

Research on Remote Sensing Image Fusion Algorithm Based on Compressed Sensing

Qiang Yang^{1,2,a}, Hua Jun Wang^{1,b} and Xuegang Luo^{1,c}

¹College of Geophysical, ChengDu University of Technology, China

²College of Computer and Information Engineering, Yibin University, China

^a scyangqiang@163.com, ^b 63918058@qq.com, ^c 543884841@qq.com

Abstract

The traditional image fusion algorithm completed the fusion based on all pixel information. The time and space requirements are higher. The improved fusion algorithm used the theory of compressed sensing (CS) for the processing of remote sensing image fusion. Firstly, the source images using wavelet transform for sparse representation, then, the improved fusion algorithm used the observation matrix for image dimension measurement, and completed the image fusion in CS domain. Finally, the algorithm used the improved OMP algorithm to reconstruct the fused image. The improved fusion algorithm is only applied with a few measurement data of the compressed sensing, and overcame the shortcomings of traditional pixel level fusion, the fusion algorithm achieved good experimental data.

Key Words: Compressed Sensing, Image Fusion, Remote Sensing Image, Sparse Matrix

1. Introduction

With the development of modern remote sensing technology, each kind of new sensor is constantly emerging. In the same area, we can get more and more remote sensing image. However, single remote sensing image data are ambiguous, incomplete and error limitation in resolution, geometric feature and image spectrum *etc.* And a plurality of remote sensing images of the same area is redundancy, complementary and cooperative. Therefore, used the multi-source remote sensing image data to fusion, and obtained the high spatial resolution image of feature enhancement, which can effectively improve the remote sensing image in the feature extraction and classification of the effectiveness. At the same time, it can effectively improve the robustness of image data of remote sensing applications.

The concept of information fusion occurred at about 70 of the last century, in 1979, Daily *et. al.* first used the radar image and Landsat-MSS image to fusion [1]. In 1981, Laner *et. al.* make the fusion test using the image data of Landsat-RBV and MSS [2]. In 1994, Genderen and Pohi gived the image fusion is a simple and intuitive definition [3]. Aftet 1990, with the launching of a plurality of remote sensing radar satellite JERS-1, ERS-1, RadaISat, and remote sensing image fusion technology has become a research hotspot in remote sensing image processing and analysis.

At present, the image fusion is divided into three levels, such as pixel level, feature level and decision level [5]. Pixel level fusion is the main research content of the current image fusion, the pixel level fusion levels mainly based on space domain fusion and fusion algorithm based on transform domain. The typical algorithm is a fusion algorithm based on multiscale transform, for example, wavelet transforms, sparse representation and ICA based image fusion algorithm. However, pixel level fusion algorithm for image processing, the time and space complexity requirements are higher. With the increasing of the remote sensing image data fusion algorithm, the real-time of fusion algorithm faces challenges. Recently, Candès, Tao and Donoho *et. al.* proposed compressed sensing (CS) theory

provides a new idea to solve this problem[6]. CS theory to prove that if the signal in a transform matrix is sparse, we can find the random measurement matrix be not related to a baseline of sparse transform, and we can analyze signal measured by the measurement matrix dimensionality reduction. For a small amount of observed data, we can use the nonlinear optimization algorithms; it can effectively reconstruct the original signal.

CS theory breaks through the traditional Nyquist requirements of sampling theorem, CS theory can be used to well below the Nyquist sampling of the signal sampling rate, it reduces the signal sampling rate, thus greatly reducing the data transmission, processing and storage requirements. The CS theory provides new ideas for the integration of large data of high resolution remote sensing image. In this paper, remote sensing image fusion algorithm based on compressed sensing overcomes the limitation of traditional image fusion algorithm; the traditional algorithm requires all the pixels in the source image based on the information, which has more demand for time and space.

In this paper, the improved fusion algorithm remote sensing image in CS domain is only applied to the compressed sensing sampling after a few measurement data, and through the improved reconstruction algorithm of CS theory to obtain the fused image.

2. The Compressed Sensing Theory

Compressed sensing theory mainly includes three aspects. (1) The sparse representation of signals. The signal F (length of N) is K -sparse in the orthogonal matrix. (2) Reduced dimension measurement. By measuring the matrix structure of stable (not related to orthogonal matrix and sparse representation), obtaining one-dimensional measurements of M length value Y , (M is much less than that of N , $M \ll N$). (3) Signal reconstruction. By using the optimization algorithm to reconstruct the original signal with high probability from the measured value of Y [6, 7, 8].

2.1. The Sparse Representation of Signals

Suppose F is the real valued discrete signal of size $N \times 1$, $F = \{f_1, f_2, \dots, f_N\}$, the input vector of F for linear representation[6], through a set of orthogonal basis vectors ψ , $\Psi = \{\psi_1, \psi_2, \dots, \psi_N\}$, That is:

$$F = \sum_{i=1}^N \psi_i u_i = \Psi u \quad (1)$$

In the formula 1, F and u are the same equivalent signal representation in different domain. Clearly, F is the signal in the time domain representation, and u is the signal in the domain ψ representation, and u is the sparse transform coefficient. If the u is only K of the non zero vector (K less than N , $K \ll N$), then F is K - sparse.

2.2. Reduced Dimension Measurement

Constructing the matrix Φ of two-dimensional $M \times N$, $\Phi = \{\varphi_1, \varphi_2, \dots, \varphi_M\}$, where M is much less than that of N ($M \ll N$), linear projection of the sparse signal u , obtain the linear measurement of the new value of X , $X = \{x_1, x_2, \dots, x_m\}$, that is:

$$X = \Phi F = \Phi \Psi u = \Theta u \quad (2)$$

In the formula 2, Θ called the observation matrix $M \times N$.

2.3. Signal Reconstruction

As the source signal with K - sparse, signal reconstruction of inverse problems can be solved by the l_0 -minimum norm [6]:

$$\hat{a} = \arg \min \|a'\|_0 \quad s.t \quad \Theta a' = X \quad (3)$$

In the formula 3, \hat{a} is the reconstruction of signal, $\|a'\|_0$ is the 0-norm of signal. Using the minimum l_0 -norm reconstruction of source signals is complicated to compute and solve the problem of poor stability. At the same time, it is also a NP-Hard problem. In 2006, D. Donoho proved that the minimum l_0 -norm can be used instead of the minimum l_1 -norm [6].

$$\hat{a} = \arg \min \|a'\|_1 \quad s.t \quad \Theta a' = X \quad (4)$$

Using the minimum l_1 -norm can reconstruct the K -sparse source signal with high probability.

Signal reconstruction is a convex optimization problem with a solution of minimum l_1 -norm. Signal reconstruction can be used matching pursuit (MP) algorithm to achieve. The basic idea of the algorithm is the N-iteration, selection and signal margin best matched atoms from the measurement matrix in Φ .

The realization of the basic steps is as follows:

Step1: Suppose $n = 1, \hat{f}^0 = 0$, the candidate subset Γ^0 is empty. The observation signal margin r^0 for the initial observation signal x .

Step2: To solve the best matching atom I^n , the calculation of I^n as shown in formula 5, the best matching atom I^n added to the candidate subset of Γ .

$$I^n = \arg \min_{i \in \Gamma, 2N} \|x - \Phi_i\|_2 \leq r^{n-1} \quad (5)$$

Step3: To solve the update candidate subset Γ ($\Gamma^n = \Gamma^{n-1} \cup I^n$), solving a new estimation signal:

$$\hat{f}^n = \Phi_{\Gamma^n}^+ x \quad (6)$$

In the formula 6, $\Phi_{\Gamma^n}^+$ for the Φ_{Γ^n} Moore-Penrose pseudo inverse [11].

Step4: To solve the observation signal margin r^n , $r^n = \|x - \Phi_{\Gamma^n} \hat{f}^n\|_2$.

Step5: Repeat the step 2、3、4. The calculation of \hat{f} and r , to $r = \sigma^2$ end of iteration (σ^2 is noise variance).

3. Analysis of Several Problems in Application of Compressed Sensing

3.1. Analysis of Signal Sparse Representation Problem

We can use the signal in very small amounts to represent the most or the entire source signals from an over complete dictionary. The sparse representation of the image can be used in a variety of ways to linear representation. At present, wavelet transform, discrete FuLiye transform, Hadamard transform, and singular value decomposition are the common form of image sparse transform. The image signals of different structure can be used in different transform the sparse representation, there is no transformation mode of fixed. The image color changes in the larger signal can be expressed using wavelet transform. For the smoother image signal can be sparse representation using finite difference transformation. For the details of the changes and the edge of the image information need to block, with sparse matrix different sparse representation.

In the practical application of image processing, due to the reasons of human eye

observation, K - sparse requirements need not be the most coefficient is 0 value, as long as the transformed coefficient is close to 0, *i.e.* $\|u - u_k\|_2 < \varepsilon$, ε is close to the normal number 0.

3.2. Analysis of Measurement Matrix Problems

Because of the measuring value of M dimension of X is much smaller than N , therefore, solving the inverse problem of the type 3 is a NP-hard ill-posed problem. That is: $a = \Theta^{-1} X$ is a NP-hard problem. For N unknowns numbers, only M constraint equations the underdetermined problem cannot be solved exactly. The number of solutions has numerous results. In order to be able to precisely reconstruct the recover source signals, E. Candes and T. Tao proposed RIP concept, namely the restriction moment constraints [7,8], RIP standards are as follows:

$$1 - \varepsilon \leq \frac{\|\Theta a\|_2}{\|a\|_2} \leq 1 + \varepsilon \quad (7)$$

The equivalent RIP criterion expression is sparse matrix ψ and the measurement matrix Φ is not relevant[9]. That is to say, $\Phi = \{\varphi_1, \varphi_2, \dots, \varphi_M\}$ cannot be represented by a sparse matrix vector $\Psi = \{\psi_1, \psi_2, \dots, \psi_N\}$.

The following matrix can be used as the measurement matrix, one is the FuLiyue random measurement matrix, Scrambling of FuLiyue matrix and radial FuLiyue matrix, the two is the Gauss / Bernoulli type of random measurement matrix, including the Gauss type stochastic matrix, Bernoulli random matrices and sub-Gauss random matrix, and other matrix such as partial Hadamard matrix, structured random matrices, topLiz matrix [10].

3.3. Analysis of Signal Reconstruction Problem

Because of the measuring value of M dimension of X is much smaller than N , reverse solution to reconstruct the signal can have a variety of results. But as the source signal F itself has the redundancy, F is compressible, this makes the vector reconstruction signal source with less than its N dimension is possible. As the source signal is K - sparse, equivalent to the source signal for setting a priori constraints, such as Al_0 -norm (non 0 elements in a minimum norm) and l_2 -norm (least square). In a variety of solutions, only with the prior conditions of the solution are the correct results of signal reconstruction. D. Donoho *et. al.* have confirmed that we can be used with a minimum l_0 - norm method to reconstruct the sparse solution, also can use the minimum l_1 -norm instead of the minimum l_0 -norm[7].

The l_1 - norm minimization model can be solved using convex optimization algorithm, we can use convex optimization algorithm based tracking method (BP), the gradient projection method (GPSR), greedy algorithm, Bregman iteration algorithm[11,12,13]. Greedy algorithm is a good solution to the sparse signal reconstruction algorithm, calculation of greedy algorithm has low complexity, fast calculation speed, and the simple geometric interpretation. The greedy algorithm mainly through iterative recursive solution for the signal reconstruction algorithm by seeking maximum and signal matching atom (as far as possible the orthogonal matching) so as to minimize the residual signal, to obtain the best approximation of the source signals in the recursive process. Because signal sparsity is low; the greedy algorithm has better performance.

4. Remote Sensing Image Fusion Based on Compressed Sensing

In this paper, the remote sensing image fusion algorithm based on compressed sensing make the fusion image registration and then use the measurement matrix to reduce the dimensionality of the transform of two images, obtain the measured values of the two images, then make the coefficient fusion for the measured value in CS domain. Finally, reconstruct the fused image according to the theory of compressed sensing. Implementation framework as shown in Figure 1.

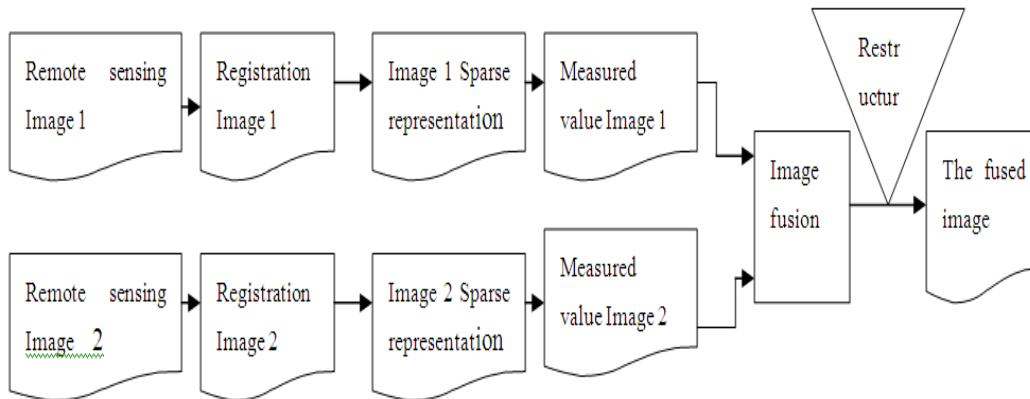


Figure 1. The Remote Sensing Image Fusion Framework Based on CS

The improved fusion algorithm steps are as follows:

- Step 1. Firstly, make the precise registration on two remote sensing image 1 and image 2.
- Step 2. And then using the orthogonal wavelet transform for remote sensing image sparse changes on F1 and F2 remote sensing image. For a remote sensing image F:

$$F(x, y) = \begin{bmatrix} f_{0,0} & f_{0,1} & \cdots & f_{0,2K-1} \\ f_{1,0} & f_{1,1} & \cdots & f_{1,2K-1} \\ \cdots & \cdots & \cdots & \cdots \\ f_{2K-1,0} & f_{2K-1,1} & \cdots & f_{2K-1,2K-1} \end{bmatrix}$$

Wavelet sparse transform for remote sensing image with the Mallat algorithm, the formula shown as follows [14]:

$$LL_{x,y} = \frac{1}{4} \sum_{k_1, k_2=0}^l p_{k_1} p_{k_2} i_{k_1+2x, k_2+2y} = \frac{1}{4} (i_{2x, 2y} + i_{2x, 2y+1} + i_{2x+1, y} + i_{2x+1, 2y+1}) \quad (8)$$

$$LH_{x,y} = \frac{1}{4} \sum_{k_1, k_2=0}^l p_{k_1} q_{k_2} i_{k_1+2x, k_2+2y} = \frac{1}{4} (i_{2x, 2y} - i_{2x, 2y+1} + i_{2x+1, y} - i_{2x+1, 2y+1}) \quad (9)$$

$$HL_{x,y} = \frac{1}{4} \sum_{k_1, k_2=0}^l q_{k_1} p_{k_2} i_{k_1+2x, k_2+2y} = \frac{1}{4} (i_{2x, 2y} + i_{2x, 2y+1} - i_{2x+1, y} - i_{2x+1, 2y+1}) \quad (10)$$

$$HH_{x,y} = \frac{1}{4} \sum_{k_1, k_2=0}^l q_{k_1} q_{k_2} i_{k_1+2x, k_2+2y} = \frac{1}{4} (i_{2x, 2y} - i_{2x, 2y+1} - i_{2x+1, y} + i_{2x+1, 2y+1}) \quad (11)$$

- Step 3. Using the Fuliye random measurement matrix dimension measurement of remote sensing image F1 and image F2. Fuliye matrix elements represented as [12]:

$$F_{j,k} = \frac{1}{\sqrt{N}} e^{i2\pi jk/N} \quad (12)$$

Observate the remote sensing image, $X = \Phi F = \Phi \Psi u = \Theta u$, obtain the image of the measured value X_1, X_2 .

Step 4. Make the fusion on the coefficient measurement value of X_1, X_2 .

In order to achieve better fusion effect, image fusion is not only integration of source image pixel. The local area also need corresponding pixel. The local fusion region variance and entropy as the fusion parameters [15]:

$$Dev(X) = \frac{1}{D_1 \times D_2} \sum_{i=0}^{D_1-1} \sum_{j=0}^{D_2-1} (G(X_{i,j}) - \overline{G}(X))^2 \quad (13)$$

$$E(X) = - \sum_{i=0}^{L-1} P_i \log(P_i) \quad (14)$$

In the formula 13, $Dev(X)$ is the variance X of region $D_1 \times D_2$ (D_1 and D_2 are the smaller integer), $G(X_{i,j})$ is the region in X (m, n) measurements, the $\overline{G}(X)$ represent the mean regional X measurements. In the formula 14, $E(X)$ is the entropy, P_i is the gray region X value for I point ratio, L is the gray level.

At the measurement fusion time, from which the regional variance $Dev(X)$ or entropy $E(X)$ is larger, the algorithm will measure the regional center pixel values as the fusion value of results.

Step 5. The fused measurement value for image reconstruction.

In this paper, using the improved orthogonal matching pursuit algorithm (OMP) to realize the fusion image reconstructed [10]. The improved algorithm steps is:

(a) Parameter initialization, where $t = 1, r_0 = y, \Lambda_0 = \Phi$.

(b) Seek the parameters λ_t to meet the following conditions: $\lambda_t = \arg \max | \langle r_{t-1}, \Phi_j \rangle |$.

(c) Set $\Lambda_t = \Lambda_{t-1} \cup \{ \lambda_t \}$, Calculation the value P_t, P_t is the the orthogonal projection space $\{ \Phi_\lambda : \lambda \in \Lambda_t \}$. According to the orthogonal projection of P_t calculation of the approximate values of $x_t = P_t y$ and residual new $r_t = y - x_t$.

(d) Update iterations, if $t=K$, then output the result, if $t < K$, then $t=t+1$, return (b) to continue.

(e) To obtain the Λ_K position of the non zero \hat{s}_λ , expressed as a measurement

vector approximation: $X_K = \sum_{\lambda \in \Lambda_K} s_\lambda \Phi_\lambda$.

5. The Experimental Analysis

The experiment selected the two pieces of image 256*256, as shown in Figure 2, figure 2 (a) and (b) for registration of remote sensing images of Yibin city. The image gray level is 256. Figure 3 represents a remote sensing source image F1 wavelet sparse change. From Figure 3, layer wavelet decomposition sparse representation can be seen in a large number of black pixels (0 points), if the multi-layer wavelet decomposition, the sparse representation of the effect will be better. Figure 4 is the remote sensing image F1 and F2 according to the CS algorithm based on the fusion results.



(a) Images after Registration F1



(b) Images after Registration F2

Figure 2. Remote Sensing Image After Registration

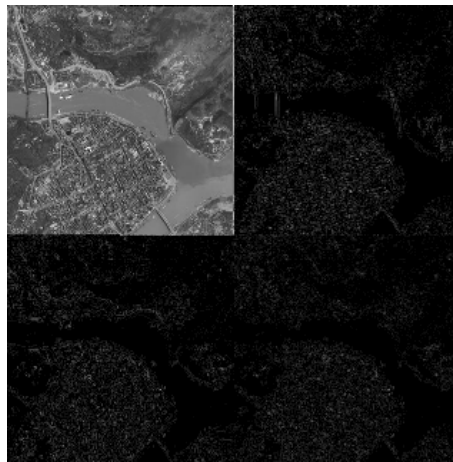


Figure 3 Wavelet sparse transform of F1 image



Figure 4. The fusion image of F

The image fusion rarely have the standard as a reference image, this paper cannot compare and analyze the standard image and the image of fusion result. Evaluation of

the effect of the traditional image processing parameters, such as the mean square error (MSE), peak signal to noise ratio (PSNR), normalized minimum variance (NSLE) *etc.* This cannot be used for objective evaluation of the experimental effect. The current commonly used no reference image evaluation method for the analysis of the experimental results, the main information entropy (H_F), standard deviation (S_F) and cross entropy (C) [5].

The evaluation formula of information entropy H_F is:

$$H_F = -\sum_{i=0}^{N-1} p_F(i) \log_2 p_F(i) \quad (15)$$

In the formula 15, p_F is to evaluate the fused image pixel value distribution. N is the fusion image of the total gray level. $p_F = \{p_F(0), p_F(1), \dots, p_F(N-1)\}$. Information entropy H_F represents the amount of information including an image of the value, Larger values of H_F , which means the image information more abundant, the visual effect is better.

The evaluation formula of the standard deviation S_F is:

$$S_F = \sqrt{\sum_{i=0}^{N-1} (i - Av_F)^2 p_F(i)} \quad (16)$$

In the formula 16, Av_F is the pixel gray mean image, p_F is the image pixel value distribution. The standard deviation of S_F reflects the image contrast, larger values of S_F , the image contrast is stronger, the visual effect is better.

The cross entropy formula for the C evaluation is :

$$C = \sum_{i=0}^{N-1} p_i \log \frac{p_i}{q_i} \quad (17)$$

In the formula 17, p_i is the gray level distribution of the source image, q_i is the gray distribution of image fusion, Cross entropy C is the pixel difference of two images. Image difference is small, the more amount of information extraction, cross entropy C can better evaluation of image fusion.

The above three formula with meaning please consult reference 5. Evaluation parameters in this experimental results as shown in table 1.

Table 1. Parameters Analysis and Evaluation of Image Fusion

Image	Evaluation parameters		
	H_x	S_F	C
The source image F1	7.232	44.902	1.230
The source image F2	6.519	41.015	0.852
The fused image	7.301	43.580	-----

Experiments indicate that the evaluation parameters of remote sensing image fusion is well based on CS, the information entropy evaluation parameters H_x is well, Cross entropy evaluation parameters C is well. The experiment has obtained the very good fusion effect.

6. Summary

Compressed sensing theory has been widely used in recent years in the information processing, image analysis and other aspects. Image fusion has important application in the field of image processing. This paper presented the theory of compressed sensing and

used for processing of remote sensing image fusion. The improved algorithm overcame the traditional image fusion algorithm needs a great demand for the limitation of time and the space of all the pixels in the source image fusion based on the information. Remote sensing image fusion is performed in the CS domain, a few measurement data fusion algorithm is only applied to the compressed sensing after sampling, obtain the fusion image to get through CS theory to reconstruct the fused data. The algorithm achieved good experimental effect. However, because of the variety of the image content types, it is difficult to have a universal fit better sparse representation method and measure matrix, reconstruction algorithm of compressed sensing in time also needs to be improved.

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Authors



Qiang Yang, is Associate professor. He graduated from the Chengdu University of Technology and received a master's degree in 2007. In 2013, He went to Germany to study at Rheinisch Westfaelische Technische Hochschule Aachen University. Now, he is the doctoral student at Chengdu University of Technology. His main research direction is image processing and pattern recognition.



Huajun Wang, is Ph. D and Professor, He is the Sichuan Province Science and technology leader. In 2001, He graduated from the University of Electronic Science and technology and received doctor degree in engineering. In 2002-2003, he went to Russia to study at the Moscow State University as a senior visiting scholar. He is currently researching on millimeter wave, microwave technology and digital image processing technology.



Xuegang Luo, received the bachelor's degree in computer science and technology from Huazhong Agricultural University and the M.Eng degree in software engineering from University of Electronic Science and Technology of China. He is currently working toward the Ph.D. degree. He is currently researching on Computer Vision and Image Understanding.