

# Magnetotactic Bacterium Multi-objective Optimization Algorithm

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## **Abstract**

*In this paper, based on Magnetotactic Bacteria Optimization Algorithm(MBOA), magnetotactic bacterium multi-objective optimization algorithm (MBMOA) is proposed for solving multi-objective optimization problems(MOPs). Magnetotactic bacterium optimization algorithm is a novel random search algorithm which simulates the process of magnetotactic bacteria (MTB) producing magnetosomes(MTS) to regulate cell moment and makes the magnetostatic energy reach the minimum .The algorithm MBOA proposed three operators named by MTS producing, MTS amplification and MTS replacement by imitating the development process of magnetosomes, the adjustment process of magnetosomes moment and the replacement process of magnetosome with worse moment. In MBMOA, MBOA is applied to produce the next population, while non-dominated feasible solutions gained by MBOA are conserved in the archive, then the evaluation method of SPEA2 is adopted to update the archive, at the last through benchmark functions test and classic algorithm comparison, the simulation results show that the MBMOA is feasible and effective for solving multi-objective optimization problems.*

**Keywords:** *Magnetotactic bacteria optimization algorithm, magnetosomes regulation, multi-objective optimization*

## **1. Introduction**

Many real-world problems require the simultaneous optimization of several conflicting objectives which lead their optimal solutions to become a set of Pareto-optimal solutions[1]. Many kinds of evolutionary algorithms (EAs) were proved successful for multi-objective optimization problems(MOPs) because they can attain multiple Pareto-optimal solutions in a run and were not restricted to the features of research region, such as genetic algorithm (GA), which includes non-dominated sorting genetic algorithm NSGA[2]and NSGAI[3], Multi-Objective genetic algorithm (MOGA)[4], Strength Pareto evolutionary algorithm SPEA[5] and SPEA2[6] and the Pareto archived evolution strategy (PAES)[7]. Besides that, swarm intelligence had also been applied to solve MOPs, for example CMOMOPSO[8], CLPSO[9], PAMOPSO[10] and FMOPSO[11] of particle swarm optimization (PSO) algorithms. In those algorithms, different schemes were presented to choose the global best positions for each particle of the swarm from Pareto optimal solutions set. Because of different optimization strategies inspired by biology systems, they exhibited a varying degree of success for MOPs.

In this paper, Magnetotactic bacterium optimization algorithm(MBOA) is a new random search algorithm. In the single objective optimization problems and real applications, MBOA presents certain advantages especially in the convergence [12]-[13]. However it has no reports on MBOA for solving multi-objective optimization problems, In MBOA, the operator of producing MTS has good exploitation ability in the population information, MTS amplification has good exploration ability and MTS replacement may enhance the diversity of the population. Based on those characters, a novel multi-objective optimization algorithm named by MBMOA is proposed.

The rest of the paper is organized as follows. Section 2 introduces the basic idea of Magnetotactic bacteria optimization algorithm (MBOA) for single objective

optimization. Section 3 describes the process of the proposed algorithm MBMOA. Section 4 shows the simulation results and analyzes the algorithm MBMOA. Finally, conclusions are drawn in Section 5.

## 2. Magnetotactic Bacteria Optimization Algorithm (MBOA)

Magnetotactic bacteria (MTB) [14] are a kind of bacteria which can swim by certain direction under the effect of external magnetic field, and it can produce magnetic particles named by magnetosomes (MTS) inside bacteria. These MTS have strong sensitivity to magnetic field, hence they can adjust the movement of the magnetotactic bacteria, while their size, shape and quantity have important effect on the interaction energy and efficiency between magnetotactic bacteria and the magnetic field. In fact, some magnetotactic bacteria have been able to adapt to the magnetic field to exist. However, there are some magnetotactic bacteria containing magnetosomes, as they make the magnetic lines curved nearby magnetosome, in order to survive, must adjust the magnetic moment to reduce the magnetostatic energy. Each magnetosome of magnetic bacteria can generate magnetic moment; the total magnetic moment is the sum of moment produced by all the magnetosomes. By imitating the process of magnetostatic energy minimization, Mo[12]-[13]proposed magnetotactic bacterium optimization algorithm (MBOA). By imitating magnetotactic bacteria producing moment and magnetosomes, In MBOA, the minimum magnetostatic energy state corresponds to solutions of optimization problems, a cell corresponding to the feasible solution, magnetosomes moment corresponding to decision variable of each cell. The corresponding relation MTB and MBOA is shown in table 1.

**Table 1. The Corresponding Relation between MTB and MBOA**

MBOA	MTB
feasible solution	cell
decision variable	magnetosome moment
optimal solution	the state of minim magnetostatic energy

The process that MBOA solving the optimization problems corresponds to the process that MTB producing magnetosomes adapting to the earth's magnetic field. MBOA mainly get good candidate solutions (with smaller magnetostatic energy) by regulating the moment of each magnetosomes. Firstly, the distance between any two cells is computed to achieve the interaction energy between them, and then the moment of each cell is gained. Magnetosomes in MTB is produced based on the interaction energy between cells. By imitating the process , the MTS producing operator of MBOA is designed; After magnetosomes formation, they will adjust their volume size to obtain a better moment, the process is MTS amplification operator of MBOA; Lastly, worse magnetosomes will be replaced which corresponds to MTS replacement of MBOA. MBOA mainly adopts MTS producing, MTS amplification and MTS replacement to generate offspring population, the detail process of MBOA is described as follows.

### 2.1. MTS Producing

Inside the Magnetotactic bacteria, there are some magnetosomes that they make the cell interaction to adapt to the geomagnetic field. Any two cells  $X_i = (x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{iL})$  and  $X_r = (x_{r1}, x_{r2}, \dots, x_{rj}, \dots, x_{rL})$  are randomly selected from the population, and  $r$  are any integer in  $\{1, \dots, N\}$ ,  $L$  is the dimension of decision variables. Let  $D_i = (X_i - X_r) = (d_{i1}, d_{i2}, \dots, d_{ij}, \dots, d_{iL})$ , where  $d_{ij} = x_{ij} - x_{rj}$ . The distance of the two cells is defined as

$$D(X_i, X_r) = \|D_i\|_2 \quad (1)$$

Where  $\|D_i\|_2 = \sqrt{\sum_{k=1}^L d_{ik}^2}$  is the Euclidean distance of  $X_i, X_r$ . Let their interaction energy is  $E_i = (e_{i1}, e_{i2}, \dots, e_{iL})$ , where  $e_{ij}(t)$  denotes

$$e_{ij}(t) = \left( \frac{d_{ij}(t)}{1 + c_1 * \|D_i\|_2 + c_2 * d_{pq}(t)} \right)^3 \quad (2)$$

$p$  is any integer in  $\{1, \dots, N\}$ ,  $q$  is a random integer from  $\{1, 2, \dots, L\}$ ,  $c_1, c_2$  is two constants,  $d_{pq} = x_{pq} - x_{rq}$ . Because the magnetotactic bacteria exist in the magnetic field, each cell has certain energy to adapt to the field, the energy is determined by the magnetic field and the total moment. Supposed  $E_i$  is the energy of the  $i$ th cell, Ignoring the direction between the magnetic field and the moment, which assumes that the angle between them  $\cos \theta = 1$ , then total moment  $M_i$  is

$$M_i(t) = \frac{E_i(t)}{B} \quad (3)$$

where  $B$  is the magnetic field strength, let  $M_i = (m_{i1}, m_{i2}, \dots, m_{iL})$ , then MTS moment adjustment can be defined as

$$v_{ij}(t) = x_{ij}(t) + m_{ij}(t) * rand \quad (4)$$

where  $rand$  is a random number between 0 and 1. The moment adjustment is based on the distance, so when the cell is far from the excellent solution, the scale of moment adjustment is bigger so that these solutions approximate the optimal solutions; when the distance between cells and the excellent solution distance is smaller, the scale of moment adjustment is smaller so that the information of excellent solutions is destroyed with small probability, hence the convergence of the population is ensured.

## 2.2. MTS Amplification

After the MTS generation, they will be amplified to adopt to the magnetic field by the following way. Let magnetic field intensity probability  $mp$ , if  $rand > mp$ , MTS will be amplified by the following way

$$X_i(t+1) = V_{best}(t) + rand(1, L) * (V_{best}(t) - V_i(t)) \quad (5)$$

Otherwise, the expansion way is

$$X_i(t+1) = V_i(t) + rand(1, L) * (V_{best}(t) - V_i(t)) \quad (6)$$

Where  $rand(1, L)$  is a randomly dimension vector, each dimension values are in  $(0, 1)$ . By using the amplification mode, when the magnetic field strength probability is smaller, the new individual will be produced near the optimal solutions; when the magnetic field strength probability is larger, the new individual will be produced in the vicinity of the original individual, the amplification have better exploration ability.

## 2.3. MTS Replacement

After the MTS extension, some MTS with poor performance will be replaced. Based on the fitness ranking of individual  $X_i(t+1)$ , the second half of individuals with smaller fitness is replaced as

$$X_i(t+1) = m_{pq}(t) * ((rand(1, L) - 1) * rand(1, L)) \quad (7)$$

The substitution mechanism can preserve the better individual, while the worse individuals have higher chance to transform the better ones.

#### 2.4. The Process of MBOA

MBOA is mainly based on the above MTS producing, MTS amplification and MTS replacement to produce the offspring population, the detail algorithm process is as follows.

##### The process of MBOA algorithm

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Step1. Parameters initialization.

Step2. Randomly generate initial population

Step3. While stop criteria is not met

According to (1) - (3) to calculate the distance between individuals, the energy and moment

Implement MTS producing according to (4),

Evaluate the fitness and implement MTS amplification according to (5) and (6)

Evaluate the fitness and implement MTS replacement according to (7), two optimal solutions are saved and are updated

Step4. End while

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### 3. Magnetotactic Bacterium Multi-objective Optimization Algorithm (MBMOA)

MBMOA mainly adopts the three operator of MBOA to generate the offspring population, and the archive is applied to conserve the non-dominated feasible solutions gained by MBOA, when the size of the archive exceed the fixed size, the evaluation method of individual in SPEA2 is used to delete the individuals with small density value. Based on the merits of MBOA, MBMOA can achieve the Pareto optimal solutions set with better convergence and uniform distribution. The process of MBMOA is described as follows:

Step1: The parameters setting, the size of the population P is N, the size of the archive A is M, the magnetic field strength B.

Step2: Initialization population  $P_0 = (p_1, p_2, \dots, p_N)$ , individual  $p_i$  adopts real coding and is denoted as  $p_i = (p_{i,1}, p_{i,2}, \dots, p_{i,d})$ , where  $p_{i,j} = l_j + rand \times (u_j - l_j)$  is the  $j$ th variable of the individual  $p_i$ ,  $l_j, u_j$  is the upper and lower bounds,  $d$  is the dimension of decision variable,  $rand$  is a random number in  $(0,1)$ . The archive  $A_0 = \phi$ , iterative times  $t = 0$ .

Step3: Rank the individuals in  $P_t$  and  $A_t$  by non-dominated relation, the non-dominated solutions are conserved in the archive  $A_{t+1}$ , if the size of  $A_{t+1}$  is larger than M, compute the nearest neighbor distance of individual in  $P_t \cup A_t$ , adopt the method of SPEA2 to delete the crowding individuals until the size of  $A_{t+1}$  is M.

Step4: if  $t > g_{max}$ , output  $A_{t+1}$  and stop, otherwise go to Step5.

Step5: Implement MTS producing, MTS amplification and MTS replacement of MBOA on  $A_{t+1}$  to produce the offspring population  $P_{t+1}$ .

Step6:  $t = t + 1$ , go to Step3.

### 4. The Simulation Experiment

#### 4.1. Benchmark Function Test

To evaluate the performance of the algorithm MBMOA, benchmark problems ZDT1, ZDT3, ZDT4 and ZDT6 are selected to validate the effectiveness of MBMOA for MOPs. ZDT1 has a convex Pareto front and converges easily. ZDT3 has a non-continuous Pareto front. Both have 30 decision variables. ZDT4 has a highly multi-modal Pareto front and a total of  $21^9$  local Pareto front. ZDT6 has solutions which are non-uniformly distributed. Both of them have 10 decision variables. Those problems are commonly used to test performance of multi-objective optimization algorithms. These test problems can effectively test if multi-objective optimization algorithm can approximate the true Pareto front and maintain good diversity and distribution. In MBMOA, real coding is used. The parameters are set follows: the population size 100, the archive size 100, and the maximum generations 100.

Firstly, to demonstrate the performance of MBMOA, ZDT1, ZDT3, ZDT4 and ZDT6 are test. The Pareto fronts gained by MBMOA are shown in Fig1, where real line denotes the true Pareto fronts, ‘\*’ denotes the optimal Pareto fronts obtained by the algorithm MBMOA. It can be seen that, for different types of test function, all the Pareto fronts gained by MBMOA can approximate the true optimal Pareto front and have good diversity and uniform distribution. Especially for ZDT4 with many local optimal solutions, MBMOA can approximate the true Pareto front which shows MBMOA has better convergence. Given all that, MBMOA is effective for solving MOPs.

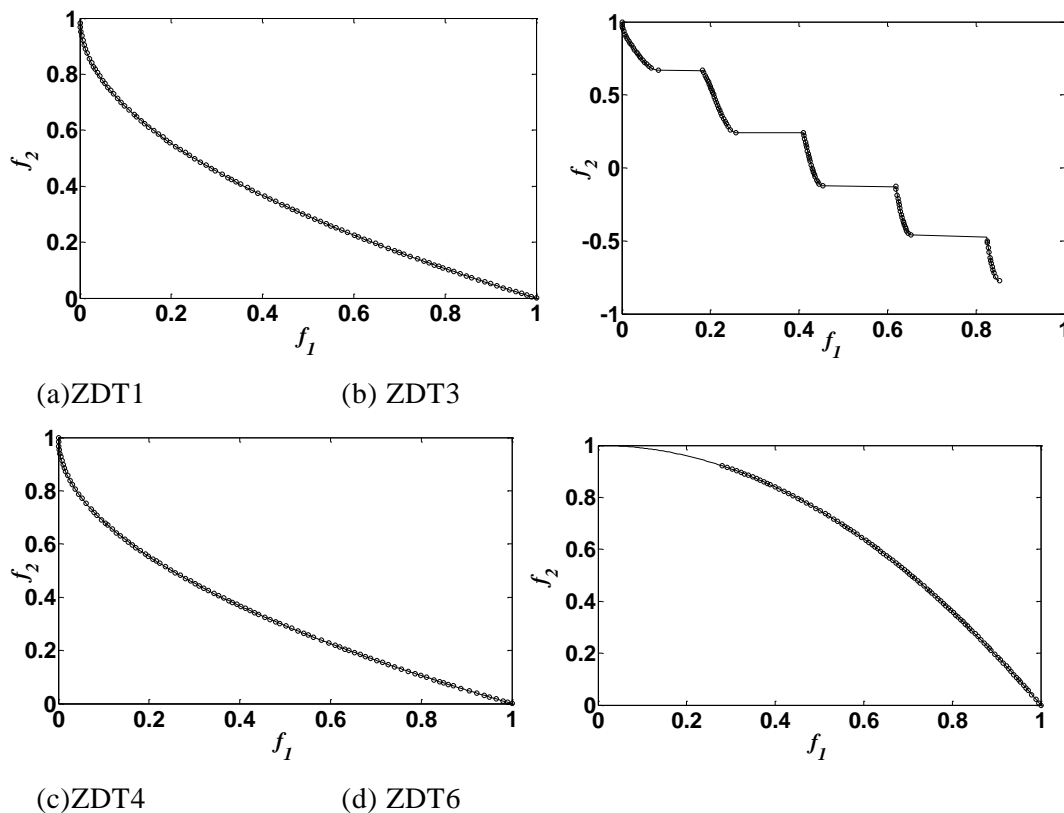


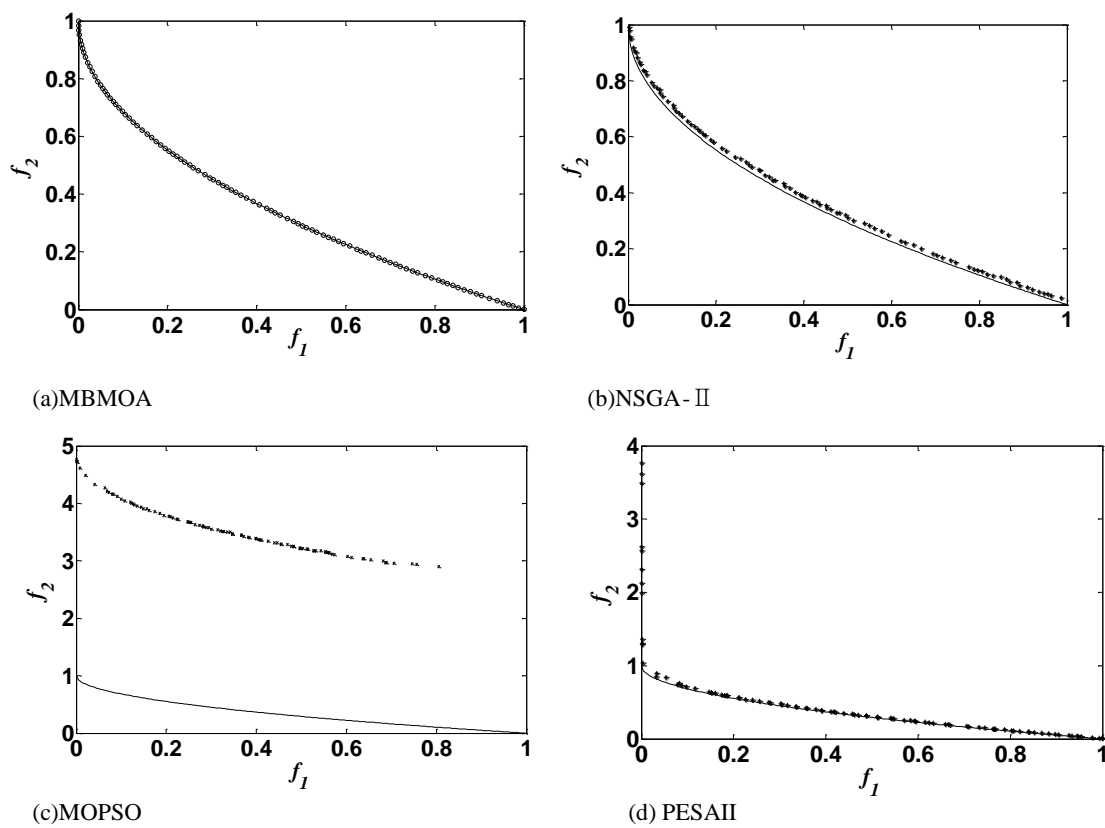
Figure1. Pareto Front of ZDT1, ZDT3, ZDT4, ZDT6 gained by MBMOA

#### 4.2. Comparison with Classic Algorithms

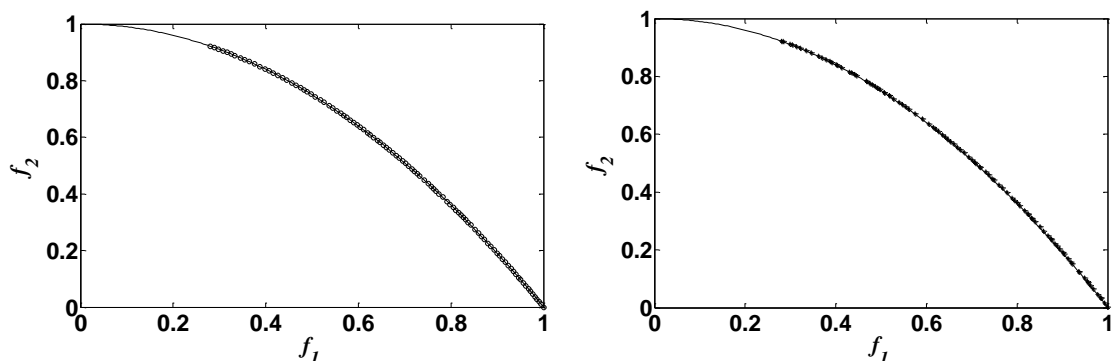
To prove the superiority of the proposed MBMOA, it is compared with the multi-objective optimization algorithm NSGAI [3], MOPSO [8] and PESAI [15]. The parameters are set the same, the maximum generation times 200. The optimal Pareto fronts gained by four algorithms are shown in Fig2-3. In Figs, real line denotes the true optimal Pareto fronts ‘\*’ denote the Pareto obtained by MBMOA, NSGAI, MOPSO and PESAI. The function

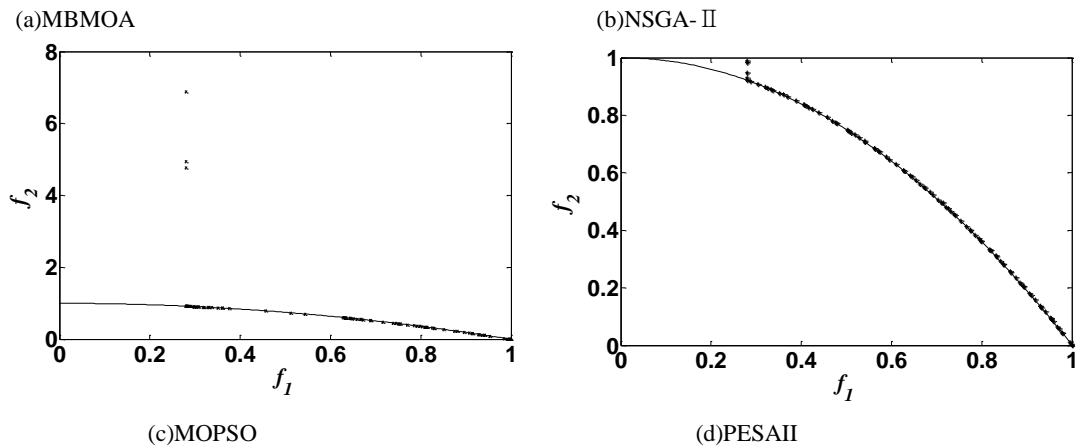
ZDT4 with  $21^9$  local Pareto fronts and ZDT6 are selected to validate the performance of MBMOA.

For ZDT4, MBMOA can escape all local Pareto fronts and approximate the true Pareto front, which indicates the algorithm MBMOA has good convergence. On the Pareto front gained by NSGAI, some individuals are more concentrated so that there are no individuals on some parts; hence the distribution of the solutions is worse, while NSGAI can not approximate the true Pareto front. The Pareto gained by MOPSO and PESAI fall into local optimal front and can not approximate the true Pareto front. For ZDT6, MBMOA can converge to the true Pareto front with uniform distribution. NSGAI also approximate the true Pareto front but its distribution is worse. For MOPSO and PESAI, there are some solutions far from Pareto front which show that they fall into local optimal solutions. Based on these experiment results, MBMOA is better than the other three algorithms in the aspect of the convergence, the diversity and the distribution of the population for ZDT4 and ZDT6. These results also show that the multi-objective optimization algorithm MBMOA can approximate the true Pareto front with good diversity and uniform distribution.



**Figure 2. Pareto Front of ZDT4 Gained by MBMOA, NSGA-II, MOPSO and PESAI**





**Figure 3. Pareto Front of ZDT4 Gained by MBMOA, NSGA-II, MOPSO and PESAI**

## 5. Conclusions

Based on the excellent performance of MBOA for Simple-objective optimization problem, the paper proposes a new multi-objective optimization algorithm MBMOA. The algorithm MBMOA adopts the real coding and the same fitness evaluation method as SPEA2. After evaluation, the operators of MBOA are applied to generate the next generation population. During the evolution, the non-dominated individuals obtained are conserved in the archive. When the size of the archive exceeds the refined size, archive truncation mechanism of SPEA2 is used to update the archive. Benchmark test results and comparison with classic algorithms show that the proposed algorithm MBMOA is feasible and effective for MOPs.

## Acknowledgements

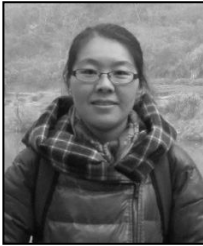
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