.

[17] A. T. Ernst and M. Krishnamoorthy, "An exact solution approach based on shortest-paths for p-hub median problems", Informs Journal on Computing, vol. 10, no. 2, **(1998b)**, pp. 149-162.

[18] M. Sasaki and M. Fukushima, "On the hub-and-spoke model with arc capacity constraints", Journal of the Operations Research Society of Japan, vol. 46, no. 4, **(2003)**, pp. 409-428.

Authors

Yanfeng Wang, is a Ph.D. candidate in management science and engineering at Shanghai Maritime University. She obtained her master degree in logistics engineering from Ocean University of China, 2012. Her main research interests are logistics strategy and planning, supply chain management.

Youfang Huang, received Ph.D. degree from Tongji University in 1998, and got his bachelor's and master's degrees in 1982 and 1989 respectively. He is currently a professor and doctoral tutor of Shanghai Maritime University. He serves as President of Shanghai Maritime University and Chairman of logistics teaching guiding committee of Chinese Ministry of Education. His main research interests are logistics strategy and planning, logistics information systems, procurement and supply chain management.