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Block Compressed Sensing of Self-adaptive Measurement and Combinatorial Optimization

Li Mingxing, Chen Xiuxin, Su Weijun and Yu Chongchong

College of Computer and Information Engneering, Beijing Technology and Business University (100048)
chenxx1979@126.com; star.li.123@163.com; swj6843@163.com; chongzhy@vip.sina.com

Abstract

The block compressed sensing has brought forth the problem that the reconstructed image is of lower quality compared with that of the compressed sensing. A new method is proposed in this paper, named as Block Compressed Sensing of Self-adaptive Measurement and Combinatorial Optimization, which capably solves the problem. According to different sparsity of each image block, we firstly measure the blocks by using different projections; then, we choose measurement with the optimal reconstruction as the final measurement. Eventually, reconstruct the original image using the optimal measurement we got. The proposed method outperforms the compressed sensing in terms of real-time and better reconstruction quality is achieved than the block compressed sensing. Our experimental results verify the superiority of the proposed method.

Keywords: *Block Compressed Sensing, Sparsity, Self-adaptive Measurement, Combinatorial Optimization, Reconstruction*

1. Introduction

In the normal methods of picture compression, pictures are firstly transformed into digital format at a high sample rate, and then they are coded by the JPEG or other coding methods. But, those methods have the limitation that they are not so applicable to some equipment such as sensors of low power and low quality. Recently, Candes and Donoho[1-4] had a break-through research and proposed a new method called as Compressed Sensing(CS) theory that is capable to avoid the limitation. The CS theory based on the sparsity of the signal can accurately reconstruct the signal by using a few measurements.

In recent years, not only in the theory, but also in application, the CS has made great progress. Many institutions in a good number of countries including China, and many famous industries like Intel, google etc. have taken part in the research of CS. For its applications, CS can be utilized in Nuclear magnetic resonance (NMR) imaging [5] of biological medicine. In the processing of Radar images [6], a great deal of data should be sampled, transferred and stored. With the application of the CS, the data amount can be dramatically reduced. In addition, the CS theory can be used to extract the features [7] of human faces so as to improve the accuracy of the recognition. However, people found that the whole image should be dealt with at once in the process of sampling of the CS, which unfortunately reduces the real-time performance of the system [8]. LU Gan proposed the Block Compressed Sensing (BCS) [9] to overcome this problem. This method divides the image into many blocks, and then simply deals with each block. It improves the real-time performance and reduces the complexity of computation in projection and reconstruction of the CS. Because the blocks of the image can greatly reduce the dimension of the image, this method can also make store easily. But in accuracy point of view to reconstruction, the BCS is lower than the CS.

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Based on the theory of the BCS, a brand new algorithm named BCS of Self-Adaptive Measurement Combinatorial Optimization (AMCO-BCS) is brought into being in this paper. On the basis of the Restricted Isometry Property, AMCO-BCS self-adaptively selects the dimension of projection by using different projection matrixes subjected to the conditions of different sparstity of the blocks. And then use different measurements of each block to reconstruct the block, choose the measurements that are most accurate as the final measurement of the block. By this way of combinatorial optimization, we achieve still better measurements. Finally we use the measurement to reconstruct the image. The first part of this paper is the introduction that focuses on the application of the CS and its advantages and disadvantages. The next part primarily describes the CS theory in the aspect of sparisty, uncorrelated projection and reconstruction at length. The following part is the AMCO-BCS which is the core content and its flow; whereas the forth part is the experiment and analysis which show the advantages and the final part is the conclusion and setting forth the following up work to be done.

2. Compressed Sensing

The CS mainly includes three parts: the sparsity of the signal, uncorrelated measurement and reconstruction using non-linear optimization. The sparsity of the signal is the precondition and the method of reconstruction is non-linear optimization. The uncorrelated measurement is the key to the CS.

2.1. The Sparse Represent of Signal

Suppose the signal $x \in R^N$, and may be not sparse. But by using the orthogonal basis $\{\psi_i\}_{i=1}^N$ （ψ_i is a column vector of N dimensions）to transfer the signal, it can be sparse, as shown in the following:

$$\mathrm{x} = \sum_{i=1}^{N} \theta_i \Psi_i \tag{1}$$

In which $\theta_i = \langle x, \psi_i \rangle = \psi_i^T x$ .the matrix form is indicated as follows:

$$\mathrm{x} = \Psi \theta \tag{2}$$

Where the $\psi = [\psi_1, \psi_2, \ldots \psi_N] \in R^{N \times N}$ is orthogonal matrix, and the coefficient

vector $\theta = [\theta_1, \theta_2, \ldots \theta_N]^T$ is K （$K << N$）sparse.

2.2. Non-linear Optimization Reconstruction

Suppose ϕ ： $M \times N(M << N)$ is the measurement matrix which is irrelevant to the ψ ,and the ϕ satisfies the Restricted Isometry Property(RIP)[10-11]. So use ϕ to project the signal x :

$$\mathrm{y} = \phi x \tag{3}$$

Where the ϕ is multiplied by the signal to get the projection $y \in R^M (M << N)$. Those few linear projections contain enough information that can be used to reconstruct the x .

However, from equation (3), it seems impossible to reconstruct x from y , because the equation is an underdetermined equation sets whose number is less than that of unknown numbers, which lead to infinite solutions. But, let formula (2) into (3), and suppose $A^{CS} = \phi\psi$, we shall get：

$$y = \phi\psi\theta = A^{CS}\theta \tag{4}$$

Although reconstruction θ from y is also an underdetermined equation, the number of unknown numbers become much less because θ is sparse, which make the reconstruction possible. Based on this, some scholars proposed the ℓ_0 minimum algorithm [2], but the algorithm is proved to be a NP-hard question. So some other scholars proposed the ℓ_1 minimum algorithm [12] which is equivalent to the ℓ_0 minimum algorithm. According to the two algorithms, the convex optimization and the greedy algorithm are the main algorithms which include Basis Pursuit (BP), Gradient Projection for Sparse Reconstruction (GPSR), Iteration Threshold (IT)[13]and Regularize Adaptive Matching Pursuit (RAMP), Compressed Sampling Matching Pursuit(CoSaMP) [14]and so on. Utilizing the measurements to reconstruct the original signal as much as possible is the target of the reconstruction. So the measurement of the signal is the key to the CS, which determines the compress degree and the quality of the reconstruction.

2.3. The Irrelevant Measurement

This paper mainly makes an improvement for the irrelevant measurement. In the light of the CS theory, the measurement matrix must satisfy some conditions to make sure that the original signal $x \in R^N$ can be reconstructed from $y \in R^M \left(M << N\right)$. But, the equation is underdetermined that leads to the solution impossible. If x is sparse and the position of the non-zero coefficient α is known, we shall get the solutions of the equations. Suppose $M \geq K$ and for any vector v and the constant $\delta > 0$, these are necessary and sufficient conditions that make the equation be solved is as follows:

$$1 - \delta \leqslant \frac{\left\|\Theta \ \mathrm{v}\right\|_2}{\left\|\mathrm{v}\right\|_2} \leqslant 1 + \delta \tag{5}$$

In other words, the matrix $\Theta = \phi\psi$ must reserve the length of those special vectors which is K sparse. Of course, the position of the non-zero coefficients in α is not known in normal conditions. However, for the signal which is K sparse, one of the conditions which can solve the problem is that the matrix Θ satisfies the inequality(5) for any sparse vector which is 3K order[3]. This condition is from the Restricted Isometry Property (RIP) proposed by Candes:

For the matrix $\phi \in R^{M \times N}$, if for all the index set $I \subset \left\{1, 2, \ldots N\right\}$ that satisfy $\left|I\right| \leq m < M$ and any vector $v \in R^{|I|}$,if there is a constant $0 < \delta < 1$ that make the following formula enable.

$$(1 - \delta)\left\|v\right\|_{l2} \leq \left\|\phi_I v\right\|_{l2} \leq (1 + \delta)\left\|v\right\|_{l2} \tag{6}$$

We are in the position to determine the matrix satisfying the RIP condition. Where ϕ_I represents the sub-matrix that consisted of the column vectors in the matrix ϕ pointed by the index set $I \subset \left\{1, 2, \ldots N\right\}$.

The commonly used measurements include Gaussian random measurement matrix [15], Benoit random matrix [16], Toeplitz matrices [17], Some hadamard matrix [18], Sparse Random matrix [19] *etc.*.

Nowadays, the random measurement matrixes are widely used, such as Gaussian random measurement matrix. But the random measurement matrixes belong to non-adaptive measurement, which leads to the high complexity of computation and difficult application to large-scale questions.

J M Duarte-Carvajalino[20]proposed a method that use the adaptive measurement matrix in CS: Based on the correlation theory, the non-correlated condition of spares transformation matrix and measurements can be equal to the unit matrix approximation question of the Gramma matrix, where the Gramma matrix is

$$Gram \;:\; \left(A^{CS}\right)^{T} A^{CS} \tag{7}$$

The unit matrix approximation question can be represented as follows：

$$\begin{cases} \min\limits_{\phi} \left\| \left(A^{CS}\right)^{T} A^{CS} - I \right\|_2^2 \\ s.t.\, A^{CS} = \phi\psi \end{cases} \tag{8}$$

The basic meaning is that utilizing the base of sparse transformation matrix to transform the random measurement to an optimal measurement matrix lower coherent to the sparse transformation matrix by training. We would get the confirm measurement by adopting the K-VSD proposed by M Aharon[21][22]. The accuracy of reconstruction would be improved or the number of measurements be reduced under the same accuracy of reconstruction.

Based on the J M Duarte-Carvajalino’s method, this paper proposes an improved method. We first generate two different random measurement matrixes, then, according to the sparsity of each image block, we can self-adaptively get the determinate measurement matrix of each block. And each block corresponds to two different measurement matrixes, and then we can get the optimal measurements by the way of combinatorial optimization. This is to be detailed in the following part.

3. AMCO-BCS

The main process of the AMCO-BCS algorithm is that:

Firstly, divide image A into $B \times B$ blocks: $A_i (i = 1,2,\ldots, B \times B)$

Secondly, generate two different random measurement matrixes. According to different sparsity of the blocks, use the method proposed in 2.3 to generate two different self-adpative measurement matrixes $\phi_{1i}, \phi_{2i} (i = 1,2,\ldots, B \times B)$.

Thirdly, use the measurement matrixes of each block to project each block:

$$y_{ji} = \phi_{ji} A_i$$

Where $i = 1,2,\ldots, B \times B$, $j = 1,2$

Fourthly, the projections of each block are got.

$Y_1 = \left[y_{11}, y_{12}, \ldots, y_{1B\times B}\right]$ and $Y_2 = \left[y_{21}, y_{22}, \ldots, y_{2B\times B}\right]$

Then we reconstruct each block with the two measurements and compare the reconstruction accuracy of two measurements of each block. Choose the higher one as the optimal measurement.

Finally, Use the optimal measurement to reconstruct the original image. The method of reconstruction is mainly based on the algorithm proposed by GanF, i.e.

Linear prediction [23] based on the Minimum Mean Square Error (MMSE):

$$\min \left\| x_i - \hat{x}_i \right\|_2^2 \tag{9}$$

$$s.t.\ \hat{x}_i = \hat{\phi}_B y_i \tag{10}$$

Where in the above formula $\hat{\phi}_B = R_{xx}\phi_B^T(\phi_B R_{xx}\phi_B^T)^{-1}$, R_{xx} represents the self-correlation function of original signal. In natural images, R_{xx} adopts AR (1) model [24] [25], where the self-correlation coefficient $\rho = 0.95$. Lu GanF has made an improvement for this reconstruction method. The main point of the improved method is that using hard threshold and projection method in the convex sets [9]. And in addition, we only handle with each block in the process of reconstruction, with which saved the complexity of computation compared with the traditional CS which deal with full size of the image. The diagram of the algorithm is shown in Figure 1.

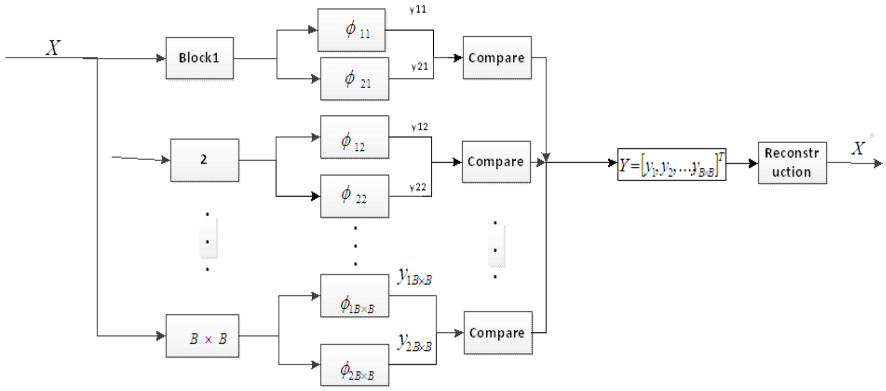

Figure 1. Diagram of AMCO-BCS Algorithm

Set X as original image and divide it into $B \times B$ blocks: $x_i (i = 1,2,..., B \times B)$. Set ψ as wavelet transform matrix of X and $\psi_i (i = 1,2,..., B \times B)$ is the wavelet transform matrix of $x_i (i = 1,2,..., B \times B)$; Set ϕ_1^i, $\phi_2^i (i = 1,2,..., B \times B)$ as the different random projection matrixes of $x_i (i = 1,2,..., B \times B)$. Set $\phi_{1i}, \phi_{2i} (i = 1,2,..., B \times B)$ as the adaptive projection matrixes of x_i generated from ϕ_1^i and ϕ_2^i. Set y_{1i}, y_{2i} as the projection of block x_i using the projection matrixes ϕ_{1i} and ϕ_{2i}; Set y as the finally projection vector. Set $PSNR_{1i}$, $PSNR_{2i}$ as the Peak Signal to Noise Ratio of the reconstruction of x_i using the projection matrixes ϕ_{1i} and ϕ_{2i} respectively.

Begin {
1. Wavelet transformation for X : $X = \psi\theta$

If θ is sparse, go on; else, end the program;
2. Divide image X into blocks: $X \rightarrow x_i (i = 1,2,..., B \times B)$;
3. For i=0 to $B \times B$ {

1) Generate $\psi_i (i = 1,2,..., B \times B)$.

2) Generate ϕ_{1i} {

Initialize ϕ_1^i;

Find the eigen-decomposition of $\phi_1^i : \omega\omega^T = V\Lambda V^T$;

Initialize $\Gamma = \phi_1^i V$;

Iteration and Update: $\min \left\| \Lambda - \Lambda\Gamma^T \Gamma\Lambda \right\|_2^2$;

Compute the optimal: $\phi_{1i} = \Gamma V^T$;

}

3) ϕ_{2i} Can be got using the same way as ϕ_{1i}.

4) Compute the projection: $y_{1i} = \phi_{1i} x_i$

$$y_{2i} = \phi_{2i} x_i$$

5) Reconstruct the block x_i:

$$x_i \leftarrow y_{1i}$$

$$x_i \leftarrow y_{2i}$$

6) Compute the PSNR1 and PSNR2;

7) If $PSNR_{1i} < PSNR_{2i}$, $y_i = y_{2i}$; else, $y_i = y_{1i}$.Save y_i.

}

4. Get the optimal $y = [y_1, y_2, ... y_i ..., y_{B\times B}]$.

5. Reconstruct X : $X \leftarrow y$.

}

4. Experiments

4.1. Experimental Environment and Index

The natural images of Lena, Cameraman and etc. are adopted in the experiment as the experimental data. Use Matlab as the simulation software to simulate the experiment. The PSNR (Peak Signal to Noise Ratio) is adopted for the experiment as the index.

4.2. Experimental Procedure

1. Transform the original image using wavelet transform. The transforming result of Lina image is shown in Figure2.

Figure 2. The Lena Image Using Wavelet Transform

As the CS theory tells, the necessary condition of the CS is that the original image or the transformed image must be sparse. It can be seen from Figure2 that the most part of the transformed image is black which means that the image is sparse. Then it can be concluded that the CS for this image is applicable.

2. Divide the image into 16 blocks, the size of each is 64*64.

Generate two different random measurement matrixes 1 and 2, then we can get the two self-adaptive measurement matrixes ϕ_{1i}, ϕ_{2i} $(i = 1,2,..., 16)$.

3. Adopt the random measurements 1 and 2 to project and reconstruct the Lena image by using BCS ,and at the same time ,adopt the ϕ_{1i}, ϕ_{2i} to project and reconstruct the Lena image by using the AMCO-BCS proposed in this paper.

The compared experimental results are shown in Figure 3.

In Figure3 the BCS1 represents that we adopt random measurement matrix 1 using BCS method and BCS2 represents that we adopt the random measurement matrix 2 using BCS method.

BCS1 BCS2 AMCO-BCS

Figure 3. The Lena Image Reconstructed with Different Methods

The PSNR of BCS1, BCS2 and AMCO-BCS is shown in Table 1.

Table 1. PSNR Using BCS1, BCS2 and AMCO-BCS

Measuring method	BCS1	BCS2	AMCO-BCS
Lena	30.9907	39.4338	31.1423
Cameraman	30.8320	30.8653	30.9126

From Figure3 and Table 1, it can be seen that the reconstruction accuracy of AMCO-BCS is better than the BCS.

The image can be divided into different number of blocks and the comparison between the BCS and AMCO-BCS in different blocks is shown in Table 2.

Table 2. Comparison between the BCS and AMCO-BCS in Different Number of Blocks

Number of blocks		4	16	64
Lena	BCS	32.4961	32.2901	31.8926
	AMCO-BCS	32.5907	32.3423	31.9832
Cameraman	BCS	32.3932	32.1132	31.7717
	AMCO-BCS	32.4012	32.272	31.8116

Table 2 also shows that the reconstruction accuracy of AMCO-BCS is better than the BCS.

In addition, the PSNR of the image of reconstruction in different sampling rates is shown in Figure4.

All these experimental results demonstrate that the AMCO-BCS proposed in this paper is better than the BCS in reconstruction accuracy, but poorer than the CS, and other experimental results that we got using other images also proved the result. As in the aspect of the real-time performance, the AMCO-BCS is superior than that of the CS, for the AMCO-BCS divides the image into blocks and deals with them which means that the image can be dealt with when it is not yet completely transmitted. But because of the AMCO-BCS is more complicated than the CS, hence its performance is inferior to the CS in terms of real-time. In other words, the AMCO-BCS is a compromised method between the CS and BCS. On the basis of better real-time performance, the quality of the reconstructed image is guaranteed.

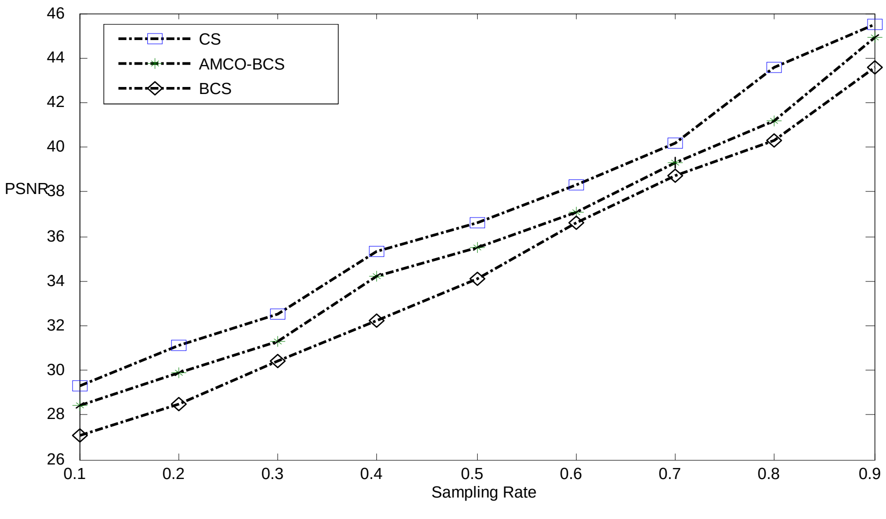

Figure 4. The PSNR in Different Sampling Rates

5. Conclusions

The Compressed Sensing theory has produced a profound impact in the fields of Signal Processing, Image Capturing and Treatment as well as Computer Vision etc. Therefore, it is of vital importance. This paper introduced the Compressed Sensing theory, the Block Compressed Sensing theory and their respective applications firstly and then emphatically introduced the whole content frame of CS theory in three parts at length. On this basis, the paper continued in detail introduction of the proposed AMCO-BCS algorithm, and further proved its superiority by experiments. The AMCO-BCS algorithm effectively avoids the defects that severely disqualify the reconstructed image when using BCS. And the greatest advantage of the AMCO-BCS is the optimal combination of measurements that enable the utmost preserve of the gained information that is required for reconstructing the image. Moreover, the measurements of the image can be taken as the features of the original images, and then we can apply the CS into the Machine Learning, which can be used in human face detection, gait recognition and so on. The method proposed in this paper optimizes the features of the images, so it has important reference value in pattern recognition and machine learning.

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References

[1] E. Candès and M. Wakin, “An introduction to compressive sampling”, IEEE Signal processing Magazine, vol. 25, no. 2, **(2008)**, pp. 21-30.
[2] E. Candès, “Compressive sampling”, Proceedings of international congress of mathematicians, **(2006)**, pp. 1433-1452.
[3] D. L. Donoho and Y. Tsaic, “Extensions of compressed sensing. Signal Processing”, vol. 86, no. 3, **(2006)**, pp. 533-55.
[4] R. Baraniuk, “Compressive sensing”, IEEE Signal Processing Magazine, vol. 24, no. 4, **(2007)**, pp. 118-12.
[5] M. Lustig, D. L. Donoho and J. M. Santos, “Compressed sensing MRI”, Signal Processing Magazine, IEEE, vol. 25, no. 2, **(2008)**, pp. 72-82.
[6] M. A. Herman and T. Strohmer, “High-resolution radar via compressed sensing”, Signal Processing, IEEE Transactions on, vol. 57, no. 6, **(2009)**, pp. 2275-2284.
[7] P. Nagesh and B. Li, “A compressive sensing approach for expression-invariant face recognition”, Computer Vision and Pattern Recognition, 2009, CVPR 2009. IEEE Conference on, IEEE, **(2009)**, pp. 1518-1525.
[8] J. A. Tropp, M. B. Wakin, M. F. Duarte, D. Baron, and R. G. Baraniuk, “Random filters for compressive sampling and reconstruction, Proceedings of International Conference of Acoustics, Speech, Signal Processing (ICASSP), France, **(2006)** May.
[9] L. Gan, “Block compressed sensing of natural images”, Proceedings of the 15th International Conference on Digital Signal Processing. Cardiff, UK. **(2007)**, pp. 403- 406.
[10] R. Baraniuk, *et. al*, “A simple proof of the restricted isometric property for random matrices”, Constructive Approximation, vol. 28, no. 3, **(2008)**, pp. 253-263.
[11] E. J. Candes, “The restricted isometric property and its implications for compressed sensing”, Competes Rendus Mathematique, vol. 346, no.9-10, **(2008)**, pp. 589-592.
[12] Y. Tsaig, D. L. Donoho, “Extensions of compressed sensing. Signal processing”, vol. 86, no. 3, **(2006)**, pp. 549-571.
[13] D. L. Donoho and Y. Tsaic, “Extensions of compressed sensing”, Signal Processing, vol. 86, no. 3, **(2006)**, pp. 533-548.
[14] D. L. Donoho, “Compressed sensing”, IEEE Transactions on Information Theory, vol. 52, no. 4, **(2006)**, pp. 1289-1306.
[15] A. M. Bruckstein, M. Elad and M. Zibulevsky, “Sparse non-negative solution of a linear system of equations is unique”, Proceedings of International Symposium on Communications, Control and Signal Processing, **(2008)**, pp. 762-767.
[16] B. Collins, E. Carlos, G. Guillén and D. Pérez-García, “Matrix Product States, Random Matrix Theory and the Principle of Maximum Entropy”, Communications in Mathematical Physics. vol. 320, Issue 3, **(2013)** June, pp 663-677.
[17] W. U. Bajw, J. D. Haupt and G. M. Raz, *et. al*. “Toeplitz-structured compressed sensing matrices Statistical Signal Processing”, 2007. SSP'07. IEEE/SP 14th Workshop on. IEEE, **(2007)**, pp. 294-298.
[18] R. Sun, H. Zhao and H. Xu, “The application of improved Hadamard measurement matrix in compressed sensing Systems and Informatics (ICSAI)”, 2012 International Conference on. IEEE, **(2012)**, pp. 1994-1997.
[19] R. Berinde and P. Indyk, “Sparse recovery using sparse random matrices”, Preprint, **(2008)**.

[20] J. M. D. Carvajalino and G. Sapiro, "Learning to sense sparse signals: simultaneous sensing matrix and scarifying dictionary optimization", IEEE Transactions on Image Processing, vol. 18, no. 7, **(2009)**, pp. 1395-1408.
[21] M. Elad and M. Aharon, "Image denoising via learned dictionaries and sparse representation", Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on. IEEE, 1, **(2006)**, pp. 895-900.
[22] M. Aharon, M. Elad and A. Bruckstein, "An algorithm for designing over complete dictionaries for sparse representation", Signal Processing, IEEE Transactions on, vol. 54, no. 11, **(2006)**, pp. 4311-4322.
[23] S. Haykin, "Adaptive filter theory", **(1986)**.
[24] T. D. Tran, J. Liang, C. Tu, "Lapped transform via time-domain pre-and post-filtering", Signal Processing, IEEE Transactions on, vol. 51, no. 6, **(2003)**, pp. 1557-1571.
[25] C. Tu and T. D. Tran, "Context-based entropy coding of block transform coefficients for image compression", International Symposium on Optical Science and Technology, International Society for Optics and Photonics, **(2001)**, pp. 377-389.

Authors

Li Mingxing, was born in Linyi of ShanDong province in December of 1989. He is a postgraduate student of Beijing Technology and Business University. His major is information engineering and Machine Learning. His current research interests are image processing and Machine Learning.

Chen Xiuxin, Chen is the correspongding author of this paper. She was born in Lingxian of Shandong province in December of 1979. She is a Ph.D of multi-media information domain in Beijing University of Technology, now. She is a lecturer of Beijing Technology and Business University. Her current research interests are multi-media processing and image/video feature extraction and match. Ms. Chen is a membership of China Association of Artificial Intelligence.

Su Weijun, Su is an associate professor of Department of Computer and Information Engineering in Beijing Technology and Business University. His main research areas are detection technology and optimal control, the theory and application of the Internet.

Yu Chongchong, was born in Dandong of Liaoning province in 1971. She is professor of Department of Computer and Information Engineering in Beijing Technology and Business University. Her current research interests are intelligent information processing and computer network. Prof. Yu is a membership of China Computer Federation and China Association for Artificial Intelligence.

