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Portfolio Selection Problem Based on Loss Aversion under Uncertain Environment

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Abstract

In investment process, investors usually have the characteristic of loss aversion. This paper discusses the portfolio selection problem with loss aversion under uncertain environment. Return rates are described as uncertain variables; uncertain measure is used to measure the uncertainty. The optimal model of maximizing expected utility based on loss aversion is established. When return rates are special uncertain variables, the model can be transformed to the crisp one, for generic uncertain return rates, hybrid intelligence algorithm integrating genetic algorithm and 99-method is designed to solve the model. Finally, numerical example is given to illustrate feasibility and validity of this method.

Keywords: *Portfolio selection, optimal model, Crisp equivalent model, Hybrid intelligence algorithm, uncertain variable*

1. Introduction

The core problem of portfolio selection is return and risk. In 1952, Markowitz initialized the mean-variance model of portfolio selection [1]; the variance of investment return was used to measure the risk in the model, which opened up the era of financial quantitative analysis. Since then, many scholars devoted themselves to the research of portfolio selection and proposed many practical models.

But the classical research of optimal model did often not consider the investors' psychology in the actual decision-making process, Kahneman and Tversky researched human action from the angle of cognitive psychology, proposed the famous Prospect Theory in 1979[2]. This theory considered that the investment actual decision-making process was affected by emotion, cognition and so on factors. They initialized the S-utility function which can replace the traditional concave utility functions [3]. S-utility function emphasized that the investors had a subjective reference value, when the price exceeded the reference value, the investors were risk avoidance, and when the price was inferior to reference value, the investors were risk-seeking. And the reaction of investors was different when they face the same gain and loss; the pain produced by loss is far greater than the happiness brined by gain that was loss aversion feature. In this paper, we used this theory to build the optimal model of portfolio selection based on loss aversion under uncertain environment, for some special uncertain variables, the model can be changed to crisp one, for generic uncertain variables, hybrid intelligence algorithm integrating genetic algorithm and 99-method is designed to solve the model.

2. Preliminaries

Uncertainty theory was founded by Liu [4] in 2007. Nowadays uncertainty theory has become a branch of mathematics based on normality, monotonicity, self-duality, countable sub additively, and product measure axioms. An important concept in uncertainty theory is uncertain measure that is used to measure the belief degree of an

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uncertain event. In order to better understand this paper, we first introduce some concepts about uncertainty theory.

Definition 1. Let Γ be a nonempty set, τ a $\sigma-algebra$ over Γ, each element Λ in the $\sigma-algebra$ τ is called an event. Uncertain measure $M\{\Lambda\}$ is a function from τ to [0, 1]. $M\{\Lambda\}$ indicates the belief degree that Λ will occur. Liu [4] proposed the following five axioms:

Axiom 1 (Normality Axiom) $M\{\Gamma\}=1$ for the universal set Γ.

Axiom 2 (Monotonicity Axiom) $M\{\Lambda_1\}\le M\{\Lambda_2\}$ whenever $\Lambda_1 \le \Lambda_2$.

Axiom 3 (Self-Duality Axiom) $M\{\Lambda\}+M\{\Lambda^c\}=1$ for any event Λ.

Axiom 4 (Countable Subadditivity Axiom) for every countable sequence of events $\{\Lambda_i\}$, we have $M\{\bigcup_{i=1}^{\infty}\Lambda_i\}\le\sum_{i=1}^{\infty}M\{\Lambda_i\}$

Axiom 5 (Product Measure Axiom) Let Γ_k be nonempty sets on which M_k are uncertain measure, $k=1,2,\cdots,n$, respectively. Then the product uncertain measure M is an uncertain measure on the product $\sigma-algebra$ $\tau_1\times\tau_2\times\cdots\times\tau_n$ satisfying

$$M\left\{\prod_{k=1}^{n}\Lambda_k\right\}=\min_{1\le k\le n}M_k\{\Lambda_k\}$$

That is, for each event $\Lambda\in\tau$, we have

$$M\{\Lambda\}=\begin{cases}\sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda}\min\limits_{1\le k\le n}M_k\{\Lambda_k\} \\ \quad if \sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\Lambda_n\subset\Lambda}\min\limits_{1\le k\le n}M_k\{\Lambda_k\}>0.5 \\ 1-\sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda^c}\min\limits_{1\le k\le n}M_k\{\Lambda_k\} \\ \quad if \sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\Lambda_n\subset\Lambda^c}\min\limits_{1\le k\le n}M_k\{\Lambda_k\}>0.5 \\ 0.5 \quad otherwise\end{cases}$$

Definition 2. Let Γ be a nonempty set, τ a $\sigma-algebra$ over Γ, and M an uncertain measure. Then the triplet (Γ,τ,M) is called an uncertainty space.

Definition 3. An uncertain variable is a measurable function ξ from an uncertainty space (Γ,τ,M) to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi\in B\}=\{r\in\Gamma\mid\xi(r)\in B\}$ is an event.

Definition 4. The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x)=M\{\xi\le x\}$ for any real number x.

Example 1 An uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x)=\begin{cases}0, & if\ x\le a\\ (x-a)/(b-a), & if\ a\le x\le b\\ 1, & if\ x\ge b\end{cases}$$

denoted by $L(a,b)$ where a and b are real numbers with $a < b$, which can be shown as Figure 1.

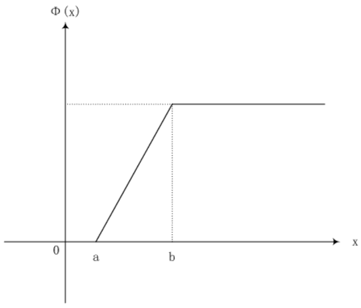

Figure 1. Linear Uncertainty Distribution

The inverse uncertainty distribution of linear uncertain variable $L(a,b)$ is $\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b$

Example 2 An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & if \quad x \le a \\ (x-a)/2(b-a), & if \quad a \le x \le b \\ (x+c-2b)/2(c-b), & if \quad b \le x \le c \\ 1, & if \quad x \ge c \end{cases}$$

denoted by $Z(a,b,c)$, where a,b,c are real numbers with $a < b < c$, which can be shown as Figure2.

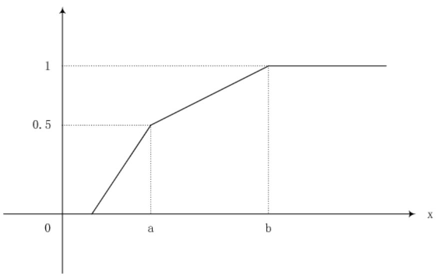

Figure 2. Zigzag Uncertainty Distribution

The inverse uncertainty distribution of zigzag uncertain variable $Z(a,b,c)$ is

$$\Phi^{-1}(\alpha) = \begin{cases} (1-2\alpha)a + 2\alpha b, & if \quad \alpha < 0.5 \\ (2-2\alpha)b + (2\alpha - 1)c, & if \quad \alpha \ge 0.5 \end{cases}$$

Example 3 An uncertain variable ξ is called normal if it has a normal uncertainty distribution.

$$\Phi(x) = (1+\exp(\frac{\pi(e-x)}{\sqrt{3}\sigma}))^{-1}, \quad x \in R$$

denoted by $N(e,\sigma)$, where e and σ are real numbers with $\sigma > 0$, which can be shown as Figure 3.

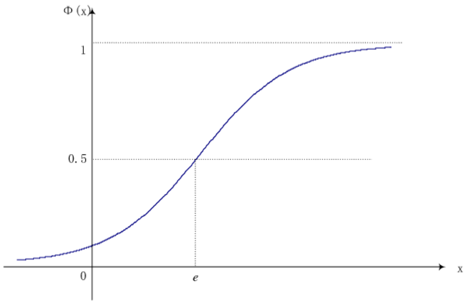

Figure 3. Normal Uncertainty Distribution

The inverse uncertainty distribution of normal uncertain variable $N(e,\sigma)$ is $\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}$

In the former research of portfolio selection, the return rates were often described as random variables or fuzzy variables [6-8], however, in many cases the portfolio selection return rates are neither like random nor like fuzzy variables. In this paper, we use uncertainty theory and describe return rates as uncertain variables, establish the optimal model to maximize expected utility based on loss aversion. The psychological study shows that the investors are more sensitive for the wealth reduction relative to the wealth increment, that is, the investors are loss aversion.

In the traditional model of maximizing expected utility, the investor's utility is usually the function of final wealth, and has not dealed with the wealth change. But Kahneman and Tversky discovered that investors most cared the change of final wealth relative to some reference value that is loss and increment of wealth. And investors more cared the loss when the final wealth was inferior to some reference value, so, the investors have the characteristic of loss aversion [9, 10].

Kahneman and Tversky defined the S-utility function as the following piecewise power function.

$$u(w) = \begin{cases} (w-w_0)^{\alpha}, & w \ge w_0 \\ -\lambda(w_0-w)^{\beta}, & w < w_0 \end{cases}$$

where $0 < \alpha \le \beta \le 1$, $\lambda \ge 1$, w_0 is the objective reference value, w is the final return, w_0 can be investors' initial wealth or the investors hoping achieved level or riskless rate, *etc*.

3. Optimal Model of Portfolio Selection Based on Loss Aversion

Supposing there are n kinds of investment projects, $x = (x_1, x_2, \cdots, x_n)$ is decision vector, x_i represents the investment proportion for the ith project, $\xi = (\xi_1, \xi_2, \cdots, \xi_n)$ is the vector that is composed of return rates of n kinds of investment projects, each $\xi_i = (i = 1,2,\cdots,n)$ is an uncertain variable, $\sum_{i=1}^{n} x_i = 1$, riskless rate ξ_f is objective reference value, the optimal model of maximizing expected utility can be described as model (1)

$$\begin{cases} \max \quad E[u(w)] \\ s.t. \\ \quad w = \sum_{i=1}^{n} \xi_i x_i \\ \quad M\left\{ \sum_{i=1}^{n} \xi_i x_i \le \gamma_1 \right\} \le \gamma_2 \\ \quad \sum_{i=1}^{n} x_i = 1 \\ \quad x_i \ge 0 \end{cases} \tag{1}$$

where $u(w) = \begin{cases} (w - \xi_f)^{\delta}, & w \ge \xi_f \\ -\lambda(\xi_f - w)^{\beta}, & w < \xi_f \end{cases}$

After repeated test, using nonlinear regression analysis, we can know that $\delta = 0.88$, $\lambda = 2.25$ best match investor's loss aversion mind[3]. In this paper, we let $\delta = 0.88$, $\lambda = 2.25$, $\beta = 1$.

4. Crisp Equivalent Model

For some special uncertain variables, we can transform the model (1) into crisp equivalent model; Liu [13] gave the following theorems.

Theorem 1 Assume the objective function $f(x, \xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the expected objective $\mathrm{E}[f(x, \xi_1, \xi_2, \cdots, \xi_n)]$ is equal to

$$\int_0^1 f(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha))\, d\alpha$$

Theorem 2 Assume the constraint function $g(x, \xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_k$ and strictly decreasing with respect to $\xi_{k+1}, \xi_{k+2}, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the chance constraint

$$M\{g(x,\xi_1,\xi_2,\cdots,\xi_n)\le 0\}\ge\alpha$$

holds if and only if $g(x,\Phi_1^{-1}(\alpha),\cdots,\Phi_k^{-1}(\alpha),\Phi_{k+1}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha))\le 0$

Theorem 3 Assume that ξ_1 and ξ_2 are independent linear uncertain variables $L(a_1,b_1)$ and $L(a_2,b_2)$, respectively. Then the sum $\xi_1+\xi_2$ is also a linear uncertain variable $L(a_1+a_2,b_1+b_2)$, *i.e.*

$$L(a_1+b_1)+L(a_2+b_2)=L(a_1+a_2,b_1+b_2)$$

The produce of a linear uncertain variables $L(a,b)$ and a scalar number $k>0$ is also a linear uncertain variable $L(ka,kb)$,*i.e.*.

$$k\cdot L(a,b)=L(ka,kb)$$

For model(1), supposing the return rates $\xi_1,\xi_2,\cdots,\xi_n$ are the same special uncertainty distributions $\Phi_1,\Phi_2,\cdots,\Phi_n$, respectively, $w=\sum_{i=1}^{n}\xi_i x_i$, so we can solve the uncertainty distribution Φ of w, when $w\ge\xi_f$, $u(w)=(w-\xi_f)^{\delta}$, and when $w<\xi_f$, $u(w)=-\lambda(\xi_f-w)^{\delta}$, so $u(w)$ is strictly increasing with respect to $\xi_1,\xi_2,\cdots,\xi_n$, from Theorem 1, the expected objective function $E[u(w)]$ is equal to

$$\int_0^{\Phi(\xi_f)}-\lambda(\xi_f-\sum_{i=1}^{n}\Phi_i^{-1}(\alpha)x_i)^{\beta}d\alpha+\int_{\Phi(\xi_f)}^{1}(\sum_{i=1}^{n}\Phi_i^{-1}(\alpha)x_i-\xi_f)^{\delta}d\alpha$$

The chance constraint $M\{\sum_{i=1}^{n}\xi_i x_i\le\gamma_1\}\le\gamma_2$ is equal to $M\{(\gamma_1-\sum_{i=1}^{n}\xi_i x_i)\le 0\}\ge 1-\gamma_2$, $\gamma_1-\sum_{i=1}^{n}\xi_i x_i$ is strictly decreasing with respect to $\xi_1,\xi_2,\cdots,\xi_n$, from Theorem 2, the chance constraint $M\{(\gamma_1-\sum_{i=1}^{n}\xi_i x_i)\le 0\}\ge 1-\gamma_2$ holds if and only if $\gamma_1-\sum_{i=1}^{n}\Phi_i^{-1}(\gamma_2)x_i\le 0$

So, model (1) is equivalent to the following crisp model (2)

$$\begin{cases}\max\ \int_0^{\Phi(\xi_f)}-\lambda(\xi_f-\sum_{i=1}^{n}\Phi_i^{-1}(\alpha)x_i)^{\beta}d\alpha+\int_{\Phi(\xi_f)}^{1}(\sum_{i=1}^{n}\Phi_i^{-1}(\alpha)x_i-\xi_f)^{\delta}d\alpha\\ s.t.\\ \quad\gamma_1-\sum_{i=1}^{n}\Phi_i^{-1}(\gamma_2)x_i\le 0\\ \quad\sum_{i=1}^{n}x_i=1\\ \quad x_i\ge 0\end{cases}\qquad(2)$$

For example, assume the return rates $\xi_1, \xi_2, \cdots, \xi_n$ are independent linear uncertain variables $L(a_1, b_1)$, $L(a_2, b_2)$, $\cdots$, $L(a_n, b_n)$, respectively, because $w = \sum_{i=1}^{n} \xi_i x_i$, from Theorem 3, we can know that w is also a linear uncertain variable $L(\sum_{i=1}^{n} a_i x_i, \sum_{i=1}^{n} b_i x_i)$, its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha) = (1-\alpha)\sum_{i=1}^{n} a_i x_i + \alpha \sum_{i=1}^{n} b_i x_i$$

We can convert the model (2) to the following model (3).

$$\begin{cases} \max \int_0^{\Phi(\xi_f)} -\lambda(\xi_f - \sum_{i=1}^{n}((1-\alpha)a_i + \alpha b_i)x_i)^{\beta} d\alpha + \int_{\Phi(\xi_f)}^{1} (\sum_{i=1}^{n}((1-\alpha)a_i + \alpha b_i)x_i - \xi_f)^{\delta} d\alpha \\ s.t. \\ \gamma_1 - \sum_{i=1}^{n}((1-\alpha)a_i + \alpha b_i)x_i \le 0 \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \ge 0 \end{cases} \tag{3}$$

After converting the uncertain programming model to a crisp one, we can solve it by traditional method.

5. Hybrid Intelligent Algorithm

When return rates are not special uncertain variables, it is difficult to solve model (1) in traditional method, so we design a hybrid intelligent algorithm integrating genetic algorithm (GA) and 99-method to solve model (1).

5.1. 99-Method

99-method was given by Liu [4]. It is suggested that an uncertain variable ξ with uncertainty distribution Φ is represented by a 99-table, which can be shown as Table 1.

Table 1. 99-Table

0.01	0.02	0.03	$\cdots$	0.99
y_1	y_2	y_3		y_{99}

Where $0.01, 0.02, 0.03, \cdots, 0.99$ in the first row are the values of uncertainty distribution Φ, and $y_1, y_2, y_3, \cdots, y_{99}$ in the second row are the corresponding values of $\Phi^{-1}(0.01)$, $\Phi^{-1}(0.02)$, $\Phi^{-1}(0.03)$, $\cdots$, $\Phi^{-1}(0.99)$. The 99-table is a discrete representation of uncertainty distribution Φ. Then for any strictly increasing function $f(y)$, the uncertain variable $f(y)$ has a 99-table as Table 2.

Table 2. 99-table of $f(\xi)$

0.01	0.02	0.03	…	0.99
$f(y_1)$	$f(y_2)$	$f(y_3)$		$f(y_{99})$

5.2. Genetic Algorithm

Representation structure: A solution $X = (x_1, x_2, \cdots, x_n)$ is represented by the chromosome $V = (v_1, v_2, \cdots, v_n)$, where the genes $v_1, v_2, \cdots, v_n$ are randomly generated in the interval [0,1], and the relation between X and V is formulated as follows: $x_i = v_i / (v_1 + v_2 + \cdots + v_n)$, $i = 1, 2, \cdots, n$, which ensure that $\sum_{i=1}^{n} x_i = 1$ always holds. [11, 12].

Initialization process: We randomly initialize $pop-size$ number of chromosomes by generating points $v_1, v_2, \cdots, v_n$ from the hypercube $[0,1]^n$ $pop-size$ times. Then solve X according to the relation between X and V. If X satisfies the constraint, it is a feasible solution.

Selection process: We select chromosomes by spinning the roulette wheel such that the better chromosomes will have. The selection process is as follows: [12]

Firstly, if there are $pop-size$ chromosomes $V_1, V_2, \cdots, V_{pop\_size}$ at the current generation, we can order these chromosomes from good to bad, the better the chromosomes is, the smaller the ordinal number it has. Let a parameter $a \in (0,1)$ in the genetic system be given, we can define the rank-based evaluation function as follows:

$$eval(V_i) = a(1-a)^{i-1}, \ i = 1, 2, \cdots, pop\_size$$

Note that $i = 1$ means the best chromosome, $i = pop-size$ means the worst one.

Secondly, calculate the cumulative probability q_i for each chromosome V_i,

$$q_0 = 0, \ q_i = \sum_{j=1}^{i} Eval(V_j), \ i = 1, 2, \cdots, pop-size$$

where $Eval(V)$ is evaluation function.

Thirdly, generate a random number r in$(0, q_{pop-size}]$, and select the chromosome V_i if r satisfies $q_{i-1} < r < q_i$.

Fourthly, repeat the third Step $pop-size$ times and obtain $pop-size$ copies of chromosome.

Crossover operation: A crossover parameter p_c is defined first [12]. Repeating the following process from $i = 1$ to $pop-size$: generating a random number r from the interval $[0,1]$, the chromosome V_i is selected as a parent if $r < p_c$. We denote the selected parents by $V_1^{'}, V_2^{'}, V_3^{'}, \cdots$, and divide them into the following pairs: $(V_1^{'}, V_2^{1})$, $(V_3^{'}, V_4^{1})$, $(V_5^{'}, V_6^{1})$, ⋯, The crossover operation on each pair is illustrated by $(V_1^{'}, V_2^{1})$. At first, we generate a random number c from the open interval $(0,1)$, then the operator on $V_1^{'}$ and $V_2^{'}$ will product two children X and Y as follows:

$$X = cV_1^{'} + (1-c)V_2^{'}, \ Y = (1-c)V_1^{'} + cV_2^{'}$$

If $X=(x_1,x_2,\cdots,x_n)$, $Y=(y_1,y_2,\cdots,y_n)$, let $x_i=x_i/(x_1+x_2+\cdots+x_n)$, $y_i=y_i/(y_1+y_2+\cdots+y_n)$, $i=1,2,\cdots,n$ which ensure that $\sum_{i=1}^{n}x_i=1$ and $\sum_{i=1}^{n}y_i=1$ always holds.

Checking whether $M\{\sum_{i=1}^{n}x_i\xi_i\le r_1\}\le r_2$ and $M\{\sum_{i=1}^{n}y_i\xi_i\le r_1\}\le r_2$ through 99-method, if both children are feasible, then we replace the parents with them. If not, we keep the feasible one if it exists, and then redo the crossover operator by regenerating a random number c until two feasible children are obtained or a given number of cycles is finished.

Mutation operation: We define a parameter p_m as the probability of mutation. This probability gives us the expected number of $p_m\cdot pop-size$ of chromosomes undergoing the mutation operations. Repeating the following Steps from $i=1$ to $pop-size$: generating a random number r from the interval $[0,1]$, the chromosome V_i is selected as a parent for mutation if $r<p_m$. For each selected parents V_i, we mutate it in the following way. Let M be an appropriate large positive number. We choose a mutation direction d in R^n randomly. Let $X=V+M\cdot d$, if $X=(x_1,x_2,\cdots,x_n)$, checking the feasibility through 99-method, if X is not feasible, we set M as a random number between 0 and M until it is feasible. If the above process cannot find a feasible solution in a predetermined number of iterations, then we set $M=0$.

5.3. Hybrid Intelligent Algorithm

The hybrid intelligent algorithm integrating genetic algorithm (GA) and 99-method is as follows:

Step 1 Determine the population size $pop-size$, crossover probability, mutation probability in genetic algorithm.

Step 2 Initialize $pop-size$ chromosomes whose feasibility may be checked by 99-method.

Step 3 Update the chromosomes by crossover and mutation operation in which the 99-method may be employed to check the feasibility of off springs.

Step 4 Calculate the objective values for all chromosomes by the 99-method.

Step 5 Compute the fitness of each chromosome based on the objective values.

Step 6 Select the chromosomes by spinning the roulette wheel.

Step 7 Repeat the third to sixth Steps a given number of cycles.

Step 8 Report the best chromosome as the optimal solution.

Number Example

To illustrate the modeling idea and to demonstrate the effectiveness of the proposed algorithm, one numerical example is presented here.

Supposing there are five kinds of investment project, each project's return rate is $\xi_i(i=1,2,\cdots,5)$, ξ_i is uncertain variable, where ξ_1,ξ_2 are linear uncertain variables, $\xi_1\sim L(0.57,3.75)$, $\xi_2\sim L(-0.85,4.93)$, ξ_1,ξ_4,ξ_5 are zigzag uncertain variables, $\xi_3\sim Z(-0.134,1.23,2.658)$, $\xi_4\sim Z(-0.5,2.13,4.658)$, $\xi_5\sim Z(-0.95,2.13,3.88)$, riskless return rate $\xi_f = 1.115$, $\gamma_1 = -0.02$, $\gamma_2 = 0.15$, $X = (x_1, x_2, \cdots, x_n)$ is decision vector, x_i represents the investment proportion for the ith project, each ξ_i is represented by 99-table as Table 3.

Table 3. 99-Table of ξ_i

0.01	0.02	0.03	…	0.99
b_1^i	b_2^i	b_3^i	…	b_{99}^i

The return rate of portfolio selection is represented by 99-table as Table 4.

Table 4. 99-Table of Portfolio Selection Return Rate

0.01	0.02	0.03	…	0.99
$\sum_{i=1}^{5} b_1^i x_i$	$\sum_{i=1}^{5} b_2^i x_i$	$\sum_{i=1}^{5} b_3^i x_i$	…	$\sum_{i=1}^{5} b_{99}^i x_i$

Because $u(w)$ is strictly increasing with respect to $\xi_i (i = 1, 2, \cdots, 5)$, the utility of portfolio selection return rate is represented by 99-table as Table 5.

Table 5. 99-Table of Utility Function

0.01	0.02	0.03	…	0.99
$u(\sum_{i=1}^{5} b_1^i x_i)$	$u(\sum_{i=1}^{5} b_2^i x_i)$	$u(\sum_{i=1}^{5} b_3^i x_i)$	…	$u(\sum_{i=1}^{5} b_{99}^i x_i)$

The model is solved by hybrid intelligent algorithm, the parameters in the algorithm are set as follows: $\delta = 0.88$, $\lambda = 2.25$, $\beta = 1$, 600 generations in GA, the population size $pop-size = 30$, the crossover probability $p_c = 0.3$, the mutation probability $p_m = 0.2$. The run of the hybrid intelligent algorithm shows the best decision plan is $X^* = (0.0016, 0.1110, 0.0002, 0.8772, 0.0100)$, the maximal expect utility of return rates is $E[u(w)] = 1.1769$. The genetic process of algorithm is shown as Figure 4.

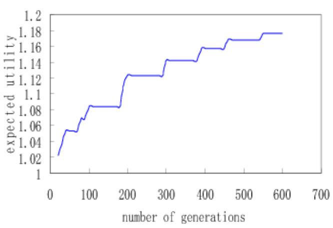

Figure 4. Genetic Process of Algorithm

6. Conclusion

This paper researches the portfolio selection model based on loss aversion under uncertain environment, describes investment return rate as uncertain variable. According to investors having the psychological characteristic of loss aversion, an optimal model of portfolio selection based on loss aversion is established, for some special uncertain return rates, crisp model can be obtained, for commonly situation, a hybrid intelligent algorithm is designed to solve the model. And a numerical example is given to show that the designed algorithm is effective for solving the optimization problem. This method can provide some science instruction for investor's investment decision.

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