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Robust Adaptive and Sliding Mode Synchronization of Uncertain Chaotic Systems

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Abstract

Parameter uncertainty of chaotic systems brings the complexity to the design of control law in many past research papers. In order to simplify the design of adaptive PID composite synchronous rule further and use the robust control method to deal with uncertainty conveniently, two kind of robust adaptive and nonlinear sliding mode synchronization algorithms are proposed for uncertain chaotic systems in this paper. The design strategy of the algorithm lies in using the boundedness of chaotic systems. The unknown information of driven system and response system is limited to be a given uncertainty range described by a bounded function. The robust adaptive controller is designed to complete synchronization. At last, it can be seen from the simulation that chaotic systems can achieve synchronization and eliminate the vibration phenomenon at the same time. The only disadvantage is that the precision of synchronization is limited by the control gain.

***Keywords**: Chaotic systems, nonlinear, robust control, adaptive synchronization, uncertainty, desired trajectory*

1. Introduction

Chaos systems have complex dynamical behaviors that possess some special features such as being extremely sensitive to tiny variations of initial conditions, and having bounded trajectories with a positive leading Lyapunov exponent and so on [1-7]. Synchronization of chaos systems with unknown parameters was investigated widely by researchers from various fields. The stability of tracking problem of a kind of single input and single output nonlinear systems, which can be transformed into strict feedback form, was researched in [8, 9] under the situation that there exist unknown parameters and uncertain nonlinear functions [10-28]. But it needs the assumption that the bounds of unknown parameters and bound functions of uncertain nonlinear functions are known.

Robust control is a strong measure to deal with uncertain problems, and the measure has been sufficiently developed. Robust control and self-adaptive control is used by many systems at once and they are interdependent. Good results can not be acquired if only robust control or self-adaptive control is used alone. Based on Lyapunov function method and model reference adaptive control theory, Hou yanze [29] put forward and analysed a robust adaptive switching control program that a controller has dead zone nonlinearity for uncertain switching systems whose switching sign is dependent on independent decision variable, and the simulation results indicate that the system can track the expected track

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quickly. Based on the passivity method, Ke haisen [30] put forward a saturated robust adaptive controller for uncertain and non-holonomic mobile robots that satisfy matched condition, and the controller need not know the upper bound of uncertain interference. Guan Xin-ping [31] researched the robust adaptive synchronization problem of two systems that exist interference. The method can effectively overcome the damage that is caused by uncertainty, and also it has a good effect to synchronization. A numerical calculation is conducted of Lorenz system, and the result of the numerical calculation shows the effectiveness of the method. Wang hong-wei [33] use Lyapunov stability theory to confirm weight updating rules of orthogonal neural network controller, the error of weight and track is guaranteed to be bounded. Based on Chebbyshev orthogonal neural network, an uncertain chaotic system robust adaptive synchronized method is designed to solve the uncertainty problem.

Through the research on adaptive PID synchronous control law and consider the complexity of control law design caused by the uncertainty and corresponding hypothesis, the design of the adaptive PID composite synchronous rule is simplified in this paper. Considering to abstract simplify model uncertainty, In order to use the robust control method to deal with uncertainty, the model uncertainty is abstracted and simplified. So, in view of the model uncertainty, the uncertainty is assumed to first-order bounded compared with state error. And because most chaotic systems are bounded, the assumption is met easily by general chaos system [34-47].

The synchronization problem of chaotic systems, which can be transformed into single input and single output nonlinear dynamic system, was researched under the conditions that there are both unknown parameters and unknown nonlinear functions. But only the single input and single output situation is concerned in those references [48-57]. In this paper, the multi-input and multi-output problem is considered and a robust adaptive synchronization law is designed for a three dimension super chaotic system based on Lyapunov stability theorem.

2. Problem Description

Considering the following driver system and response system, where parameter of response system is known and there exists unknown parameters and uncertain nonlinear function in driven system.

The driven system can be written as

$$\dot{x} = f_x(x) + F_x(x)\theta_x + \Delta(x,t) \tag{1}$$

The response system can be written as

$$\dot{y} = f_y(y) + bu \tag{2}$$

Take a three dimension system as a example, the main driven system can be written as

$$\dot{x}_1 = f_{x1}(x_1,\cdots,x_4) + \sum_{j=1}^{p_1} F_{x1j}(x_1,\cdots,x_4)\theta_{x1j} + \sum_{j=1}^{q_1} \Delta_{x1j}(x,t) \tag{3}$$

$$\dot{x}_2 = f_{x2}(x_1,\cdots,x_4) + \sum_{j=1}^{p_2} F_{x2j}(x_1,\cdots,x_4)\theta_{x2j} + \sum_{j=1}^{q_2} \Delta_{x2j}(x,t) \tag{4}$$

$$\dot{x}_3 = f_{x3}(x_1,\cdots,x_4) + \sum_{j=1}^{p_3} F_{x3j}(x_1,\cdots,x_4)\theta_{x3j} + \sum_{j=1}^{q_3} \Delta_{x3j}(x,t) \tag{5}$$

And the slave response system can be written as

$$\dot{y}_1 = f_{y1}(y_1,\cdots,y_4) + b_1u_1 \tag{6}$$

$$\dot{y}_2 = f_{y2}(y_1,\cdots,y_4) + b_2u_2 \tag{7}$$

$$\dot{y}_3 = f_{y3}(y_1,\cdots,y_4) + b_3u_3 \tag{8}$$

When θ_x is unknown parameter, and the number of unknown parameter is $\sum_{i=1}^{n} p_i$, and the number of uncertain nonlinear function is $\sum_{i=1}^{n} q_i$, b_i is a known constant [8-10].

So the robust adaptive control target for chaotic system with unknown parameter and uncertain nonlinear function is to design the control $u = u(x, y, \hat{r}_{ij})$, $\dot{\hat{r}}_{ij} = f(z_1, z_2, z_3)$ such that the state of slave system can trace state of main system, such as $y \to x$.

Considering the above adaptive PID control is too complex, so the design of robust adaptive synchronization need benefit the boundedness of chaotic system. And use a bounded function to describe the uncertain region of the unknown information of driven system and response system, then design a robust adaptive controller to synchronize two systems [11-14].

3. Assumption

Two assumptions are built for the above system to simplify the analysis.

Assumption 1: the driven system and response system have the same structure, it means that it has the same dimension.

Assumption 2: the nonlinear function satisfies the below conditions , for $1 \le i \le n$, $1 \le j \le p_2$, there exists a unknown positive constant $r_{ij} \le d_{ij}$ such that

$$\begin{aligned} & f_{yi}(y_1,\cdots,y_4) - f_{xi}(x_1,\cdots,x_4) - \sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\theta_{xij} - \sum_{j=1}^{q_1} \Delta_{xij}(x,t) \\ & \le r_{i1}|z_1| + r_{i2}|z_2| + r_{i3}|z_3| \end{aligned} \tag{9}$$

where d_{ij} is a known constant. Because the chaotic system is bounded, so it is easy to be satisfied for many chaotic systems[15-17].

4. Robust Adaptive Synchronization Law Design

Define the error variable as $z_i = y_i - x_i$, where the error system can be written as

$$\begin{aligned} \dot{z}_i = & f_{yi}(y_1,\cdots,y_4) - f_{xi}(x_1,\cdots,x_4) \\ & - \sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\theta_{xij} - \sum_{j=1}^{p_2} \Delta_{xij}(x,t) + b_i u_i \end{aligned} \tag{10}$$

Then it also has

$$z_i \dot{z}_i \le r_{i1}|z_1||z_i| + r_{i2}|z_2||z_i| + r_{i3}|z_3||z_i| + z_i b_i u_i \tag{11}$$

Use the adaptive method to design the control u_i as

$$u_i = b_i^{-1}[\hat{r}_{i1}|z_1| + \hat{r}_{i2}|z_2| + \hat{r}_{i3}|z_3|]\operatorname{sgn}(z_i) \tag{12}$$

The error of assumption is defined as

$$\tilde{r}_{ij} = r_{ij} - \hat{r}_{ij} \tag{13}$$

Solve its derivative as

$$\dot{\tilde{r}}_{ij} = -\dot{\hat{r}}_{ij} \tag{14}$$

Where the parameter adaptive law is designed as

$$\dot{\hat{r}}_{ij} = -|z_i z_j| \tag{15}$$

Choose the Lyapunov function as

$$V_1 = \sum_{i=1}^{n} z_i^2 + \sum_{j=1}^{j=n}\sum_{i=1}^{i=n} \tilde{r}_{ij} \tag{16}$$

It is easy to get

$$\dot{V}_1 \le 0 \tag{17}$$

So the system is stable and synchronization is fulfilled [18-21].

The adaptive synchronization controller can be designed according to above process and to make the research more convincible, a kind of nonlinear sliding mode synchronization method is proposed as follows.

5. Design of Nonlinear Sliding Mode Synchronization Controller

Choose a nonlinear sliding mode as bellows:

$$S = az + f_1\left(\int_0^t zdt\right) = 0 \tag{18}$$

Where $f_1(x)$ is a sigmoid function and it is defined as

$$f_1(x) = \frac{1-e^{-\tau x}}{1+e^{-\tau x}} \tag{19}$$

It is obvious that if $x > 0$, then $f_1(x) > 0$. Also if $x < 0$, then $f_1(x) < 0$.

It is necessary to prove that if the system state can be converged to the below nonlinear sliding mode, then the system state can converged to expected value x_1^d. Then the above sliding mode can be proved to be reasonable. It can be proved as bellows:

First , define a new variable as

$$w_1 = \int_0^t zdt \tag{20}$$

Then it satisfies:

$$\dot{w}_1 = z \tag{21}$$

If the system converged to the sliding mode surface, then it holds

$$S = 0 \tag{22}$$

Then

$$az = -f_1\left(\int zdt\right) \tag{23}$$

Choose a Lyapunov function as

$$V = \frac{1}{2} w_1^2 \tag{24}$$

Then

$$\dot{V} = w_1 \dot{w}_1 \tag{25}$$

And it can be written as

$$\begin{aligned} \dot{V} &= w_1 z \\ &= -\frac{1}{a} w_1 f_1(w_1) \end{aligned} \tag{26}$$

Choose parameter of sliding mode a to be a positive number, then it is easy to prove that

$$\dot{V} \le 0 \tag{27}$$

According to Lyapunov stability theory, the error z can converged to zero.

Then last part of this diagram will introduce how to use this nonlinear sliding mode and how to design a controller such that the system state can converged to the sliding mode.

Also define an error variable as $z_i = y_i - x_i$, then the above drive - response system can be transformed as an error response of system as

$$\dot{z}_i = f_{yi}(y_1,\cdots,y_4) - f_{xi}(x_1,\cdots,x_4) - \sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\theta_{xij} - \sum_{j=1}^{p_2} \Delta_{xij}(x,t) + b_i u_i$$

And choose a sliding mode surface as

$$S_i = z_i + a_i f_1(\int_0^t z_i dt) \tag{28}$$

Then solve its derivative it satisfies:

$$\dot{S}_i = a_i \dot{z}_i + \dot{f}_1(\int z_i dt) z_i \tag{29}$$

Define

$$w_i = \int z_i dt \tag{30}$$

where

$$\dot{f}_1(w_i) = \frac{2\tau w_i e^{-\tau w_i}(1+e^{-\tau w_i})}{\left(1+e^{-\tau w_i}\right)^2} \tag{31}$$

Then

$$\dot{S}_i = a_i \dot{z}_i + \dot{f}_1(w_i) z_i \tag{32}$$

And consider the chaotic dynamic, it can be written as

$$\dot{S}_i = \dot{z}_i + \dot{f}_1(w_i) z_i a_i \tag{33}$$

And it can be written as

$$\begin{aligned}\dot{S}_i = {} & f_{yi}(y_1,\cdots,y_4) - f_{xi}(x_1,\cdots,x_4) - \sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\theta_{xij} \\ & - \sum_{j=1}^{p_2} \Delta_{xij}(x,t) + b_i u_i + \dot{f}_1(w_i) z_i a_i\end{aligned} \tag{34}$$

Then And design the control u_i as

$$\begin{aligned}u_i = {} & \frac{1}{b_i}[-f_{yi}(y_1,\cdots,y_4) + f_{xi}(x_1,\cdots,x_4) - \dot{f}_1(w_i) z_i a_i \\ & + \sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\hat{\theta}_{xij} + \sum_{j=1}^{p_2} \hat{q}_{ij}\psi_{ij}(x) + f_2(z_i)]\end{aligned} \tag{35}$$

where

$$f_2(z_i) = -z_i(k_{i1}S_i + k_{i2}\frac{S_i}{|S_i| + \varepsilon_{i1}} + k_{i3}\frac{3}{2} z_i^{1/3} \exp(S_i^{2/3}) + k_{i4} sign(S_i)) \tag{36}$$

Substitute the control law to above equation, it satisfies:

$$\dot{S}_i = -\sum_{j=1}^{p_1} F_{xij}(x_1,\cdots,x_4)\tilde{\theta}_{xij} + \sum_{j=1}^{p_2}[\hat{q}_{ij}\psi_{ij}(x) - \Delta_{xij}(x,t)] + f_2(z_1) \tag{37}$$

Where

$$\tilde{\theta}_{xij} = \theta_{xij} - \hat{\theta}_{xij} \tag{38}$$

Since

$$\dot{\tilde{\theta}}_{xij} = -\dot{\hat{\theta}}_{xij} \tag{39}$$

Then design

$$\dot{\hat{\theta}}_{xij} = -F_{xij}(x_1, \cdots, x_4) S_i \tag{40}$$

Then choose a Lyapunov function as

$$V_1 = \sum_{i=1}^{n} \sum_{j=1}^{p_1} \frac{1}{2} (\tilde{\theta}_{xij})^2 \tag{41}$$

Then

$$\dot{V}_1 - \sum_{i=1}^{n} \left(\sum_{j=1}^{p_1} F_{xij}(x_1, \cdots, x_4) \tilde{\theta}_{xij} L S_i \right) = 0 \tag{42}$$

Since

$$\begin{aligned} &\hat{q}_{ij} \psi_{ij}(x) S_i - \Delta_{xij}(x,t) S_i \le \hat{q}_{ij} \psi_{ij}(x) S_i - q_{ij}^* \psi_{ij}(X) |S_i| \\ &= [\hat{q}_{ij} - q_{ij}^* sign(S_i)] \psi_{ij}(x) S_i \\ &= [\hat{q}_{ij} sign(S_i) - q_{ij}^*] \psi_{ij}(x) |S_i| \end{aligned} \tag{43}$$

Define

$$\tilde{q}_{ij} = -\hat{q}_{ij} sign(S_i) + q_{ij}^* \tag{44}$$

Then

$$\hat{q}_{ij} \psi_{ij}(x) S_i - \Delta_{xij}(x,t) S_i \le -\tilde{q}_{ij} \psi_{ij}(x) |S_i| \tag{45}$$

And solve the derivative of the estimation of unknown parameter , then it holds

$$\dot{\tilde{q}}_{ij} = -\dot{\hat{q}}_{ij} sign(S_i) \tag{46}$$

Design the turning law as

$$\dot{\hat{q}}_{ij} = -\psi_{ij}(x) S_i \tag{47}$$

Choose a Lyapunov function as

$$V_2 = \sum_{i=1}^{n} \sum_{j=1}^{p_2} \frac{1}{2} (\tilde{q}_{ij})^2 \tag{48}$$

Then

$$\dot{V}_2 + \sum_{i=1}^{n} \sum_{j=1}^{p_2} [\hat{q}_{ij} \psi_{ij}(x) S_i - \Delta_{xij}(x,t) S_i] \le 0 \tag{49}$$

Choose a Lyapunov function as

$$V_3 = \sum_{i=1}^{n} S_i^2 \tag{50}$$

Then solve its derivative as

$$\dot{V}_3 = \sum_{i=1}^{n} S_i \dot{S}_i \tag{51}$$

Then choose a big Lyapunov function for the whole system as

$$V = \sum_{i=1}^{3} V_i = \sum_{i=1}^{n} S_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{p_1} \frac{1}{2} (\tilde{\theta}_{xij})^2 + \sum_{i=1}^{n} \sum_{j=1}^{p_2} \frac{1}{2} (\tilde{q}_{ij})^2 \tag{52}$$

Solve its derivative as

$$\dot{V} = S_i f_2(z_i) \tag{53}$$

Then it can be simplified as

$$\dot{V} \le -z_i[k_{i1}S_iS_i + k_{i2}\frac{S_iS_i}{|S_i|+\varepsilon_{i1}} + k_{i3}S_i\frac{3}{2}z_i^{1/3}\exp(S_i^{2/3}) + k_{i4}|S_i|]\frac{2\tau z_i e^{-\tau z_i}(1+e^{-\tau z_i})}{\left(1+e^{-\tau z_i}\right)^2} \quad (54)$$

It is easy to prove that

$$\dot{V} \le 0 \quad (55)$$

According to Lyapunov stability theory, the system is stable. Then the sliding mode can be achieved and according to the proof above, the state z_i can converge to zero.

6. Numerical Simulation

Also use a three dimension system as a example

$$\dot{x}_1 = a(x_2 - x_1) + k_{lb}x_3\cos x_2 \quad (56)$$

$$\dot{x}_2 = bx_1 + cx_2 - x_1x_3 + k_{lb}x_3\cos x_2 \quad (57)$$

$$\dot{x}_3 = x_2^2 - hx_3 + k_{lb}(1+\sin(x_2x_3))x_2 \quad (58)$$

If $a = 20, b = 14, c = 10.6, h = 2.8, k_{lb} = 0$, the system has an attractor. Set a,b,c,h as unknown parameters as system and k_{lb} is coefficient of nonlinear function. The structure of response system is known as follows:

$$\dot{y}_1 = a_y(y_2 - y_1) + u_1 \quad (59)$$

$$\dot{y}_2 = b_yy_1 - k_yy_1y_3 + u_2 \quad (60)$$

$$\dot{y}_3 = -c_yy_3 + h_yy_1^2 + u_3 \quad (61)$$

Choose parameter as $(a_y, b_y, c_y, k_y, h_y) = (10, 40, 2.5, 1, 4)$, and the initial state of response system is $(y_1, y_2, y_3) = (1, -1, 2)$, use above robust adaptive strategy, the simulation result without considering the nonlinear functions is as follows:

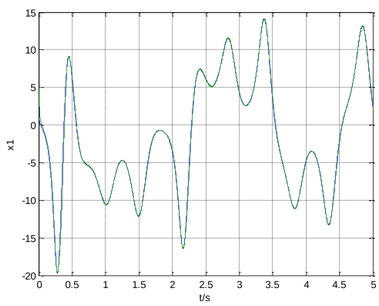

Figure 1. State x1 and y1

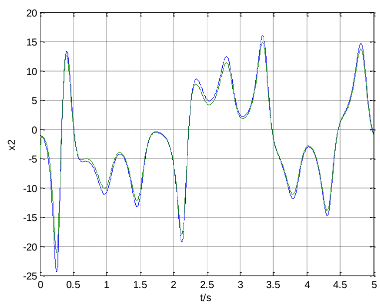

Figure 2. State x2 and y2

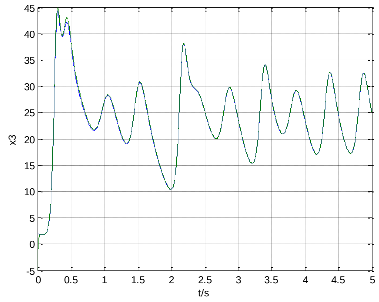

Figure 3. State x3 and y3

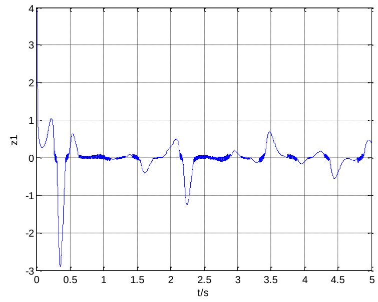

Figure 4. The Curve of Error z1

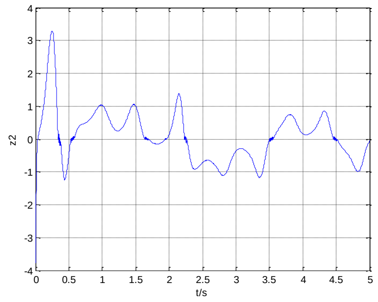

Figure 5. The Curve of the Error z2

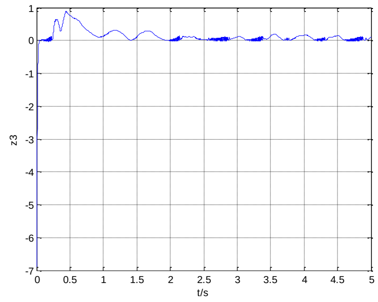

Figure 6. The Curve of the Error z3

To reduce the oscillations, use soft function replace of the sign function, the simulation result is as follows:

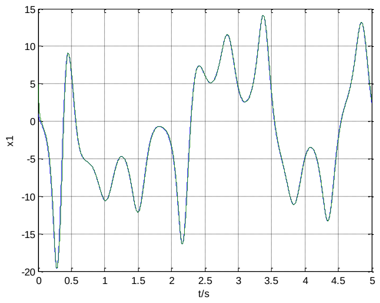

Figure 7. State x1 and y1

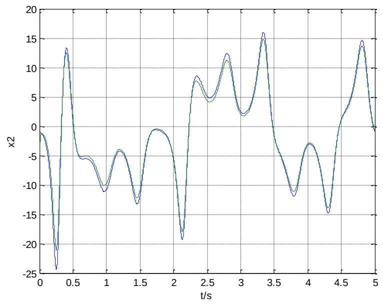

Figure 8. State x2 and y2

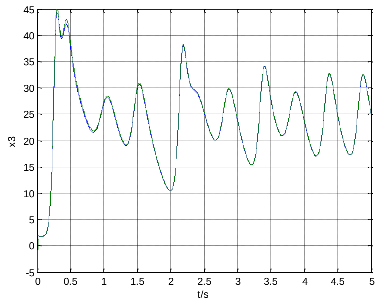

Figure 9. State x3 and y3

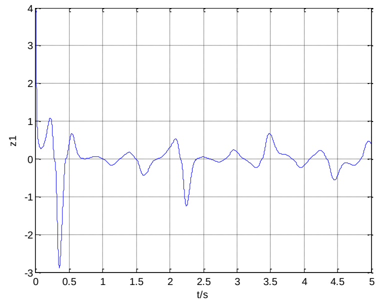

Figure 10. The Curve of Error z1

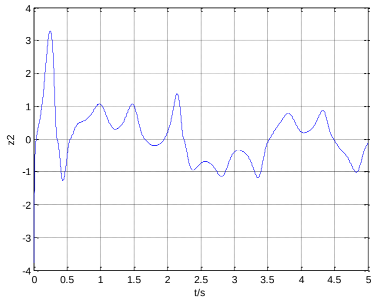

Figure 11. The Curve of the Error z2

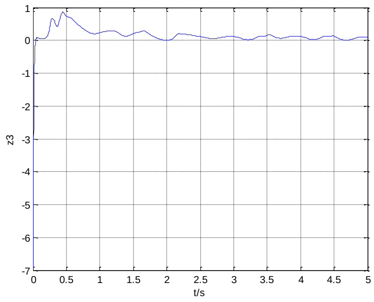

Figure 12. The Curve of the Error z3

So we can make a conclusion that the synchronization can be fulfilled and oscillation can be reduced. And the disadvantage is that the synchronization error is not easy to eliminate. And the advantage is that the structure of system is not necessary to know accurately. And the synchronization of the system can be realized no matter there are nonlinear function or not.

7. Conclusion

The uncertain chaotic systems with robust adaptive and sliding mode synchronization algorithms are proposed in this paper. The design strategy of the algorithm lies in using the boundedness of chaotic systems. The unknown information of driven system and response system is given uncertainty range by the hypothesis presence bounded function. The robust adaptive controller is designed to complete synchronization. At last, it can be seen from the simulation that chaotic systems can achieve synchronization and eliminate the vibration phenomenon at the same time under the condition that control system need not be known precisely. But the precision of synchronization is limited.

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