Personalized Educational Process Models with Spacing Effect

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Abstract

Teaching new material and reviewing what has already been taught are two important activities in the educational process. Different students with different learning abilities need to review at different rates, that is to say, a personalized teaching process is necessary. Review has a spacing effect, namely it is most useful only if it is executed neither too soon nor too late. How should designers of educational software schedule the learning process to satisfy the need that different students learn different material? We present a mathematical model to capture the issue in idealized form. The learning abilities of students and the spacing effect of review are modeled as some input parameters' constraints on the schedule. Our results include the optimal scheduling in accordance with which tutoring systems teach one student specific material, characteristics of the rate at which new material can be introduced under different conditions, and the reproduction of ladder-like educational process.

Keywords: Educational process model, Spacing effect, Teaching/learning strategies, Personalized

1. Introduction

The invention of "learning software as effective as a private tutor" is not only one expectation of future scientific research of President Barack Obama [1], but also one possible thing that will affect human life by a commitment to scientific research. Current research focuses on the empirical research on how people learn [2-7]. Such research endeavors to provide principles for how to learn more effectively, including organization of learning content, content presentation, feedback and control of educational process and so on. Tutoring systems need to incorporate these empirical results to realize the aim that learning software is as effective as a private tutor [8-11]. Timothy P. N. etc. have done the first attempt in this direction [12]. They are motivated by the goal of creating different schedules for different students and imagine that the designers of the educational software can firstly specify a set of spacing constraints to capture the need of students.

In real-life educational processes, however, different students learn the same material at different rates, while the same student learns different material at different rates. Can educational software realize personalized tutoring? How does learning abilities of students affect the educational process? How do different types of material affect the process?

Here we develop and analyze an idealized mathematical model for personalized tutoring systems which incorporate the spacing effect between adjacent presentation of learning material into educational software. This model mainly considers the following problem: the tradeoff between teaching new material and reviewing what has already been taught for different students. For example, the software for learning foreign languages must determine when to learn new words and when to review old words. The issue is similar to that faced by a foreign language teacher deciding how to arrange and organize his course according to students' knowledge and teaching goals.

Review is an important activity in the learning process and it is useful only if it is neither too soon nor too late. This phenomenon is called spacing effect in psychology. The spacing effect is among the best known phenomena in cognitive psychology, and many theoretical explanations have been suggested and many empirical studies have been made [2-7]. Jeroen G. R. presents a model of spacing effect by generalizing the SAM model [4]. And Philip I. P. etc. present an activation-based model of spacing effect based on practice and forgetting effects on vocabulary memory [3]. Philip's model is better than Jeroen's model [3], and we adopt Philip's model to describe spacing effect in this paper. For further results and background, see reference [2-7].

Can we develop a unified scheduling method for different students learning different material? One unified scheduling method is more practical and effective than many different scheduling methods for different students in educational software. With such educational software in mind, we envision a system in which the software designer only needs to determine the feature of students and material, and the optimal scheduling for each student can be implemented by the unified scheduling method. What we find, however, is that the scheduling problems are mathematically subtle: Existing techniques do not handle united scheduling problems.

Our main contribution is to develop an approach for reasoning about the feasibility of united scheduling under spacing effect. We begin by introducing a stylized mathematical model for learning ability and spacing effect of review, and then consider the design of optimal scheduling.

2. Models

In general, we model the educational process as a sequence of abstract educational units, and the learning ability and the spacing effect of review on different material are determined by four equations with five input parameters. The design goals of educational software are classified into two types: infinite perfect learning and cramming. The details are as follows.

2.1 The Educational Process

The key of educational software is to implement the real-life educational process. We consider the process as a sequence of abstract educational units which represent all kinds of learning contents, such as the definitions of vocabulary words. The sequence describes the order in which educational units are presented including introduction and review. The first emergence of one unit in the sequence indicates that the unit is introduced at the corresponding step. The subsequent emergence of the same unit means that repeated review occurs at different steps. For example, the sequence u1, u2, u1, u3, u2, u3, u1,..., indicates that educational unit u1 was introduced at the first step and repeatedly reviewed at the third step and the seventh step.

The real-life educational process is complex, nuanced and context sensitive. But in order to obtain the interest of generality as well as mathematical tractability, this model regards all learning units as equal, and it ignores all possible relationships between units. This kind of simplification doesn't affect the relationship essence of the tradeoff between teaching new material and reviewing, and helps us to create a formalism that captures the spacing effect of review.

2.2 Learning Ability and Review Spacing

Effective review can contribute to building up the ability to recall the material longer and longer without seeing the material. Premature review will reduce the benefit of review, and too late review may mean that students have forgotten and they may need to relearn. In fact, an optimal schedule of review exists in the learning process and much empirical work in psychology was focused on the optimal review in the past one hundred years. Many principles for the review on expanded retrieval have been well established in the reference [3], more generally, see reference [2-7].

To capture the need to review one unit in the sequence, we need a simple formalism that can easily depict the learning ability difference of students and the decay property of learning material [2]. We wish to leave some tunable parameters under the control of the software designer, motivated by the goal of creating suitable scheduling for different students and different material. Therefore, the learning ability of students is defined as Equation (1), which shows that the mastery degree of knowledge changes with time if each review occurs exactly on the optimal step. In Equation (1), b and d are constants, which are defined by the software designers according to the long term performance of students learning. Note that the variable t begins from the introduction time of one unit.

$$g(t) = 1 - b \cdot t^{-d}$$
 (1)

How should we describe the spacing effect of review? We model this problem as a recursive process with Equation (2) and Equation (3) [3]. Equation (2) is the decay rate of one unit after k - 1 times of review. In Equation (2), c is the decay scale parameter and a is the initial decay rate on the introduction of one unit, these two parameters are also determined by the software designers according to different learning material. Equation (3) is the effect factor of review after k times of review, which depends mainly on ti, namely the spacing from the ith presentation time to the current time. For example, one sequence including three times of reviewing one unit is (0, 2, 6, 14), which means that the unit is introduced at time 0, and the first, second and third review occur at time 2, 6, 14 respectively, we can thus get t1=14, t2=12 and t3=8. For the introduction of any unit, r1 = a because m0 is equal to negative infinity.

$$r_{k}(m_{k-1}) = c \cdot e^{m_{k-1}} + a$$

$$m_{k}(t_{1},...,t_{k}) = \ln\left(\sum_{i=1}^{k} t_{i}^{-r_{i}}\right)$$
(3)

According to Equation (2) and (3), we can know that the more the review of one unit occurs, the greater the review effect factor and the smaller the decay rate. This is consistent with actual learning processes. We define the retention rate of one unit after time Δt from the kth review as Equation (4). We transform Equation (4) into Equation (5) with time t expressing Δt , noting that t begins from the introduction of the unit in the sequence. Ti denotes the time internal between the (i-1)th review and ith review. And the parameter f is controlled by the software designer according to different material. Given all parameters related to students and learning material, we can get the optimal review time of one unit after the kth review is the intersection of the learning ability function curve and the retention rate function curve.

$$p(\Delta t) = f \cdot (\Delta t)^{-r_{k}}$$

$$p(t) = f \cdot \left(t - \sum_{i=1}^{k} \mathbf{T}_{i}\right)^{-r_{k}}$$
(4)
(5)

This model is, by design, a simplification of learning ability and spacing effect of review in the learning process. A more nuanced model might define a better learning ability function and a better retention rate function. Our simple model, however, captures the essence of the tradeoff between teaching new material and spacing review, and will allow for mathematical analysis that elucidates the mechanics of review scheduling in an optimal way.

2.3 Educational Goals

Infinite learning and bounded learning are two natural goals for the designers of educational software. Infinite learning is a sort of "lifelong learning", and in which students can grow their knowledge without bound, never forgetting anything along the education way of the software. Bounded learning, also known as "cramming", is a studying technique that gets students familiar with a certain set of educational units by a particular point in time, regardless of whether they are destined to forget what they learned quickly. These two goals are all considered in this paper.

We model infinite learning as an infinite sequence including infinite units, each of which occurs infinitely often. With respect to infinite learning, we will be interested in the existence condition of infinite learning and the introduction rate of new material at which the student would learn new units. So we define the introduction time function: For a given schedule of educational units, let In denote the position in the schedule where the nth distinct educational unit is introduced. We can know that the slower the growth of In, the faster new units are being introduced.

For bounded learning we consider a finite sequence including n distinct units. The need of students in bounded learning can be captured by the number n of distinct units and the sequence length L, namely the limit of total time. It is obvious that the number of reviewing distinct units in the sequence is not equal, and the sooner one unit is introduced, the more the unit is reviewed.

3. Optimal Scheduling

Different students usually need different schedules which are the most suitable for them, such as the recap method and the slow flashcard schedule [12]. This phenomenon increases the design difficulty of educational software. Can we find a united scheduling policy which is suitable for all students? The answer is positive in our models.

The learning ability function is associated with the introduction time of one unit and the interval between the introduction time and current time. And it is monotonically increasing from 0 to 1 with time. The retention rate function is associated with the last presentation time and the interval between the last presentation time and current time. This retention function, however, is monotonically decreasing from the value of parameter f to 0 with time. As mentioned in the previous section, the optimal review time of one unit after the kth review is the intersection of the learning ability function curve and the retention rate function curve.

In fact, more than one unit may need to be reviewed at the same time step in our models because there is not one unit, but many units which have been introduced before the current time. How can we choose the most suitable unit to be reviewed in the case of the conflict situation in which several units need to be reviewed at one step? We choose neither the oldest unit nor the newest unit. Our solution is to calculate the ratio (g(t)-g(t))/(1-g(t)) of each unit which has been introduced before the current time, in which the functions g(t) and p(t) are defined in Equation (1) and Equation (5). Then we compare the ratios and choose the unit whose ratio is the max. This solution is very fair to all units. For example, one actual sequence (b = 0.4, d = 0.4, a = 0.5, c = -0.7 and f = 1) is as follows.

 $\label{eq:u0,u1,u2,u3,u0,u1,u2,u0,u1,u2,u3,u4,u0,u1,u2,u3,u4,u5,u3,u4,u6,u5,u7,u8,u5,u4,u6,u7,u8,u5,u6,u7,u8,u9,u6,u7,u8,u9,u3,u10,u9,u11,u12,u4,u10,u9,u11,u10,u12,u11,u0,u1,u10,u2,u11,u12,u13,u14,u6,u12,u13,u14,u15,u13,u14,u7,u8,u5,u13,u14,u12,u15,u16,u17,u15,u18,u16,u15,u17,u16,u18,u17,u10,u9,u16,u11,u17,u18,u19,u20,u18,u21,u19,u20,u22,u19,u20,u21,u22,u23$

In the above sequence, the unit u2 is introduced at time 3, and is reviewed at time 7, 10, 15, 54 respectively. The total of reviewing each unit from u0 to u23 is not equal, and it changes within the range of 0 to 5, and the spacing between two adjacent reviews of one

unit becomes bigger and bigger with time growing. This phenomenon is consistent with the actual learning situation.

This schedule scheme is intuitive and effective. The complexity of the schedule scheme is only O(nL), in which n is the total of units and L is the length of learning time.

4. Results and Discussion

The learning ability of students and the spacing effect of review are defined by four equations with five parameters, namely a, b, c, d and f. In this section, we describe how choices of these parameters affect the rate at which new educational units can be introduced into the schedule and under what parameter conditions students can achieve special educational goals.

4.1 Overview of Results

We begin with the first main issue: how do the parameters of students and learning material affect the rate In of new units being introduced in the schedule? To explain it more clearly, we classify students into flexible students and slow students, and we classify learning material into memory learning material and understanding learning material. Can all students achieve infinite perfect learning of all kinds of material? We answer the question in the negative.

When students learn all kinds of material, the curves of the rate In are not linear, but jagged with many ladder-like steps. In the plat parts of the curves, no new units are introduced and the time is spent on reviewing the units which have been introduced. Although the learning ability of students are determined by two parameters b and d, but it depends on the parameter d mainly. Regardless of the learning ability, all students can achieve infinite perfect learning if learning material hasn't too large decay rate.

When the same student learns different material, the parameters a and c of learning material have more important effect on the rate In than the parameter f. There is an upper bound of units when students learn memory learning material, and students can't achieve infinite perfect learning when they learn some memory learning material.

Following these results, we consider the second main issue: is cramming always effective for all students learning all kinds of material? We answer this question in affirmative.

4.2 Flexible Students and Slow Students

What's the difference of the rate In at which new educational units can be introduced to flexible students and slow students? What is the highest rate of In for flexible students? And can slow students also achieve infinite perfect learning? Here we explore the effect of learning ability parameters, namely b and d in Equation (1), on the rate In of new units being introduced. In the following simulation, we suppose that the learning material for all students is the same. Without loss of generality, we assume that a = 0.8, c=-0.7 and f=3.

According to Equation (1), it is referred that the smaller the parameter b and the larger the parameter d, the more flexible the students and the quicker they study, conversely, the larger the parameter b and the smaller the parameter d, the slower the students study. The effect of the parameters b and d on the rate In is described in Figure 1 (a) and (b) separately. In the figures, the abscissa denotes learning time and the ordinate is the total of distinct units which have been introduced. The curves of the rate In are jagged with many ladder-like steps. In the plat parts of the curves, no new units are introduced and the time is spent on reviewing the units which have been introduced. When the parameter d is fixed and d = 0.4, we change the parameter b from 0.1 to 0.9 and we get the curves in Figure 1 (a). When the parameter b is fixed and b = 0.8, we change the parameter d from

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0.1 to 0.5 and we get the curves in Figure 1 (b). Comparing these two figures, we can know that the parameter d have a more important effect on the rate In function than the parameter b. The learning ability of one student is mainly determined by the parameter d. Moreover, when the parameter d is larger, the student will spend much time on reviewing, instead of learning new units.



Figure 1. The Effect of Learning Ability Parameters on the Rate of New Units Being Introduced

It takes less review for flexible students to grasp one unit. They have a higher rate In of new units being introduced. How can we combine the parameters b and d to get the highest rate In for flexible students? The rate In for flexible students is described with the different combination of parameters in Figure 2. The Figure shows that the higher rate In can be gotten by the smaller b and the larger d, but when the parameter b is very small, the parameter d has less effect on the rate In, just as the blue curve in the Figure.



Figure 2. The Rate of New Units Being Introduced to Flexible Students

Slow students spend much time on reviewing what has already been taught. The rate of new units being introduced to slow students is shown in Figure 3. When they are introduced a certain number of units, slow students can't learning new units any more in a long period of time. After they have grasped what have been introduced, they begin to learn new units once again. The period in which they can't learn new units is determined by the parameter d. The parameter b has less effect on the rate In for slow students. And the finicky slow students can also achieve the infinite perfect learning so long as the learning material can be grasped by them. As shown in the next section, some learning material can't be grasped by some students.



Figure 3. The Rate of New Units being Introduced to Slow Students

4.3 Memory Learning Material and Understanding Learning Material

Different types of learning material have different decay rate. We can retain understanding learning material for a longer time than memory learning material. In Equation (2) and (5), we can get different decay rates for different types of learning material by adjusting three parameters a, c and f. In this section, we explore the effect of learning material on the rate In of new units being introduced. In order to observe the results better, we assume that the students are neither flexible students nor slow students, but normal students, whose parameters of learning abilities are b = 0.8 and d = 0.4.

Based on Equation (2) and (5), we know that the larger the parameters a, c and f, the quicker the learning material decay, on the contrary, the smaller the parameters a, c and f, the slower the learning material decay. These three parameters have different effects on the rate In, which are illustrated in Figure 4 (a), (b) and (c). The curves of the rate In are also jagged with many ladder-like steps in the figures. When the parameters c and f are fixed (c = -0.7, f = 3), we change the parameter a from 0.1 to 0.9 and we get the curves in Figure 4 (a). Similarly, when the parameters a and f are fixed (a = 0.8, f = 3), and the parameter c changes from -0.1 to -5, the curves of the rate In are shown in Figure 4 (b). Note that the parameter c must be negative. When a = 0.8 and c = -0.7, and f changes from 1 to 9, the curves are shown in Figure 4 (c). Comparing these three figures, it is shown that the parameters a and c have more important effects on the rate In than the parameter f.



Figure 4. The Effect of Learning Material Parameters on the Rate of New units being Introduced

When all the three parameters a, c and f are larger, the learning material has a larger decay rate. Students spend much time on reviewing and new units are introduced slowly. We call this type of material as memory learning material. How slow is the rate In of new units being introduced when a, c and f are very large? As shown in Figure 5, after having been introduced a certain amount of units, students can't be introduced units any more. There is an upper bound of units being introduced to students for memory learning material. It is shown that students can't achieve infinite perfect learning for some memory learning material.



Figure 5. The Effect of Memory Learning Material on the Rate of New Units being Introduced

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In contrast, when all the parameters a, c and f are smaller, the material is called understanding learning material and it has a smaller decay rate. Students have a higher rate In of new units being introduced, which is shown in Figure 6. However, the ladderlike phenomenon always occurs in the educational process no matter how high the rate In is.



Figure 6. The effect of Understanding Learning Material on the Rate of New Units being Introduced

4.4 Cramming

We focus on iinfinite perfect learning in the previous sections. We turn our attention now to cramming. In this section, we mainly consider two questions. First, how much cramming can be done when different students learn different material in a given amount of time? Second, what is the distribution of students grasping each unit which has been introduced before the final time point?

It is shown that the same student learns different material at different rates in the previous section. Figure 7 (a) illustrates the total of new units that one student with parameters b = 0.4 and d = 0.8 has learned at time t = 500. The abscissa is different conditions which have different parameters, and the ordinate is the total of distinct units that have been introduced before the given time. Within a given time, the total of distinct units that have been introduced to the same student changes in a larger range with different parameters of material. This phenomenon implies that cramming is always effective to all types of material, but different material has different effects on cramming.



Figure 7. The total of New Units being Introduced in a Given Time

We know that different students learn the same material at different rates from the previous section. Figure 7 (b) illustrates the total of new units that different students learn the same material with parameters a = 0.8, c = -0.7 and f = 3 at time t = 1000. Within a given time, the total of new units that have been introduced to different students changes in a smaller range. This shows that cramming is always effective to all students, regardless of flexible students or slow students.

Each unit is introduced at different time point in the sequence. And the decay rates of different unit are different at the same time point. The mastery of each unit is described at the finial time point in Figure 8. The abscissa is the serial number of units, and the ordinate is the value of p function at the final time point. The p function is described in Equation 5 and it represents the mastery of one unit after the decay from the recent reviewing or introduction. Figure 8 (a) describes the mastery of each unit that different students learn the same material with parameters a = 0.8, c = -0.7 and f = 1 at time t = 600. Figure 8 (b) describes the mastery of each unit that the same student with parameters b = 0.8 and d = 0.4 learns for different material at time t = 500. The curves in Figure 8 are not straight, but zigzagged. The former part of each curve fluctuates in a smaller range. However the latter part of each curve fluctuates wildly. This shows that the earlier one unit is introduced, the better one student grasps the unit. Students will forget the recent units soon because the recent units have a larger decay rate, which is shown in the latter part of curves.



5. Conclusion

With the current boom in the educational software, it is clear that the time has come to develop a theory of algorithmic education. The models presented in this paper are simple and theoretical. As long as designers of educational software determine the parameters related to students and material by some approaches, the models can work efficiently. It is our hope that work on simple theoretical models will reproduce some important phenomena in the learning process and provide the foundations of intuition for designers of educational software, in much the same way that algorithmic game theory does for engineer who work in online ad auctions and other related fields.

The research on the theory of algorithmic educations has just begun. The focuses for future work include four aspects. The first aspect is the theory of infinite perfect learning and cramming. Such a theory would include the existence condition and corresponding scheduling policy. The truly adaptive educational process is the second aspect. In this paper we assume that designers of educational software have known parameters of students and material in advance, but an alternative approach would be to test the student's knowledge throughout the process. The schedule is controlled by an online algorithm which chooses the next unit based on the answers the student has given. The third aspect is the design and analysis of models in which all units are not dependent. To describe the educational process better, we assume that all units in the sequence are dependent. In fact, the units in learning material are dependent. The mastery of one unit may help to learn another unit, and the learning process of one unit may "reinforce" the mastery of prior units. The last aspect is empirical work. According to the theoretical work, actual educational software can be created and the evaluation of theoretical work can be gotten in the practical applications. The data and feedback from real applications can help to refine the models.

Acknowledgement

This work was supported by the National Natural Science Foundation of China under Grant No. 61373009. Supported by the Science and Technology Support Project of Sichuan province under Grant No. 2013GZX0166, 2015GZ0089 and the Fundamental Research Funds for the Central Universities under Grant No. 2682014CX055.

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