

Design and Research on the LQR Controller for the Magnet Power Supply of the Accelerator based on Particle Swarm Optimization

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Abstract

Accelerator has been widely applied to high energy, low energy physics, Medical environmental and military fields, etc. The magnet power supply is one of the most important parts of the accelerator. While the magnet power supply of accelerator is taken as the control object, and the design of LQR controller of the magnet power supply based on PSO has been proposed. At the beginning, the state-space model of the magnet power supply has been established by modern control theories. Then, the PSO (Particle Swarm Optimization) algorithm has been used for the weight matrices optimization of LQR controller in order to ensure that the magnet power supply can provide a magnetic field quickly which the accelerator required. The experimental results indicate that the method that proposed in the paper can meet the requirements of fast output response of the system and each control index of the LQR controller is obviously superior to that by the traditional method.

Keywords: *Particle Swarm Optimization; LQR; Parameter Optimization; Heavy Ion Accelerator; Magnet Power Supply*

1. Introduction

The accelerator can make use of the electric field to drive the charged particles for high energy that can be broadly divided into cyclotron and linear accelerators. There are two branches in Accelerator applications. One of them has been widely applied to the scientific research on the low energy physics, high energy physics and heavy ion physics; another has been applied to Medical, industrial and agricultural, environmental and military fields. HERFL-CSR (Heavy Ion Research Facility in Lanzhou Cooling Storage Ring) is one of the most major science engineering, undertaken by Institute of Modern Physics; Chinese Academy of Sciences [1], construction began in December 1999, confirmed by the state in July 2008. Cooling Storage Ring relying on heavy ion accelerator, to carry out basic research and key technologies of heavy ion radiotherapy, deep the Heavy Ion clinical cancer experimental terminal has completed clinical trials to obtain a significant effect, making China the world's first 4 to achieve ion-clinical cancer country[2]. HERFL-CSR mainly related to the content of magnet, power supply, vacuum and high – frequency control, etc. The section of the magnet power supply plays an important role

in this project. The main function of the magnet power supply[3] is to provide magnetic field to the accelerator, in order to make the magnetic field increasing and decreasing synchronize with the movement of the ions, the PWM power has the advantages of simple structure[4], low manufacture cost and easy to be controlled, and can avoid the great impact on power grid and harmonic pollution[5-7]. Therefore, the research on the PWM power supply has very practical significance for the accelerator power supply.

LQR is an optimal control method with the quadratic performance indexes and these indexes have specify physical concepts generally[8], which has been successfully applied to many fields of industry control. At the same time, LQR has simple math disposal process and can achieve closed loop optimal control with the linear state feedback or output feedback. One of the key problems on the design of the controller is how to chose the weighed matrices Q and R , which will have a directly effect on the final control results. At present the commonly used weight matrix optimization algorithm are genetic algorithms, Variable Universe Fuzzy Control algorithm, particle swarm optimization. Reference [9] applied LQR to design the controller of inverted pendulum system based on GA. The anti-swing control system of Crane based on LQR and variable Universe Fuzzy Control is proposed in reference [10], and the PSO is also used to optimize the weight matrix for micro aerial vehicle with ducted fan [11].

Particle Swarm Optimization (PSO) is a evolutionary computation technique that has been successfully used for many years in different domain. The method that predicted the relationship between metric data and quality factors with historical data by using the optimized BP network based on PSO is proposed [12]. Reference [13] designed the PSO-PID controller for multi-leaf collimator, etc. In this paper, the weight matrices are optimized based on PSO for the magnet power supply and the simulation gives a nice result.

2. Model of the Magnet Power Supply

The magnet power supplies of Heavy ion accelerator are mainly single phase power supply, so what this dissertation mainly specializes in is the Single-phase PWM inverter model. The model of the magnet power supply is established based on the HERFL-CSR that shows in figure 1.

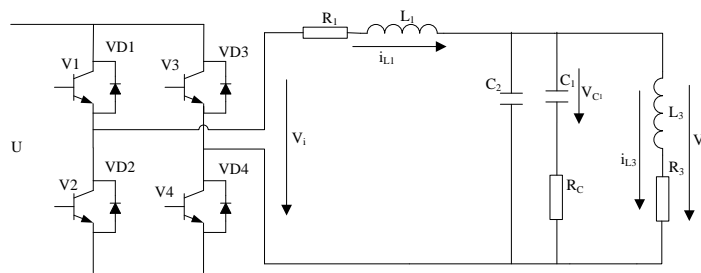


Figure 1: Accelerator Magnet Power Supply Model

When the VD1 and VD4 conduct, $v_1 = U$; when the VD2 and VD3 conduct, $v_1 = -U$. i_{L3} is the load current (exciting current), v_0 is output load voltage, c_1 , c_2 and L_1 are smoothing capacitors and filter inductor, R_c and R_1 are corresponding to the resistance of the smoothing capacitors and filter inductor. R_3 and L_3 are the load of magnet winding. Therefore, the mathematical model can be obtained as follow:

$$V_1 - V_0 = R_1 i_{L1} + L_1 \frac{d i_{L1}}{d t} \quad (1)$$

$$i_{t1} = C \frac{dV_0}{dt} + i_{t3} + C_2 \frac{dV_{C1}}{dt} \quad (2)$$

$$V_0 = i_3 R_3 + L_3 \frac{di_3}{dt} \quad (3)$$

$$V_0 = V_{C1} + R_2 C_1 \frac{dV_{C1}}{dt} \quad (4)$$

The state space equation of the proposed model is described as (5) and (6).

$$\begin{bmatrix} \dot{i}_{t1} \\ \dot{i}_{t3} \\ \dot{V}_{C2} \\ \dot{V}_0 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_3}{L_3} & 0 & \frac{1}{L_3} \\ 0 & 0 & -\frac{1}{R_2 C_2} & \frac{1}{R_2 C_2} \\ C_1 & -\frac{1}{C_1} & \frac{1}{R_2 C_2} & \frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} i_{t1} \\ i_{t3} \\ V_{C2} \\ V_0 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_1 \quad (5)$$

$$Y = [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} i_{t1} \\ i_{t3} \\ V_{C2} \\ V_0 \end{bmatrix} + [0] V_1 \quad (6)$$

Where the chosen values of each component parameter is as follows:

$$\begin{aligned} L_1 &= 0.3 \text{ mH}; \\ R_1 &= 0.01 \Omega; \\ L_3 &= 91.4 \text{ mH}; \\ R_3 &= 0.0796 \Omega; \\ C_1 &= 10 \mu\text{F}; \\ C_2 &= 47 \mu\text{F}; \\ R_2 &= 1 \Omega. \end{aligned}$$

The specific mathematical model of the state space equation is shown in (7) and (8).

$$\begin{bmatrix} \dot{i}_{t1} \\ \dot{i}_{t3} \\ \dot{V}_{C2} \\ \dot{V}_0 \end{bmatrix} = \begin{bmatrix} -33.3333 & 0 & 0 & -3.3333e+03 \\ 0 & -0.8709 & 0 & 10.9409 \\ 0 & 0 & -2.1277e+04 & 2.1277e+04 \\ 1.0000e+05 & -1.0000e+05 & 1.0000e+05 & -1.0000e+05 \end{bmatrix} \begin{bmatrix} i_{t1} \\ i_{t3} \\ V_{C2} \\ V_0 \end{bmatrix} + \begin{bmatrix} 3.3333 * e + 003 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_1 \quad (7)$$

$$Y = [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} i_{t1} \\ i_{t3} \\ V_{C2} \\ V_0 \end{bmatrix} + [0] V \quad (8)$$

3. LQR controller

The control object of LQR controller is the linear system with state-space equation in modern control theories[14], of which the objective function is the integral of the quadratic function of state variables and control variables.

Given the liner systems presented as $\dot{X} = AX + BU$, $Y = CX + DU$, the optimization of feedback control rules of LQR controller can be determinated as $U^* = -KU$, which minimizes performance index described as (9).

$$J = \int_0^{\infty} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \quad (9)$$

Where $K = R^{-1}B^T P$, P is a definite positive symmetric matrix that can satisfy the equation Riccati $PA + A^T P - PBR^{-1}B^T P + Q = 0$. Matrices Q and R are respectively the weight matrices of state variables and input vectors. Matrix Q is required to be a symmetric and semi-definite positive matrix, and Matrix R is required to be a symmetric and definite positive matrix. The pair $(A B)$ must be controllable. Then, matrices Q and R has a dimension 4×4 and 1×1 respectively. Since they are symmetry so there are 10 distinct in Q and 1 in R for a total of 11 distinct elements need to be selected. And also those 11 elements should satisfy the positive definitions. One practical method is to set Q and R to be diagonal matrix ($Q = \text{diag}(Q(1,1), Q(2,2), Q(3,3), Q(4,4))$, $R = r$) such that only 5 elements need to be decided[15,16]. The value of the elements in Q and R is related to its contribution to the cost function J.

The values of K depend on the choosing of matrices Q and R , which is close related to the location of closed loop pole of the system and the performance index of response in time domain[17,18]. So, the choosing of matrices Q and R is the key problem in the design of the optimal controller with the linear and quadratic performance index. As the traditional method is realized by trial and error, the proposed method has adopted the optimization algorithm PSO in the paper. Once the feedback factor $K = [k_1 \quad k_2 \quad k_3 \quad k_4]$ is determinated, the state-space equation of this closed loop control system is obtained as (10):

$$\begin{cases} \dot{X} = A'X + B'U \\ Y = C'X + D'U \end{cases} \quad (10)$$

where

$$A' = A - B * K ; B' = B * k_1 ; C' = C ; D' = D .$$

4. Design of PSO-LQR Controller

4.1. Review of PSO Algorithm

Particle Swarm Optimization(PSO) is a evolutionary computation technique presented by Kennedy and Eberhart in1995[19,20]. With the original idea coming from the social behavior of biology in nature, such as flocks of birds and schools of fish. In PSO algorithm[21,22], each particle in swarm represents a solution to D dimension, which is defined with its position and velocity, the mathematical description of the basic particle swarm optimization is as follows. Supposed the scale of swam is N , the position of particle i can be expressed as:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{iD}) \quad (11)$$

The velocity of particle is defined as the distance of particle movement in each iteration, described as (12).

$$v_i = (v_{i1}, v_{i2}, \dots, v_{iD}) \quad (12)$$

Then, the velocity of the particle $i(i = 1, 2, \dots, N)$ in the $d(d = 1, 2, \dots, D)$ - dimensional space can be adjusted according to (13) as:

$$v_{id} = v_{id} + c_1 rand_1() (p_{id} - x_{id}) + c_2 rand_2() (p_{gd} - x_{id}) \quad (13)$$

$$\begin{cases} v_{id} = v_{\max}, & \text{if } v_{id} > v_{\max} \\ v_{id} = -v_{\max}, & \text{if } v_{id} < -v_{\max} \end{cases}$$

Finally, the particle can adjust its position according to (14) is:

$$x_{id} = x_{id} + v_{id} \quad (14)$$

where N is the number of particles in the group, d is the dimension, v_{id} is the velocity of particle i , c_1 and c_2 is the acceleration constant, $rand_1()$ and $rand_2()$ is the random number between 0 and 1, x_{id} is the current position of particle i , p_{id} is the best previous position of the i th particle, p_{gd} is the best particle among all the particles in the population.

4.2. PSO-LQR controller

The structure diagram of LQR control system based on PSO is shown in figure 2.

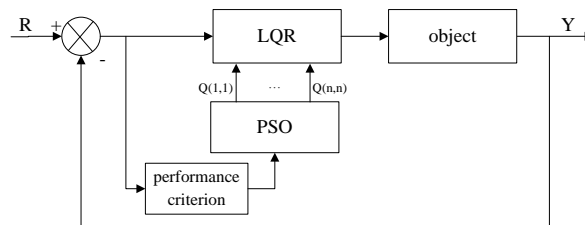


Figure 2: Structure Diagram Of LQR Control System Based On PSO

The design of the controller involves the following two components: the LQR controller for the object with close-loop control and the optimization module with PSO algorithm.

According to the operating state of the system, the module of PSO can optimize the weight matrices of the LQR controller to meet the performance requirements, and the output of this module will provide the optimized weight matrices of LQR controller.

And now, the performance criteria that also is the fitness function of the PSO in this design is defined as (15):

$$J = (1 - \exp(-1)) \times (M_p + M_{p1}) + \exp(-1) \times (t_s - t_r) \quad (15)$$

Where t_r is the rise time of the output current, t_s is the settling time of the output current, M_p is the overshoot of the output current, and M_{p1} is the overshoot of the output voltage.

4.3. Implementation of PSO-LQR Controller

With the above analysis, the specific implementation steps of the parameter optimization of PID controller based on PSO can be divided as follow:

- step1: to generate initial population;
- step2: to determine the fitness value of each particle using the performance criteria;
- step3: to evaluate the fitness value of each particle, and update the global optimum position value;
- step4: to update the velocity and position of the particles;
- step5: if the maximum iteration number comes to the end or the performance criteria is satisfactory, the system gets the optimal solution. Otherwise, it returns to step 2.

The specific method is as below:

1. Initialize the number of the particle(Population size): Population size affects the performance of the PSO algorithm. It is easy to get local optimal solution, if the population size is small. If the population size is too large, it is difficult and time-consuming to realize, which will exponentially increase the complexity of the algorithm. In this paper, the population size is set to 20.

2. Initialize the particle dimension: the number of the particle dimension is determined by the optimized object. The output of the PSO module are the values of the elements on the digonal of the weighted matrices, such as $Q(1,1)$, $Q(2,2)$, $Q(3,3)$, $Q(4,4)$, so the particle dimension is set to 4, and the matrix R is set to 1.

3. Initialize the range of particles: in order to accelerate the calculation speed, the elements on the digonal of the weight matrices have been adjusted by trial and error. Ultimately, the range of the four parameters has been respectively set as follows, $Q(1,1) \in [10,100]$, $Q(2,2) \in [2,100]$, $Q(3,3) \in [1,100]$ and $Q(4,4) \in [1,100]$.

4. Determine the fitness value: calculate the fitness value of each particle through the (15).

5. Simulation and Results

As previously mentioned, the particle dimension is set to 4, the population size is set to 20, the other parameters are chosen as follows: ω (Inertia weight factor)=0.6; the acceleration constant $c_1 = 2$ and $c_2 = 2$; the velocity maximum of particle $v_{max} = 1$, the velocity minimum of particle $v_{min} = -1$; the maximal iteration number Iteration = 50, and the initial value of the particles are randomly generated within a certain space. The design takes the step signal as input signal to achieve the step response of the control system. The optimized value of the weight matrix is $Q = \text{diag}(100, 1, 1, 1.5506)$. The curve of the optimal fitness value, which has been achieved by the proposed method, is shown in figure 3, and the curves of the element value of weight matrix Q are shown in figure 4, figure 5, figure 6 and figure 7 respectively.

Another example of the trial and error for LQR controller is given to illustrate the proposed design is effective. The results of the three methods are shown in figure 8. From the simulation results, it can be conclude that the output response time and the steady time of the system, of which the LQR controller has been optimized by PSO, are much shorter with better effects.

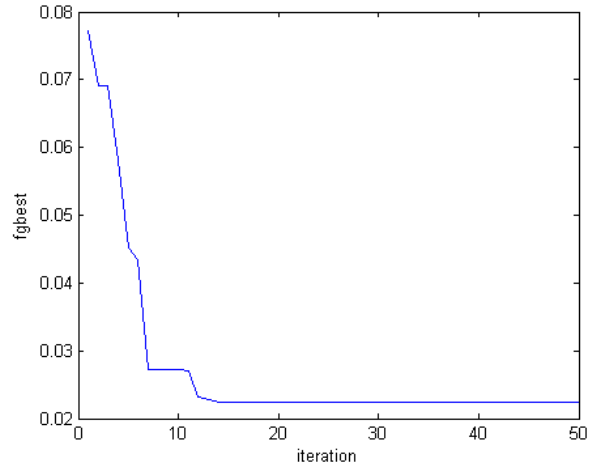


Figure 3: Curve Of The Global Optimal Fitness Value

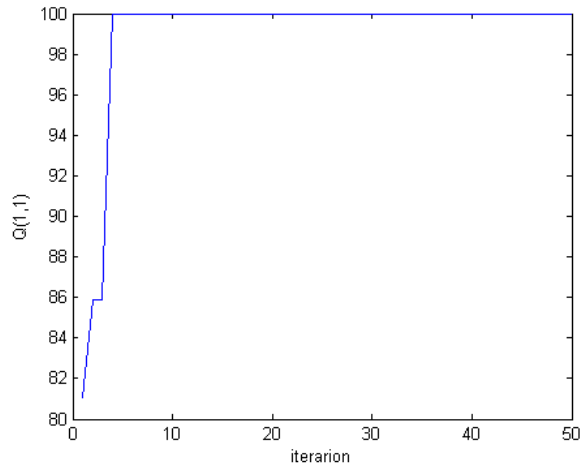


Figure 4: Curve Of The Q(1,1)Based On PSO

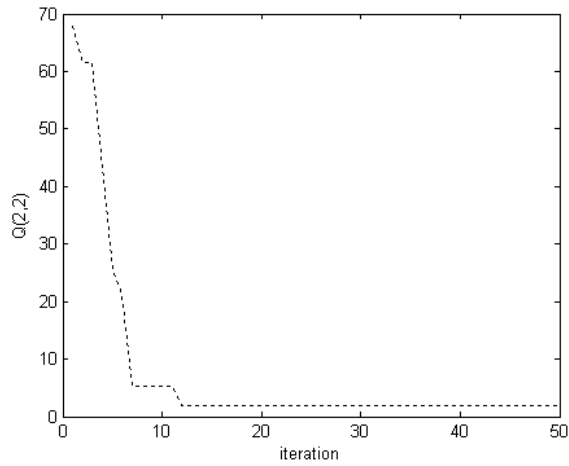


Figure 5: Curve Of The Q(2,2)Based On PSO

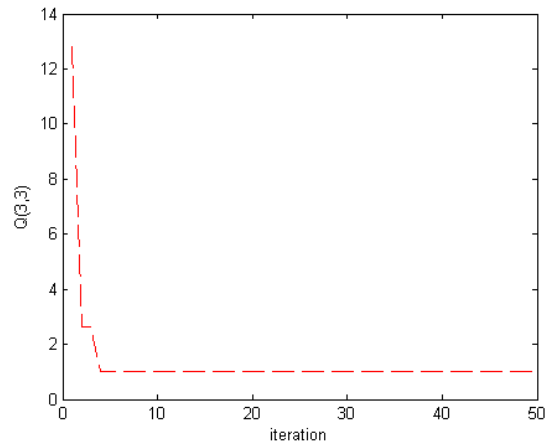


Figure 6: Curve Of The Q (3,3)Based On PSO

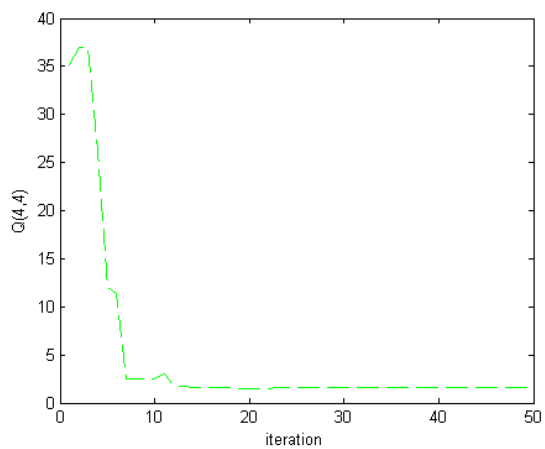


Figure 7: Curve Of The Q (4,4)Based On PSO

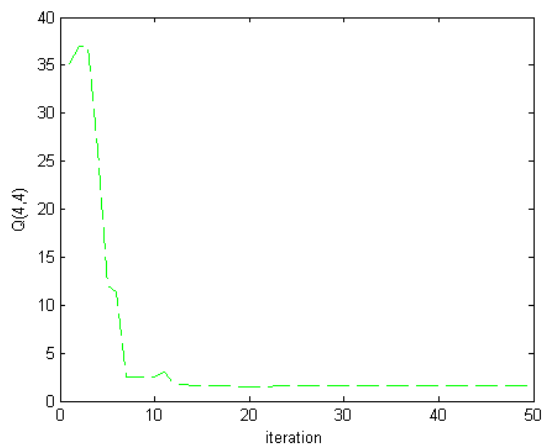


Figure 8: Curve Of The Q (4,4)Based On PSO

6. Conclusions

Particle Swarm algorithm is a robust, simple and very efficient optimization algorithm. In this paper, the optimization of the weighed matrix of the LQR controller has been done by PSO, which has avoided choosing the matrice Q and R artificially when the optimal LQR controller of the magnet power supply for the accelerator has

been devised. The research and results show that the parameter optimization of the LQR controller with PSO has an satisfactory control effect, which is feasible, universal and practical. The method overcomes the shortcomings of the weighted matrices selected difficultly. The result indicates that the output current can stably run according to the requirements of the accelerator.

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