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Triangular Fuzzy Multi-attribute Decision Making Based on Risk Attitude of Decision Maker

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Abstract

With respect to multi-attribute decision making problems in which the attribute values and weights of attributes are expressed with triangular fuzzy numbers, a new decision analysis method considering the risk attitude of the decision maker is proposed. In the method, firstly, risk attitude parameters of the decision maker are used to transfer the decision problem with triangular fuzzy numbers into the traditional decision problem with point values. Then, based on TOPSIS, a new decision making method is given. Finally, sensitivity analysis is made by selecting the different values of risk attitude parameter.

Keywords: *multi-attribute decision making; triangular fuzzy number; risk attitude factor*

1. Introduction

Multi-attribute decision making (MADM), also named multi-attribute decision making with limited schemes, deals with decision problems in consideration of multiple attributes to select the best alternative or alternatives. MADM has been widely applied in many fields, such as economic management and engineering. To MADM problems, if the decision information (generally refers to the attribute weights and decision matrix) is the exact numbers, then select or ordering scheme is easier [1]. But in many actual problems, due to inaccurate estimates and measurement error, uncertainty decision information often occurs [2], for example in the selection of air conditioning system, image acquisition reliability, and security attribute values have uncertain characteristics. At the same time, the attribute weight information and the preference judgment are often uncertain. In uncertain multiple attribute decision making problems, there are several ways can be used to describe the uncertain information, such as fuzzy number, interval number and triangle fuzzy number. Among them, MADM problems with triangular fuzzy numbers are studied extensively by domestic and foreign scholars [3-7]. The paper [3] studied MADM problem in which the attribute values expressed with triangular fuzzy number and the decision-maker had a preference for the alternatives and preference information was also triangular fuzzy numbers, putting forward the sorting method of alternative based on similarity. The paper [4] analyzed uncertain MADM problem whose elements of decision-making matrix were triangular fuzzy numbers when the preference information was given by the form of preference ordered pair. With regard to the attribute evaluation information of the alternative and attribute weight are multiple attribute group decision-making (MAGDM) problem formed by fuzzy language, the paper [6] converted language information into triangular fuzzy number, raising a fuzzy MAGDM algorithm through constructing combined consistency index which concentrates decision-maker's

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authority and consensus of opinion. It also analyzed and proved the feasibility and effectiveness of the whole algorithm through the examples of firms' credit assessment and sensitivity of combined consistency index. Regarding fuzzy of attribute information in the alternative assessment, the paper [6] presented an evaluation method for the entropy weight multi-attribute alternatives based on fuzzy information only when there is fuzzy judgment matrix but no experts' weights. Through the entropy of fuzzy evaluating matrix and the distance and approach degree of triangular fuzzy number from the assessed to an ideal point, the method made optimization selection evaluation of several reasonable projects and gets the optimization project with certain reliability. The paper [7] comes up with a decision-making method based on linear programming and fuzzy vector projection, in view of the multiple attribute decision-making problem with completely unknown attribute weight and whose attribute values are triangular fuzzy numbers. The method builds up a linear programming model based on weighted attribute values maximizing deviations, getting the attribute weight through solving the model, calculating the projection of weighted attribute values of projects on the fuzzy positive and negative ideal point, and then calculating the relative closeness degree, accordingly, sorting the projects. When the attribute values and weights of attributes are expressed with triangular fuzzy numbers, a new decision analysis method is put forward. Through inducing the risk attitude of decision maker, the triangular fuzzy number decision-making information is mapped into a point of decision information, which can convert the original problem into traditional MADM problems, and further by selecting different risk attitude factor, sensitivity analysis of the ranking results.

In the actual decision problems, decision makers because of their own conditions and different external environment will have different risk attitude factor. In general, different risk attitude factor decision makers will lead to different results of the decision making. So some scholars in the study of interval multiple attribute decision making considering the effects of risk attitudes of decision makers in decision making, and made some progress, cf. [8-10]. However, no one considers the decision maker's risk attitude into MADM problem with attributes values expressed with triangular fuzzy numbers. To the MADM problem, in which the attribute weights and attribute values are expressed with triangular fuzzy numbers, this paper will put forward a new MADM method considering the decision maker's risk attitude into decision-making process.

The rest of this paper is organized as follows. Section 2 briefly introduces some basic concepts of triangular fuzzy and risk attitude factor. Section 3 gives the calculation steps of the proposed decision method for the MADM problems with triangular fuzzy numbers information. Section 4 studies a practical example to show the applicability and feasibility of the proposed method. Finally, a conclusion is given in Section 6.

2. Basic Concepts

Some basic concepts on triangular fuzzy numbers and preliminary knowledge are introduced below to facilitate future discussions.

Definition 1 [8]. Let $\tilde{a}=[a^L,a^U]$ be an interval number, $n(\tilde{a})=(a^L+a^U)/2$ and $e(\tilde{a})=a^U-a^L$. The interval mapping function considering risk attitudes of decision makers is defined as follow:

$$\varphi_\varepsilon(\tilde{a})=n(\tilde{a})+\varepsilon e(\tilde{a}) \tag{1}$$

That is

$$\varphi_\varepsilon(\tilde{a})=(0.5-\varepsilon)a^L+(0.5+\varepsilon)a^U . \tag{2}$$

Where ε $(|\varepsilon|\leq 0.5)$ is called the risk attitude factor. It reflects the decision makers' risk attitude (or degree) about the interval attribute value $\tilde{a}$.

Specifically, according to the decision maker's attitudes towards risk, we can divide it into three cases: pessimistic, neutral and optimistic, ε corresponding ranges are $-0.5 \le \varepsilon < 0$, $\varepsilon = 0$ and $0 < \varepsilon \le 0.5$ .Usually, risk attitude factor ε is given by decision makers (or negotiation and decision analysts), or in a decision analysis to take sensitivity analysis method about ε . The mapping function is given by Equation (2), which can be used to transform the interval MADM problem into the traditional MADM problem with value at risk attitude factor ε . The calculation steps of interval MADM problem are given below:

Definition 2[11]. Let $\tilde{a}$ and $\tilde{b}$ be two arbitrary fuzzy numbers, then for arbitrary $\alpha \in [0,1]$, $\tilde{a}$ and $\tilde{b}$'s α cut set are respectively defined as follows:

$$\tilde{a}_\alpha = [a^L(\alpha), a^U(\alpha)]$$

And

$$\tilde{b}_\alpha = [b^L(\alpha), b^U(\alpha)],$$

Further. The relationship of $\tilde{a}$ and $\tilde{b}$ is defined as follows:
$\tilde{a} \le \tilde{b}$, if and only if $a^L(\alpha) \le b^L(\alpha)$ and $a^U(\alpha) \le b^U(\alpha)$, for arbitrary $\alpha \in [0,1]$.

Defination 3[11]. If $\tilde{A} = (a,b,c)$, $0 \le a \le b \le c$, then $\tilde{A}$ named a triangular fuzzy number. The membership function of $\tilde{A}$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases} \tag{3}$$

Lemma1 [11]. Let $\tilde{a} = (a^L, a^M, a^U)$ be any triangular fuzzy number, then to any $\alpha \in [0,1]$, α cut sets of $\tilde{a}$ is given by the following form

$$\begin{aligned} \tilde{a}_\alpha &= [a^L(\alpha), a^U(\alpha)] \\ &= [a^L + (a^M - a^L)\alpha, a^U - (a^U - a^M)\alpha] \end{aligned} \tag{4}$$

By definition 2, the relationship of size of two any fuzzy numbers $\tilde{a}$ and $\tilde{b}$ can be measured by α cut sets, and α cut sets of $\tilde{a}$ and $\tilde{b}$ are interval numbers, which $\alpha \in [0,1]$ is an arbitrary real numbers. To eliminate the arbitrariness of α and make the comparison of triangular fuzzy numbers considering the risk attitude of decision-makers, in the following discussion, a new sort function of triangular fuzzy number is proposed.

Definition 4 Let $\tilde{a} = (a^L, a^M, a^U)$ be a triangular fuzzy number, and for arbitrary positive number $\varepsilon \in [-0.5, 0.5]$, we define a function $\varphi_\varepsilon(\tilde{a})$ as

$$\varphi_\varepsilon(\tilde{a}) = \int_0^1 (0.5-\varepsilon)a^L(\alpha) + (0.5+\varepsilon)a^U(\alpha)d\alpha \tag{5}$$

Which can be derived as follows:

$$\begin{aligned}\varphi_\varepsilon(\tilde{a}) &= \int_0^1 (0.5-\varepsilon)a^L(\alpha) + (0.5+\varepsilon)a^U(\alpha)d\alpha \\ &= \int_0^1 (0.5-\varepsilon)(a^L + (a^M - a^L)\alpha) \\ &\quad + (0.5+\varepsilon)(a^U - (a^U - a^M)\alpha)d\alpha \\ &= \frac{1}{4}(a^L + 2a^M + a^U) + \varepsilon \cdot \frac{a^U - a^L}{2}\end{aligned} \tag{6}$$

3. MADM Method Based On Risk Attitude of Decision Maker

Based on the above discussion, we give the new fuzzy MADM method based on risk attitude of decision maker, and the specific steps are given as follows:

Step1. Suppose there exist n parallel possible alternatives $x_1, x_2, \cdots, x_m$ and m attributes $o_1, o_2, \cdots, o_n$, the task of the decision maker is to select the best alternative. Suppose that the rating of alternative x_i $(i = 1, 2, \cdots, m)$ on attribute o_j $(j = 1, 2, \cdots, n)$ given by the decision maker is a triangular fuzzy number $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$. Then a MADM problem can be concisely expressed with the following decision matrix:

$$A = (\tilde{a}_{ij})_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} o_1 & o_2 & \cdots & o_n \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix} \end{array}$$

In general, attributes can be classified into two types: benefit attributes and cost attributes. In other words, the attribute index set I can be divided into two subsets: I_1 and I_2, where $I_k (k = 1, 2)$ is the subset of benefit attributes and cost attributes, respectively. That is to say, the larger the value of an alternative on the benefit attribute the better the alternative, while the smaller the value of an alternative on the cost attribute the better the alternative. Furthermore, $I = I_1 \cup I_2$ and $I_1 \cap I_2 = \varnothing$, where $\varnothing$ is empty set.

To eliminate the impact of different physical dimension on decision-making result, using the standardized approach from Xu [3] to deal with , standardized matrix $R = (\tilde{r}_{ij})_{m\times n}$ can be obtained, where $\tilde{r}_{ij} = (r_{ij}^L, r_{ij}^M, r_{ij}^U)$ are calculated by the following equations:

$$\begin{cases} r_{ij}^L = a_{ij}^L \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^U)^2} \\ r_{ij}^M = a_{ij}^M \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^M)^2} \\ r_{ij}^U = a_{ij}^U \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^L)^2} \\ i \in M, j \in I_1 \end{cases} \tag{7}$$

And

$$\begin{cases} r_{ij}^{L} = (1/a_{ij}^{U}) \Big/ \sqrt{\sum_{i=1}^{m}(1/a_{ij}^{L})^2} \\ r_{ij}^{M} = (1/a_{ij}^{M}) \Big/ \sqrt{\sum_{i=1}^{m}(1/a_{ij}^{M})^2} \\ r_{ij}^{U} = (1/a_{ij}^{U}) \Big/ \sqrt{\sum_{i=1}^{m}(1/a_{ij}^{U})^2} \\ i \in M, j \in I_2 \end{cases} \tag{8}$$

Where $M = \{1,2,...,m\}$.

Step2. As described above, fuzzy decision-making matrix $A = (\tilde{a}_{ij})_{m\times n}$ can be transformed standardized decision-making matrix $R = (\tilde{r}_{ij})_{m\times n}$ based on Equation (7) and Equation (8);

Step3. According to definition 3, we can change the triangular number fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$ into the decision matrix $\Phi(\varepsilon)$, which considers the risk attitude of decision makers and given as follow:

$$\Phi(\varepsilon) = \begin{bmatrix} \varphi_{11}(\varepsilon) & \varphi_{12}(\varepsilon) & \cdots & \varphi_{1n}(\varepsilon) \\ \varphi_{21}(\varepsilon) & \varphi_{22}(\varepsilon) & \cdots & \varphi_{2n}(\varepsilon) \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{m1}(\varepsilon) & \varphi_{m2}(\varepsilon) & \cdots & \varphi_{mn}(\varepsilon) \end{bmatrix}$$

Where

$$\varphi_{ij}(\varepsilon) = \varphi_{\varepsilon}(\tilde{r}_{ij}) = \frac{1}{4}(r_{ij}^{L} + 2r_{ij}^{M} + r_{ij}^{U}) + \varepsilon \cdot \frac{r_{ij}^{U} - r_{ij}^{L}}{2} \tag{9}$$

And $i = 1,2,\cdots,m;\ j = 1,2,....,n$ **.**

Step4. For a given risk attitude factor ε , the attribute weights are determined by using the entropy weight method, as follows

(1) Calculate

$$p_{ij}(\varepsilon) = \frac{\varphi_{ij}(\varepsilon)}{\sum_{i=1}^{m}\varphi_{ij}(\varepsilon)} (i = 1,2,...,m;\ j = 1,2,...,n) \tag{10}$$

(2) Calculate the entropy with data for each criterion, and the entropy of the set of normalized outcomes of the *j*th criterion is given as

$$e_j = -k\sum_{i=1}^{m}[p_{ij}(\varepsilon)\ln p_{ij}(\varepsilon)], \quad j = 1,2,...,n \tag{11}$$

(3) Define the entropy weight of *j*th attribute as

$$w_j = \frac{1 - e_j}{n - \sum_{j=1}^{n} e_j}, \quad j = 1,2,...,n \tag{12}$$

Step5. According to Equation (9) and Equation (12), the comprehensive evaluation value of each alternatives is calculated as

$$z_i(\varepsilon)=\sum_{j=1}^{n}[\varphi_{ij}(\varepsilon)w_i], i\in M \tag{13}$$

Step6. With different risk attitude factors, we can sort all the alternatives according to the comprehensive evaluation values $z_i(\varepsilon)$ $(i=1,2,...,m)$. The larger of $z_i(\varepsilon)$ is, the more superior of the corresponding alternative is.

4. Empirical Studies

In the following discussion, we will apply the proposed method to solve the virtual enterprises partner selection problem.

Virtual enterprise is a dynamic alliance structure which is consisted of several enterprises of different regions with different nature of the work. For a specific production tasks, business leader which owns main production resources (including human, material, equipment or technical conditions) will take this task, and it looks for partners in the globally, and then they cooperate with each other to ensure fast, efficient, low cost to complete the production task [8]. Virtual enterprise is a new form of enterprise organization, is the main form of twenty-first century enterprise production management and market competition. Partner selection is a very important and complicated process, whether to select the initial agile, competitive and compatible partners, related to the success of virtual enterprise [9]. There are eight evaluate indexes (attributes) in the virtual enterprise partner selection problem [10]. They are force (o_1), complementary (o_2), dealing (o_3), win (o_4), Focus (o_5), integration (o_6), growth (o_7) and consistency (o_8).

A virtual enterprise wants to choose a best partner from a total of four potential partners (alternatives) x_1,x_2,x_3,x_4. The experts mark for the four potential partners on the basis of the above eight indexes (attribute), and the values are expressed in the form of triangular fuzzy numbers. The details are shown in Table 1.

Table 1. Attribute Values of Each Alternative

	x_1	x_2	x_3	x_4
o_1	(0.80, 0.85, 0.90)	(0.72, 0.76, 0.80)	(0.91, 0.93, 0.96)	(0.62, 0.65, 0.68)
o_2	(0.88, 0.90, 0.93)	(0.67, 0.77, 0.83)	(0.60, 0.67, 0.70)	(0.69, 0.72, 0.75)
o_3	(0.95, 0.97, 0.98)	(0.90, 0.93, 0.95)	(0.77, 0.79, 0.82)	(0.93, 0.95, 0.96)
o_4	(0.82, 0.85, 0.88)	(0.94, 0.98, 1.00)	(0.98, 0.99, 1.00)	(0.97, 0.99, 1.00)
o_5	(0.78, 0.79, 0.81)	(0.78, 0.79, 0.81)	(0.83, 0.85, 0.88)	(0.94, 0.97, 0.99)
o_6	(0.65, 0.69, 0.72)	(0.83, 0.85, 0.88)	(0.94, 0.96, 0.99)	(0.86, 0.88, 0.90)
o_7	(0.84, 0.86, 0.89)	(0.85, 0.87, 0.90)	(0.77, 0.78, 0.80)	(0.81, 0.83, 0.88)
o_8	(0.95, 0.96, 0.98)	(0.70, 0.77, 0.78)	(0.83, 0.85, 0.89)	(0.91, 0.96, 0.98)

To sort the four alternatives using this paper's method, the calculation steps are given as follows:

Step1. Transform decision making matrix $A=(\tilde{a}_{ij})_{m\times n}$ into standardized decision-making matrix based on Equation (7) and Equation (8):

$$\tilde{R}^T = \begin{bmatrix} (0.475,0.528,0.585) & (0.428,0.473,0.520) & (0.541,0.578,0.624) & (0.368,0.404,0.442) \\ (0.545,0.585,0.648) & (0.415,0.500,0.578) & (0.372,0.435,0.488) & (0.427,0.468,0.523) \\ (0.511,0.531,0.550) & (0.484,0.510,0.534) & (0.414,0.433,0.461) & (0.500,0.520,0.539) \\ (0.422,0.445,0.469) & (0.499,0.513,0.533) & (0.504,0.519,0.533) & (0.499,0.519,0.533) \\ (0.445,0.463,0.485) & (0.445,0.463,0.485) & (0.474,0.498,0.527) & (0.537,0.568,0.593) \\ (0.370,0.406,0.435) & (0.473,0.500,0.532) & (0.535,0.564,0.599) & (0.490,0.517,0.544) \\ (0.484,0.515,0.544) & (0.489,0.521,0.550) & (0.443,0.467,0.489) & (0.466,0.497,0.538) \\ (0.521,0.540,0.575) & (0.384,0.433,0.457) & (0.455,0.478,0.522) & (0.499,0.540,0.575) \end{bmatrix}$$

Step2. For different *g*iven risk attitude factor ε , the comprehensive evaluation vector, and the ranking results in Table 2.

Table 2. Ranking Results with Different Risk Attitude Factor

ε	comprehensive evaluation vector	Ranking Result
-0.5	(0.4721,0.4388,0.4714,0.4542)	$x_1 > x_3 > x_4 > x_2$
-0.3	(0.4859,0.4567,0.4883,0.4683)	$x_3 > x_1 > x_4 > x_2$
-0.1	(0.4999,0.4741,0.5046,0.4821)	$x_3 > x_1 > x_4 > x_2$
0	(0.5070,0.4826,0.5126,0.4889)	$x_3 > x_1 > x_4 > x_2$
0.1	(0.5142,0.4910,0.5204,0.4957)	$x_3 > x_1 > x_4 > x_2$
0.3	(0.5287,0.5077,0.5357,0.5091)	$x_3 > x_1 > x_4 > x_2$
0.5	(0.5435,0.5242,0.5507,0.5225)	$x_3 > x_1 > x_2 > x_4$

We can see from Table 2, when the decision maker's risk attitude factor increased from -0.5 to 0.5, comprehensive evaluation value of each alternative is changed, thus the ranking results are also different. Such as, the best potential partners is x_1 for $\varepsilon = -0.5$; but, the best potential partner is x_3 , for $\varepsilon = -0.3,...,0.5$. We can know from the above example, the risk attitude of decision maker will have an important influence on the decision. When the decision maker is in different risk attitudes, he/she can adjust the risk attitude factor to make decisions. The method proposed in this paper takes into account the different decision maker's risk attitude factor, which is more in line with reality, has the feasibility and rationality.

5. Conclusion

By introducing decision-maker's risk attitude, this paper proposed a new fuzzy MADM method triangular fuzzy numbers MADM problem, taking full consideration of the impact of decision-makers' different risk attitude, which is more realistic. The method also can be extended into MADM problem with the attribute values are trapezoidal fuzzy numbers, dynamic fuzzy or linguistic variables.

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