

## Lead-Lag PSS Design Based on $H_\infty$ Control Theory and Genetic Algorithm

Dharam Singh noroji and Rakesh narvey

*Electrical Engineering Department  
Madhav Institute of Technology and Science  
Gwalior, India*

*dharam.ei21@gmail.com, rakeshnarvey@yahoo.com*

### **Abstract**

*The aim of this paper is to explain the way to scale back the dimension of an influence system stabilizer (PSS) supported  $H_\infty$  management theory. In recent years, sturdy PSS styles that adopt associate in nursing  $H_\infty$  controller are investigated so as to ensure the performance once the state of the system configuration and power flow modification. However the  $H_\infty$  controller has not been wide adopted into sensible use attributable to the tangled nature of its theory and structure. We tends to think about the  $H_\infty$  management downside below the condition that PSS structure is mounted to be a lead-lag compensator. Infinity norm of transfer function from disturbance to output is subjected to be minimized via searching and evolutionary computation. The resulted optimal parameters make the system stable and also guarantee robust performance. We applied the evolutionary robust controller to a pneumatic servo system. For performances comparison, three controllers; PID with derivative first order filter controller, PI controller and H- loop shaping controller are investigated. We tends to optimize the parameters of the Lead-Lag PSS by a genetic algorithm program that has associate in nursing analysis perform that takes under consideration a closed-loop system  $H_\infty$  norm and a desired response. During this manner, we tends to design a PSS that encompasses a standard managementler structure and guarantees its control performance.*

*Keywords- Power system stabilizer; Lead-lag compensation  $H_\infty$  control theory; Dimensional reduction; Genetic-Algorithm*

### 1. INTRODUCTION

To incorporate the performance specification into the robust control problem, various techniques such as  $H_2/H_\infty$  optimal control, mixed sensitivity function, H-loop shaping, u-synthesis, etc., were proposed. Most of these techniques design optimal robust controller via solving of two Riccati equations [1]. Successful practical works of  $H_\infty$  control were shown in several research works [2-5]. However, controllers Thai are designed by these H control techniques result in complicated structure and high order. It is well known that high order or complicated structure controller is not desired to implement in practical works. This paper presents a method to design a controller, which has simple structure and lower order, and still retains robustness. We propose a fixed-structure H\_ loop shaping control that evolves by genetic algorithm. The approach is based on the concept of H loop shaping control proposed by Glover and Mc. Farlane. In this technique, performance of the controller is indicated by only a single index, stability margin ( $E$ ). In our approach, we define a controller's structure and then evaluate the control's parameters by genetic algorithm. Firstly, we shape the nominal plant by weighting functions like as the conventional procedure. Then, we define the objective function or fitness function to be maximized as the stability margin ( $E$ ) of the shaped plant [6].

However, the controllers that are generally used in industrial control systems are not  $H_\infty$  controllers but rather classical controllers, such as PID controllers. The  $H_\infty$  controller is not widely used in practice for several reasons. For example, the calculation costs of classical controllers are low because of the simplicity of their structure, and established experience and know-how can be used in controller tuning. On the other hand, the theory of  $H_\infty$  control is rather difficult to comprehend, and the structure is complex because the controller is high-dimensional. For these problems, methods for reducing the dimensions of the  $H_\infty$  controller to that of a classical controller have been proposed.

Many research works on position tracking of pneumatic servo system employed the approaches of linear and nonlinear control. Because of time-consuming in process identification and many feedback states in this approach, nonlinear control is now implemented in very precise requirement applications but not yet for general industrial applications. In linear control, linear dynamic model is obtained by applying the linearization technique around a specified operation point. The resulted linear dynamic between position output and control valve voltage input is expressed as a third order dynamic system, which contains a pole at the origin. Because of containing of a pole at the origin of pneumatic model, it is still difficult to apply standard technique to identify the parameters. Hamiti et al., [7] proposed an analog inner loop proportional controller to stabilize a pneumatic system and used a new close loop plant as a modified pneumatic plant model.

## 2. $H_\infty$ Control Theory

### A. $H_\infty$ Control Theory

For the closed loop system shown as a block diagram in Fig. 1, the state equations of the generalized plant are expressed as follows.

$$\begin{aligned} \dot{x} &= Ax + B_1w + B_2u, \\ z &= C_1x + D_{11}w + D_{12}u, \\ y &= C_2x + D_{21}w + D_{22}u, \end{aligned} \quad (1)$$

Where  $x$  represents the state variables;  $w$ , the disturbance;  $z$ , the controlled variable;  $u$ , the control input; and  $y$ , the observed variable ( $x \in R^n, w \in R^{m1}, u \in R^{m2}, z \in R^{p1}, y \in R^{p1}$ ).

The feedback control for a generalized plant  $G(s)$  is given by using the controller  $K(s)$ :

$$u = Ky. \quad (2)$$

The generalized plant  $G(s)$  is expressed as

$$G(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad (3)$$

$$G_{ij}(s) = C_i(sI - A)^{-1}B_j + D_{ij}, \quad i, j = 1, 2. \quad (4)$$

As a result, the transfer function from  $w$  to  $z$  is

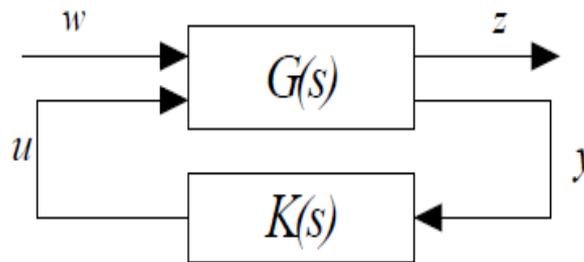
$$T_{zw} = G_{11}(s) + \frac{G_{12}(s)G_{21}(s)K(s)}{1 - G_{22}(s)K(s)}. \quad (5)$$

A controller that suppresses the value of the transfer function is required, because the aim is to suppress disturbances of the controlled value  $z$ . In  $H_\infty$  control theory, the  $H_\infty$  norm is used as an index of the size of the transfer function. The norm of a steady transfer function is defined by

$$\|T_{zw}\|_{\infty} = \sup \frac{\|z\|_2}{\|w\|_2} \quad (6)$$

$H_{\infty}$  control is defined as the problem of finding the controller  $K(s)$  that internally stabilizes the closed-loop system in Fig. 1 and satisfies (7) for a given positive number  $\gamma$ . Here, the transfer-function of the controller  $K(s)$  is an  $m \times p$  matrix.

$$\|T_{zw}\|_{\infty} < \gamma \quad (7)$$



**Figure 1. Closed Loop System**

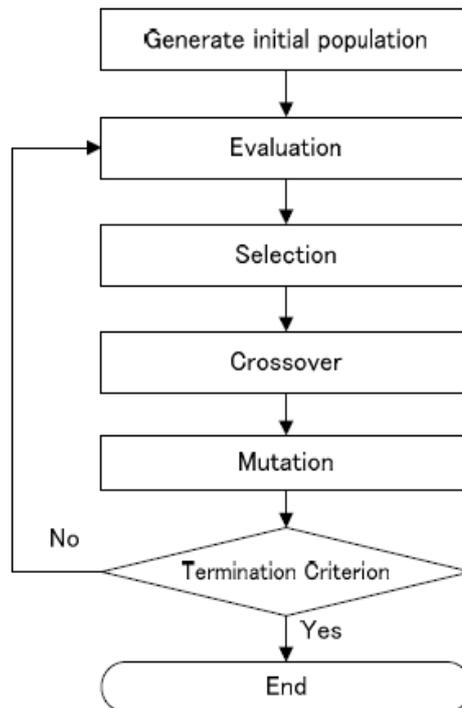
*B. Dimensional Reduction by Direct Method*

The direct method is one of the dimensional reduction methods that derive a low-dimension controller for a high dimension controlled object. In the direct method, the parameters of the controller are determined by using some objective function under the condition that the controller structure is fixed. Direct dimensional reduction of the  $H_{\infty}$  controller adopts the  $H_{\infty}$  norm shown in the previous section as the objective function. The closed loop transfer function  $T_{zw}$  and the  $H_{\infty}$  norm  $\|T_{zw}\|_{\infty}$  are derived from (5) which has the fixed-structure controller  $K(s)$ . The parameters of  $K(s)$  are adjusted to satisfy the  $H_{\infty}$  norm constraint in (7). In this paper the structure of the controller  $K(s)$  is identical to that of an existing PSS.

**3. Genetic Algorithms**

In the last few years, Genetic Algorithms (GAs) have shown their potentials in many fields, including in the field of electrical power systems. Although GAs provide robust and powerful adaptive search mechanism, they have several drawbacks (Mitchell, 1996). Some of these drawbacks include the problem of “genetic drift” which prevents GAs from maintaining diversity in its population. Once the population has converged, the crossover operator becomes ineffective in exploring new portions of the search space. Another drawback is the difficulty to optimize the GAs’ operators (such as population size, crossover and mutation rates) one at a time. These operators (or parameters) interact with one another in a nonlinear manner. In particular, optimal population size, crossover rate, and mutation rate are likely to change over the course of a single run (Baluja, 1994). From the user’s point of view, the selection of GAs’ parameters is not a trivial task. Since the ‘classical’ GA was first proposed by Holland in 1975 as an efficient, easy to use tool which can be applicable to a wide range of problems (Holland, 1975), many variant forms of GAs have been suggested often tailored to specific problems (Michalewicz, 1996). However, it is not always easy for the user to select the appropriate GAs parameters for a particular problem at hand because of the huge number of choices available. At present, there is a little theoretical guidance on how to select the suitable GAs parameters for a

particular problem (Michalewicz, 1996). Still another problem is that the natural selection strategy used by GAs is not immune from failure. To cope with the above limitations, an extremely versatile and effective function optimizer called Breeder Genetic Algorithm (BGA) was recently proposed (Muhlenbein, 1994). BGA is inspired by the science of breeding animals. The main idea is to use a selection strategy based on the concept of animal breeding instead of “natural selection” (Irhamah & Ismail, 2009). The assumption behind this strategy is as follows: “mating two individuals with high fitness is more likely to produces an offspring of high fitness than mating two randomly selected individuals” [8].



**Figure 2. Flow Chart of A GA**

## **4. Formulation of Proposed Evaluation Function for Dimensional Reduction**

### *A. Elements of the Evaluation Function*

We explain the elements of the evaluation function based on the aim of improving PSS performance.

#### *a. $H_{\infty}$ norm*

If the value of the  $H_{\infty}$  norm is larger than 1, a penalty is added to the evaluation function in order to ensure  $H_{\infty}$  performance. Here, the value of the penalty is proportional to that of the  $H_{\infty}$  norm. By the procedure, solutions with high evaluation values are searched intensively. We can also find the controller with smallest  $H_{\infty}$  norm if no controller with an  $H_{\infty}$  norm smaller than 1 exists under the constraint of a fixed structure [9].

*b. Control of Power Oscillation*

When we design the reduced-dimension PSS controller, we need to take into account the reduction of the risk of the generator tripping and prompt suppression of the power oscillation. These are evaluated based on the phase angle data obtained by transient stability calculations. We use an LMI-based  $H^\infty$  PSS for the evaluation. In the proposed method, we make the oscillation within the desired value when the  $H^\infty$  PSS is installed. As the evaluation value, we adopt the sum of the absolute values of the difference between a phase angle with the proposed and that of the  $H^\infty$  PSS.

*B. Proposed Evaluation Function*

The proposed evaluation function is

$$f = \sum_{time=0sec}^{10sec} |\delta_{H^\infty} - \delta_{lead-lag}| + W_f \cdot penalty \quad (9)$$

$$\begin{aligned} \|T_{zw}\|_\infty < 1 : penalty &= 0, \\ \|T_{zw}\|_\infty \geq 1 : penalty &= \|T_{zw}\|_\infty, \end{aligned}$$

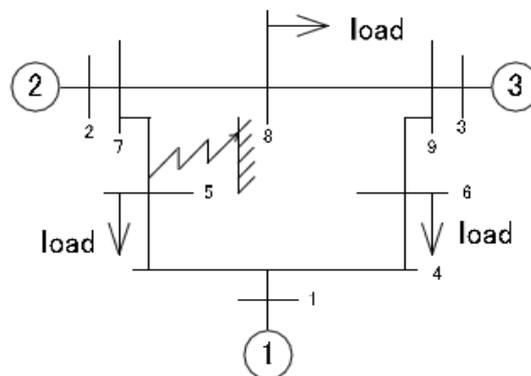
where  $\delta_{H^\infty}$  is the value of the phase angle with the  $H^\infty$  PSS,  $\delta_{lead-lag}$  is the value of the phase angle with the proposed PSS, and  $W_f$  is a weight of evaluation for the penalty term of the function. We use the  $W_f = 10$  in this paper as an example. The penalty is added if the value of the  $H^\infty$  norm is larger than 1.

**5. Design of the Proposed PSS**

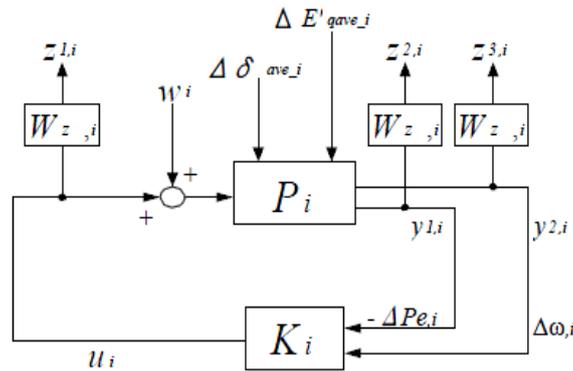
In this paper, we design a two-input ( $\Delta P + \Delta \omega$  type) PSS that can control power swings of both long and short terms. The procedure for designing the proposed controller can be divided into five parts, as follows.

*A. Definition of Control Target*

A 3-machine 9-bus system model shown in Fig. 3 is used as a model for the proposed control design. In the system, generator 2 is the control target; therefore the designed PSS is installed at that generator. The system data and the generator models (AVR, GOV, etc.) are given in Ref. [10]. We are designing a PSS for a distributed control system. Thus, we divide the 3-machine 9-bus system into 3 subsystems, and we make generator control system models for each subsystem. The generator control system model is shown in Fig. 4.



**Figure 3 Model of 3-Machine 9-Bus System**



**Figure 4. Generator Control System Model of Each Subsystem**

**B. Definition of Weight Functions**

In Figure 4  $W_{z1,i}$  is the weight function for reducing the modelling error.  $W_{z2,i}$  and  $W_{z3,i}$  are the weight functions for reducing the control error. They are the sensitivity to active power deviations and angular velocity deviations, respectively. Generally, the frequency response of the modeling error is large in a high frequency band and that of the sensitivity for the control error is large in low frequency bands. Therefore,  $W_{z1,i}$  is set to be large in high frequency bands, whereas  $W_{z2,i}$  and  $W_{z3,i}$  are set to be large in low frequency bands. The weight functions need to be adjusted by trial and error. In this paper, we select  $W_{z2,i}$  and  $W_{z3,i}$  when we design the  $H^\infty$  PSS which is used to obtain the desired value, and we make use of the weight functions for the proposed reduced-dimension PSS. Each weight function is given by the following equation.

**BI.1. Definition of the Generalized Plant**

The equations of the generalized plant are

$$\begin{aligned} \dot{x} &= Ax + B_1w + B_2u, \\ z &= C_1x + D_{11}w + D_{12}u, \\ y &= C_2x + D_{21}w + D_{22}u, \end{aligned} \tag{11}$$

where  $x$  represents the state variables;  $w$ , the disturbances;  $z$ , the controlled variables;  $y$ , the observed variables; and  $u$ , the control input.

The equations of the generalized plant in a  $\Delta P + \Delta\omega$  type  $H^\infty$  control system are given in (12)–(14). These equations are applied to the subsystem of the  $i$ -th generator. The mutual interference signals from other generators are treated as disturbance inputs to the  $i$ -th generator because a decentralized control is introduced

$$\begin{bmatrix} \Delta\dot{\delta}_i \\ \Delta\dot{\omega}_i \\ \Delta\dot{E}'_{qi} \\ \Delta\dot{E}'_{fdi} \end{bmatrix} = \begin{bmatrix} 0 & \omega_b & 0 & 0 \\ -\frac{K_{1,ii}}{M_i} & -\frac{D_i}{M_i} & -\frac{K_{2,ii}}{M_i} & 0 \\ \frac{K_{4,ii}}{\tau_{doi}} & 0 & -\frac{C_{3,ii}}{\tau_{doi}} & \frac{1}{\tau_{doi}} \\ -\frac{K_{Ai}}{T_{Ai}} & -\frac{K_{5,ii}}{T_{Ai}} & 0 & -\frac{K_{6,ii}}{T_{Ai}} \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E'_{qi} \\ \Delta E'_{fdi} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_{Ai}}{T_{Ai}} \end{bmatrix} u_i + \begin{bmatrix} 0 & 0 \\ \sum_{j \neq i} \frac{K_{L,ij}}{M_i} & 0 \\ -\sum_{j \neq i} \frac{K_{4,ij}}{\tau_{doi}} & 0 \\ \sum_{j \neq i} \frac{K_{Ai}}{T_{Ai}} K_{5,ij} & \frac{K_{Ai}}{T_{Ai}} - \sum_{j \neq i} \frac{K_{Ai}}{T_{Ai}} K_{6,ij} \end{bmatrix} \begin{bmatrix} \Delta \delta_{ave_i} \\ w_i \\ \Delta E_{qave_i} \end{bmatrix}, \quad (14)$$

$$\begin{bmatrix} -\Delta P_q \\ \Delta q \\ u_i \end{bmatrix} = \begin{bmatrix} -K_{1,ii} & 0 & -K_{2,ii} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta q \\ \Delta E_{qi} \\ \Delta E_{fdi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_{ave_i} \\ w_i \\ \Delta E_{qave_i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_i, \quad (15)$$

$$\begin{bmatrix} -\Delta P_q \\ \Delta q \end{bmatrix} = \begin{bmatrix} -K_{1,ii} & 0 & -K_{2,ii} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta q \\ \Delta E_{qi} \\ \Delta E_{fdi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_{ave_i} \\ w_i \\ \Delta E_{qave_i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_i, \quad (16)$$

where  $\Delta$  is the minimal change;  $\delta$ , the rotor angle;  $\omega$ , the angular velocity;  $E'q$ , the q-axis component of the internal voltage;  $E_{fd}$ , the field voltage;  $\omega\theta$ , the nominal angular velocity ( $2\pi f\theta$ );  $M$ , the inertia constant;  $K1-K6$ , the generator constants;  $KA$ , the AVR gain;  $TA$ , the AVR time constant;  $D$ ,

the damping coefficient; and  $\tau'd0$ , the open circuit time constant.

### BI.2. Evaluation of the Controller

The proposed PSS is evaluated using the  $H_\infty$  norm and transient stability calculations. We carried out 10 s simulations for three-phase to ground faults using the 3-machine 9-bus system. The fault occurs 0.01 s after the beginning of the simulation, one circuit of the double circuit lines is opened 0.07 s after the fault, and the re-closing is performed 0.6 s later.

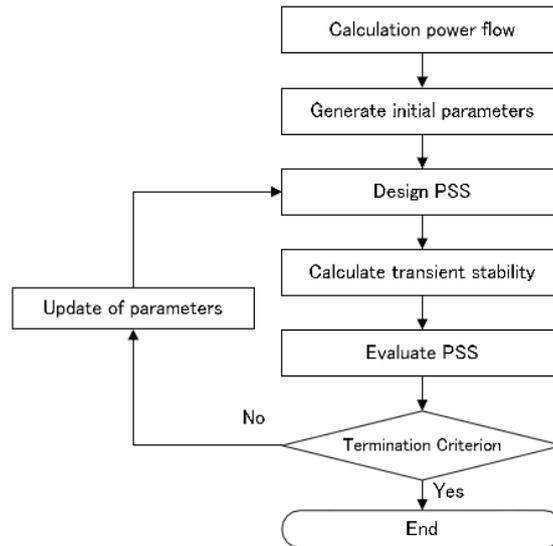
## 6. Parameter Determination for Low-dimension PSS by GA

### A. Parameter Optimization using GA

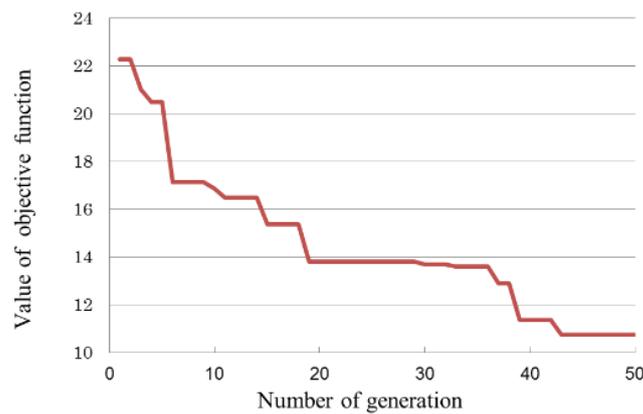
According to the flow chart shown in Fig. 5 we optimize the parameters of the PSS at nominal loading using a GA. We confirm that the value of the evaluation function decreases with the number of generations. The change of the value of the evaluation function is shown in Fig. 6. In the simulation, the total number of chromosomes is 100 and the maximum number of generation is 50. In the figure, the value of the evaluation function is the best value of all the chromosomes [11].

### B. Evaluation of the Reduced-Dimension PSS

The parameters of the PSS are show in Table I. The phase angles of generator 2 with the  $H_\infty$  PSS used for the target response are shown in Fig. 7, together with those of the proposed PSS. The value of the  $H_\infty$  norm is  $\|T_{zw}\|_\infty = 0.5936$ . From the results, we can confirm that the proposed PS achieves  $H_\infty$  performance and suppresses the oscillation adequately.



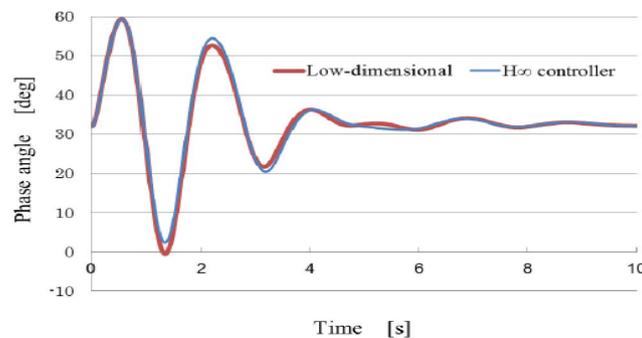
**Figure 5. Flow of the Proposed PSS Parameter Tuning**



**Figure 6. The Transition of the Value of the Objective Function**

TABLE I  
 SAMPLES LOW-DIMENSION PSS PARAMETER

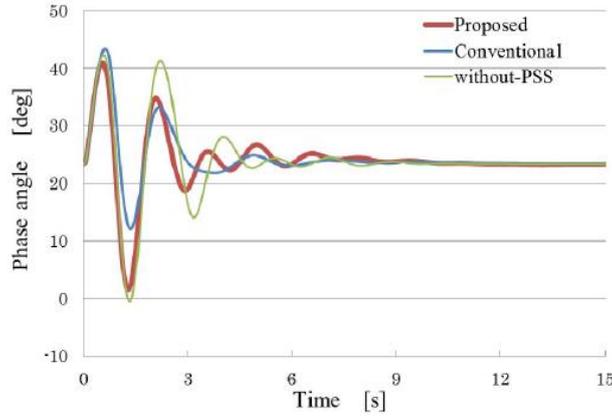
Input	$K$	$T_1$	$T_2$	$T_3$	$T_4$
$\Delta P$	0.1589	0.1984	0.2368	0.0200	0.1362
$\Delta \omega$	0.5918	0.3736	0.8136	1.0000	0.0968



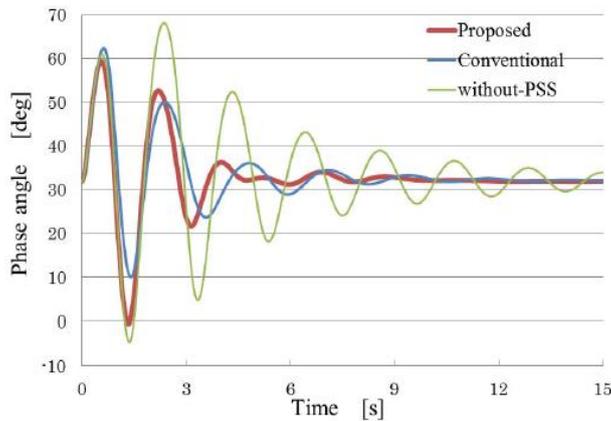
**Figure 7. Evaluation of the Phase Angle at Nominal Loading (Generator 2)**

## 7. Simulations

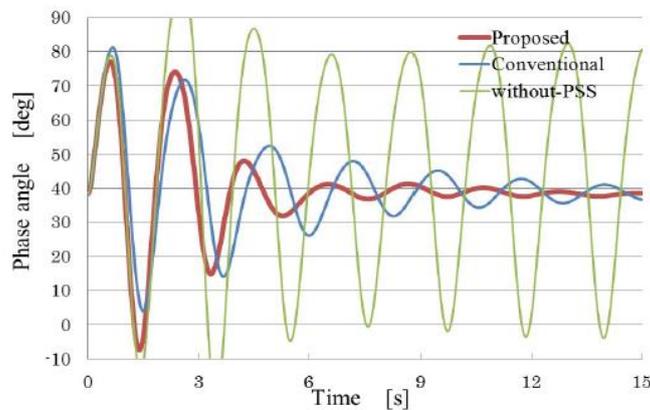
The simulation results of the phase angles (after the outputs are changed by  $-30$ ,  $0$ , or  $+30\%$ ) are shown in Figures 8–10. And the outputs of PSS are shown in Figures 11–13. The values of the evaluation indices for changes of the operating points are shown in Figures 14 and 15.



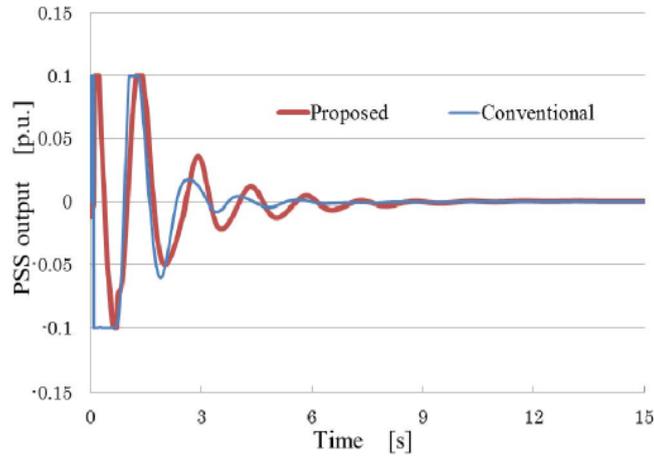
**Figure 8. Phase Angles When The Generator Outputs Decrease By 30%(Generator 2)**



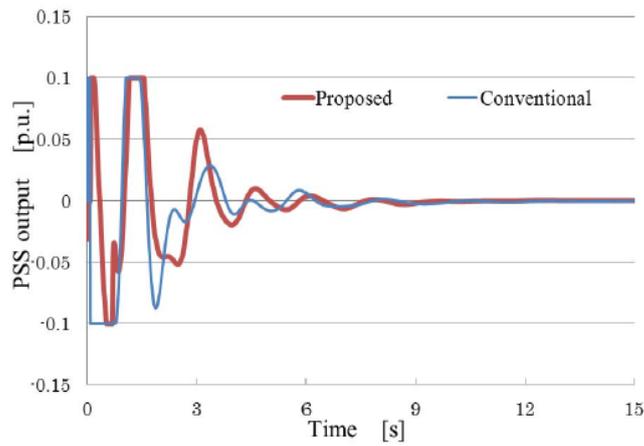
**Figure 9. Phase Angles at Nominal Loading (Generator 2)**



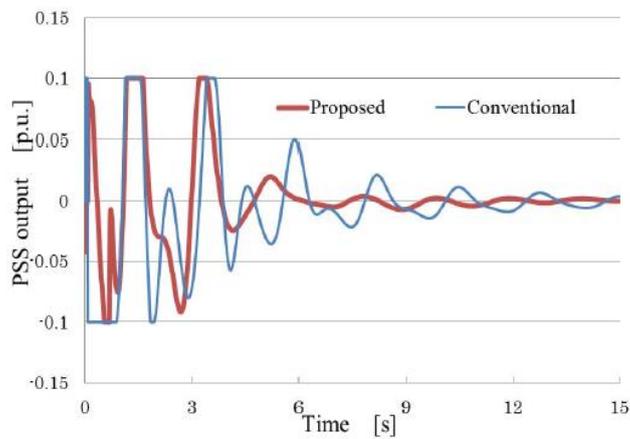
**Figure 10. Phase Angles When the Generator Outputs Increase By 30% (Generator 2)**



**Figure 11. Outputs of PSS When the Generator Outputs Decrease By 30% (Generator 2)**



**Figure 12. Outputs of PSS at Nominal Loading (Generator 2)**



**Figure 13. Phase Angles When the Generator Outputs Increase By 30% (Generator 2)**

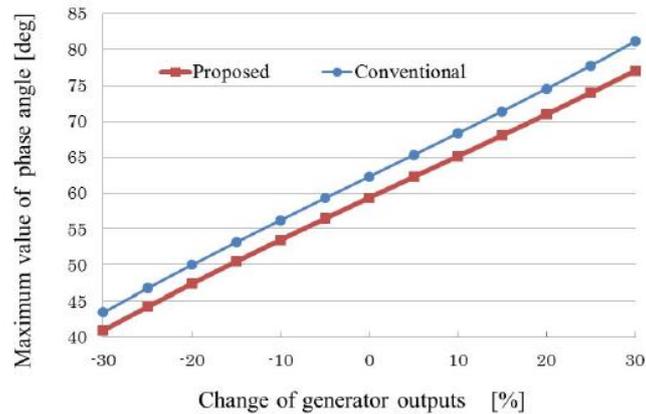


Figure 14. Maximum Values of Phase Angle at Each Operating Point

## 8. Conclusion

In this paper, we tend to planned a style methodology of a lead-lag PSS victimisation direct dimensional reduction. we tend to optimized the PSS parameters by a GA whose analysis operate considers the  $H_{\infty}$  norm and therefore the oscillation of the phase. we tend to meted out simulations for three-phase to ground faults at some in operation points to verify the hardness of the planned PSS. From the simulation results, we tend to verified the effectiveness of the planned PSS for the nominal model and below severe system conditions. We will succeed  $H_{\infty}$  performance and thereby solve the issues that come back from the quality and issue of a system with high dimensional controller.

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