

# Analysis and Synthesis of Geneva Mechanism with Elliptic Crank

Han Jiguang Yu Kang

Jiangsu Normal University, Xuzhou, Jiangsu, China, 221116  
[hjg@jsnu.edu.cn](mailto:hjg@jsnu.edu.cn)

## Abstract

*For both internal and external Geneva mechanism, the kinematics coefficient of the Geneva mechanism is a constant if the groove number of the Geneva wheel is a constant. The elliptic crank using as the drive crank of the Geneva wheel is equal to the mechanism which has a variable length and a variable speed along the elliptical moving crank. Therefore the kinematics coefficient of the Geneva mechanism can be changed. In this paper the analysis method of the combined Geneva mechanism is presented. The synthesis method of the combined Geneva mechanism is put forward based upon the kinematics coefficients. The calculation method of the extreme kinematics coefficient is proposed. In the end, the design example is given.*

**Keywords:** Combined mechanism, Geneva, elliptic generator, mechanism synthesis, kinematics coefficient

## 1. Introduction

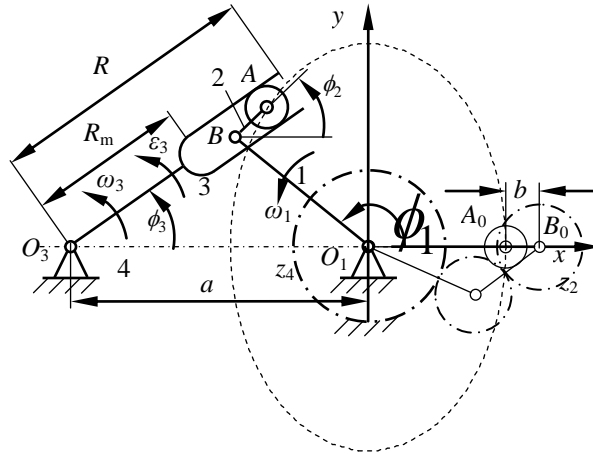
Geneva mechanism is one of the most widely used stepping mechanisms because of its simple structure, reliability and accuracy in controlling the kinematic angles of the driven parts and *etc.* However, for the fixed groove number, the kinematic coefficient - the ratio of the Geneva kinematic time and the active part kinematic time in one circle is a constant, which limits its applications. In order to effectively improve the Geneva kinematic characteristics, combined mechanisms of the elliptic gear and the Geneva wheel were studied by literature researcher [1-3]. However, a difficulty can be found for manufacturing the elliptic gears. As we know, for a plant mechanism composed by a fixed centre wheel  $z_4$ , a hold bar 1 and a plant wheel  $z_2$ , the track of the point A on the plant wheel is the cycloid curve [4]. When the relative transmission ratio of the plant wheel  $z_2$  and fixed centre wheel  $z_4$  relative to hold bar  $O_1B$  (active part 1) is  $i = z_4/z_2 = 2$ , the cycloid curve will change to elliptic curve as shown in Figure 1 (dotted line). These plant mechanisms can be called elliptic cranks.

The moving point A has different radius and speed in different positions. Similar to the movement of elliptic gear Geneva combination mechanism, such as change the kinematic coefficient, can be achieved by using the moving point A as the drive round pin to drive the Geneva wheel. This paper presents the analysis and combined method of this new combined mechanism.

## 2. Kinematic Analysis of the Combined Geneva Mechanism

As shown in Figure 1, the Geneva mechanism can be simplified as an oscillating bar with a groove. The inner Geneva and the outer Geneva have nearly the same structure types [7]. Minor arc in the left of the ellipse is used to drive the outer Geneva and the right major arc is used to drive the inner Geneva. Therefore, we can use the uniform kinematic function to describe the inner and the outer Geneva

mechanism. Supposed at the initial moment  $\phi_1 = 0$ , drive round pin point  $A_0$  is in the middle of  $O_1B_0$ . For easy analysis, we set the length of the hold bar  $O_1B$  as 1, the relative length with connecting rod  $BA$  is  $b$ , then the coordinate  $(x, y)$  of round pin  $A$  can be expressed as:



**Figure 1. Ellipse Generate Mechanism and Geneva Combined Mechanism**

$$\begin{cases} x = (1-b)\cos\phi_1 \\ y = (1+b)\sin\phi_1 \end{cases} \quad (1)$$

The angular displacement  $\phi_3$  of Geneva 3 is:

$$\phi_3 = \arctan \frac{y}{x+a} \quad (2)$$

The angular velocity  $\omega_3$  of Geneva 3 is:

$$\omega_3 = \frac{y'(x+a) - yx'}{(x+a)^2 + y^2} \omega_1 \quad (3)$$

Where,  $\begin{cases} x' = -(1-b)\sin\phi_1 \\ y' = (1+b)\cos\phi_1 \end{cases}$

The angular acceleration  $\alpha_3$  of Geneva 3 is:

$$\alpha_3 = \frac{[y''(x+a) - yx'']\omega_1^2 - 2[x'(x+a) + yy']\omega_3\omega_1}{(x+a)^2 + y^2} \quad (4)$$

Where,  $\begin{cases} x'' = -(1-b)\cos\phi_1 \\ y'' = -(1+b)\sin\phi_1 \end{cases}$

The jerk  $j_3$  of Geneva 3 is:

$$j_3 = \frac{[y'''(x+a) + y''x' - y'x'' - yx''']\omega_1^3 - 2[x''(x+a) + x'^2 + y'^2 + yy'']\omega_3\omega_1^2}{(x+a)^2 + y^2} - \frac{4[x'(x+a) + yy']\alpha_3\omega_1}{(x+a)^2 + y^2} \quad (5)$$

Where,  $\begin{cases} x''' = (1-b)\sin\phi_1 \\ y''' = -(1+b)\cos\phi_1 \end{cases}$

Supposed when the round pin  $A$  is getting into the groove, the position angle of active part  $O_1B$  is  $\phi_0$ . From the mechanism symmetry, the working region of the outer Geneva round pin  $A$  is the minor arc from  $\phi_0$  to  $2\pi - \phi_0$ ; the working region of the inner Geneva round pin  $A$  is the major arc from  $-\phi_0$  to  $\phi_0$ . We introduce the Geneva coefficient  $\delta$ : outer Geneva  $\delta=1$  and inner Geneva  $\delta=-1$ . The working region of the round pin  $A$  can be expressed as:

$$\delta\phi_0 \leq \phi_1 \leq -\delta\phi_0 + (1+\delta)\pi \quad (6)$$

The kinematic curve of the combined Geneva mechanism is shown in Figure 2. In this example, some parameters are as follows: the Geneva groove number  $z=6$ ; outer Geneva kinematic coefficient is 0.35; inner Geneva kinematic coefficient is 0.65; the position angle of active part  $O_1B$  when the round pin getting into (or out of) groove is  $117^\circ$ ; the connecting rod length is 0.0624; the centre distance is 2.0652. In order to simplify the analysis in this figure, angular velocity, angular acceleration and jerk are magnified to the absolute maximum of 30. The left of the  $243^\circ$  line is outer Geneva kinematic curve, the right is the inner kinematic curve. The sum of the inner and the outer Geneva kinematic coefficient is equal to 1.

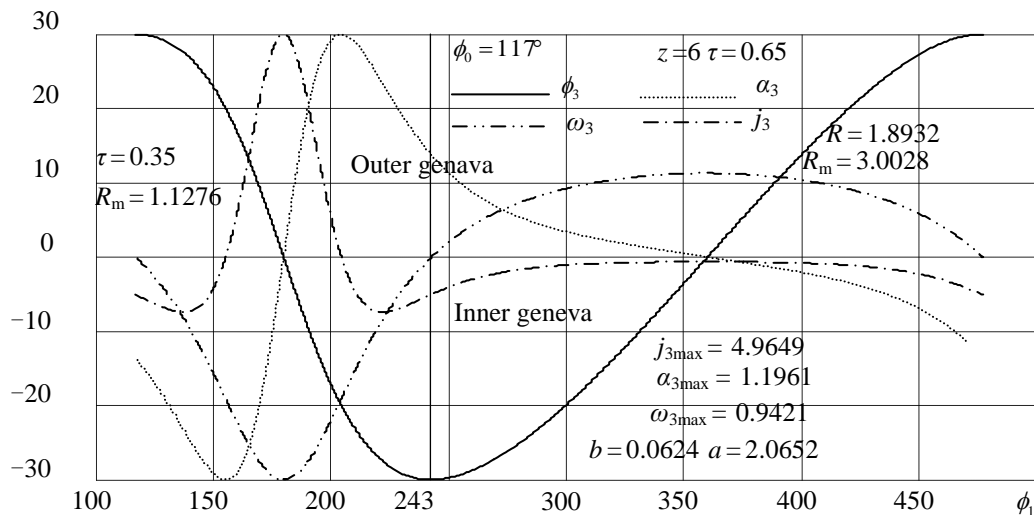


Figure 2. Kinematic Curve of the Combined Geneva Mechanism

### 3. Mechanism Combine based on the Kinematic Coefficient $\tau$

When the groove number  $z$  and the kinematic coefficient  $\tau$  are constants, according to the definition of the kinematic coefficient,

$$\phi_0 = \pi \left( \frac{1+\delta}{2} - \delta\tau \right) \quad (7)$$

When the Geneva is turning, the turning angle of active gear 1 is  $2\alpha$ , then

$$\alpha = \frac{1+\delta}{2} \pi - \delta\phi_0 \quad (8)$$

The angle between two adjacent grooves is  $2\beta$ , i.e.

$$\beta = \frac{\pi}{z} \quad (9)$$

### 3.1. Relative Length $b$ of Connecting Rod $BA$

In order to avoid the rigid impact, the round pin  $A$  should be on the tangent direction to the inter groove. The  $AO_3$  and cycloid is tangent at the point  $A(x_0, y_0)$ ,

$$\begin{cases} x_0 = (1-b)\cos\phi_0 \\ y_0 = (1+b)\sin\phi_0 \end{cases} \quad (10)$$

At this time the position angle of the groove is  $\beta$ , then

$$\text{tg}\beta = \text{tg}\phi_3 = \frac{y'_0}{x'_0} = \frac{-(1-b)\cos\phi_0}{(1+b)\sin\phi_0}$$

Then

$$b = -\frac{\cos(\phi_0 - \beta)}{\cos(\phi_0 + \beta)} \quad (11)$$

If  $b < 0$ , it means in the initial moment  $\phi_1 = 0$ , point  $A_0$  is in the outside of  $O_1B_0$ .

### 3.2. Centre Distance $a$

The center distance  $a$  can be determined based upon the tangency point  $A(x_0, y_0)$

$$a = \frac{y_0}{\text{tg}\beta} - x_0 = -\frac{2\cos\beta}{\cos(\phi_0 + \beta)} \quad (12)$$

### 3.3. Geneva Radius $R$

The Geneva radius  $R$  is the distance between rotation center  $O_3$  and round pin into groove point  $A(x_0, y_0)$ . For the outer Geneva,  $R$  is the maximum radius of the Geneva groove, for the inner Geneva,  $R$  is the minimum radius of the Geneva groove, *i.e.*,

$$R = \frac{y_0}{\sin\beta} = -\frac{2\sin^2\phi_0}{\cos(\phi_0 + \beta)} \quad (13)$$

### 3.4. Groove Bottom Radius $R_m$

According to Figure 1, the groove bottom radius of the outer Geneva  $R_m$  is the minimum distance between round pin center  $A$  and rotation center  $O_3$ . For the inner Geneva groove, the bottom radius is the maximum distance. Define  $l = AO_3$ , then

$$l^2 = (x+a)^2 + y^2 \quad (14)$$

The extreme point of  $l$  is certain at  $(l^2)' = 0$ . Combine formula (1), (12) and (14), then diff to  $\phi_1$ , we can get

$$\sin\phi_1[\cos\phi_0\cos^2\beta + (\sin^2\phi_0 - \cos^2\beta)\cos\phi_1] = 0 \quad (15)$$

Formula (15) has three roots in total, the first real root is  $\phi_1 = \pi$ , corresponding to minimum value  $-(1-b)+a$  of  $l$ , *i.e.*, is the bottom radius  $R_m$  of the outer Geneva. The second real boot is  $\phi_1 = 0$ , corresponding to the maximum value or local minimum value  $(1-b)+a$  of  $l$ . The maximum value is corresponding to groove

bottom radius  $R_m$  of the inner Geneva. These two real roots can be expressed into a uniform form as

$$R_m = -\delta(1-b) + a = -\frac{2\cos\beta(1+\delta\cos\phi_0)}{\cos(\phi_0 + \beta)} \quad (16)$$

The third root of formula (15) is

$$\phi_1 = \arccos \frac{-\cos\phi_0 \cos^2 \beta}{\sin^2 \phi_0 - \cos^2 \beta} \quad (17)$$

For  $\cos\phi_1 \leq 1$ , the existence condition of formula (17) is

$$-\cos\phi_0 \cos^2 \beta \leq \sin^2 \phi_0 - \cos^2 \beta$$

Or

$$\phi_0 \leq \arccos(-\sin^2 \beta) \quad (18)$$

Put formula (1), (12) and (17) into formula (14),

$$R_m = \frac{-2\sin^2 \phi_0 \sin \beta}{\cos(\phi_0 + \beta)\sqrt{\sin^2 \phi_0 - \cos^2 \beta}} \quad (19)$$

Referring to Figure 3, when the inequality (18) is satisfied, the root  $\phi_1 = 0$  is corresponding to the local minimum value of  $l$ , which can be proved by the positive second derivative which is omit because of its complex. Hence we calculate the groove bottom radius  $R_m$  according to formula (19) when condition (18) is in satisfied. Otherwise, we use formula (16).

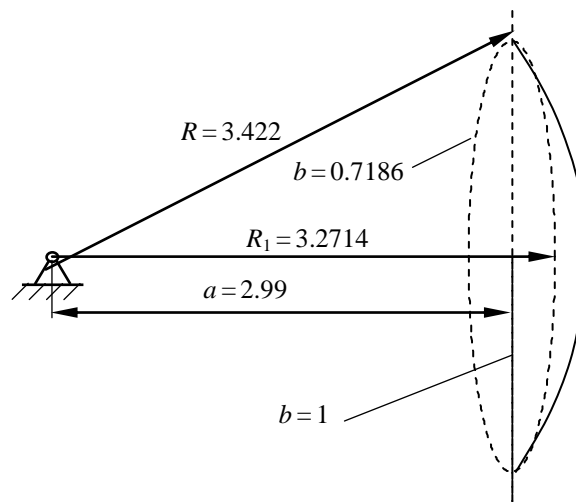


Figure 3. Working Failure caused by Flat Ellipse

### 3.5. Extreme $\tau_m$ of the Kinematic Coefficient

The more  $b$  is close to 1, the more flat the ellipse is. When  $b=1$ , the ellipse is changed into a line, as shown in the dotted line in Figure 3. Suppose  $R_1$  is the distance between the ellipse intersection point with minor right axis side and the rotation center  $O_3$ , *i.e.*,

$$R_1 = a + (1-b) = -\frac{2\cos\beta(1-\cos\phi_0)}{\cos(\phi_0 + \beta)} \quad (20)$$

If the ellipse is too flat, the groove radius  $R$  may be greater than  $R_1$ , as shown in Figure 3. For the outer groove, during the none-working period, the round pin  $A$  is still in the groove and the Geneva can't realize stop. For the inner Geneva, during the working period, the round pin  $A$  is out of the groove and can't drive the Geneva to move. Both situations should be avoided. Therefore, the extreme condition of kinematic coefficient is  $R = R_1$ . From formula (13) and (20) we can get

$$\phi_0 = \arccos(\cos\beta - 1) \quad (21)$$

From formula (8) we can get the extreme  $\tau_m$  of the kinematic coefficient

$$\tau_m = \frac{1 + \delta}{2} - \delta \frac{\arccos(\cos\beta - 1)}{\pi} \quad (22)$$

For the outer groove,  $\tau_m$  is the maximum. For the inner groove  $\tau_m$  is the minimum. The relationship map between  $\tau_m$  and groove number  $z$  is shown in Figure 4.

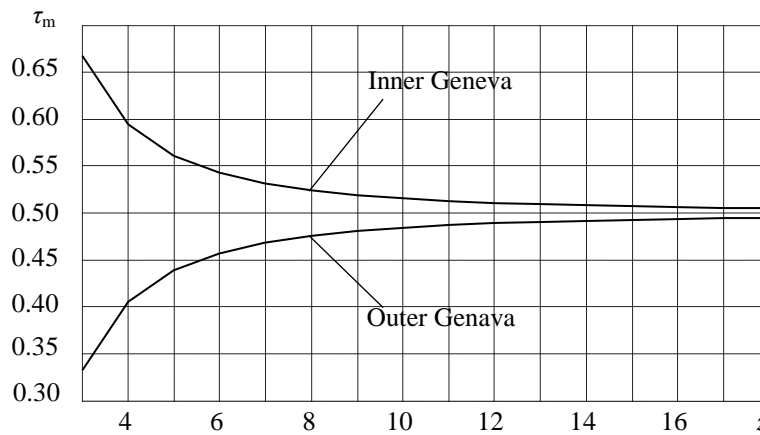


Figure 4. Extreme of the Kinematic Coefficient

#### 4. Design Examples

Design an outer Geneva mechanism with kinematic coefficient  $\tau = 0.4$  and the groove number of the Geneva wheel  $z = 4$ .

At first calculate according to formula (9) and get

$$\beta = \pi / 4$$

Take  $\delta = 1$ , calculate according to formula (19) and get

$$\tau_m = 0.4054 > 0.4$$

Because of giving  $\tau = 0.4 < \tau_m = 0.4054$ , the mechanism can be design.

Then from formula (7), calculate the groove-in angle

$$\phi_0 = 108^\circ$$

According to formula (11-14) and get

$$b = 0.5095$$

$$a = 1.5872$$

$$R = 2.0303$$

$$R_m = 0.0777$$

At last get the working region of the round pin A from the formula (6)

$$108^\circ \leq \phi_1 \leq 252^\circ$$

Take kinematic analysis according to the formulas (1-5). Using the numerical comparative method to ensure the extreme of the angular velocity, angular accelerate and jerk, respectively as

$$\omega_{3\max} = 1.3764$$

$$\alpha_{3\max} = 1.7173$$

$$j_{3\max} = 8.4379$$

The angular accelerate on the Geneva run and stop points are

$$\alpha_{30} = \mp 0.5528$$

## 5. Conclusions

As the drive mechanism of the Geneva mechanism, the elliptic crank can replace the elliptic gear mechanism in some range. For the given groove number, different Geneva mechanisms with different kinematic coefficients can be combined. Of course, the kinematic coefficients  $\tau$  must be less than  $\tau_m$  for the outer Geneva mechanism and large than  $\tau_m$  for the inner Geneva mechanism.

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## Corresponding author

Han Jiguang, Ph.D, is a professor in the School of Mechanical and Electrical Engineering, Jiangsu Normal University. His major research interests include mechanical transmission and CAD. The School of Mechanical and Electrical Engineering, Jiangsu Normal University, Xuzhou, Jiangsu, China, 221116. E-mail: hjg@jsnu.edu.cn.

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