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A Comprehensive Survey of Test Functions for Evaluating the Performance of Particle Swarm Optimization Algorithm

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Abstract

Test functions play an important role in validating and comparing the performance of optimization algorithms. The test functions should have some diverse properties, which can be useful in testing of any new algorithm. The efficiency, reliability and validation of optimization algorithms can be done by using a set of standard benchmarks or test functions. For any new optimization, it is necessary to validate its performance and compare it with other existing algorithms using a good set of test functions. Optimization problems are widely used in various fields of science and technology. Sometimes such problems can be very complex. Particle Swarm Optimization is a stochastic algorithm used for solving such optimization problems. This paper transplants some of the test functions which can be used to test the performance of Particle Swarm Optimization (PSO) algorithm, in order to improve its performance and have better results. Different test functions can be used for different types of problems. These test functions have a specific range and values, which can be applied in different situations. These functions, when applied to the PSO algorithm, can give the better comparison of results. The test functions that have been the most commonly adopted to assess performance of PSO-based algorithms and details of each of them are provided, such as the search range, the position of their known optima, and other relevant properties.

Keywords- *test functions, bird flock, inertia weight, optimum point, optimization, optimization algorithms, optimization problems, Particle swarm optimization, performance, search space, optimum*

1. Introduction

Optimization is the search for a set of variables that can either maximize or minimize a scalar cost function, fx(n). The n-dimensional decision vector(x) consists of the n decision variables over which the decision maker has control. The cost function is multi-value variable since it depends on more than one decision variable. The decision maker desires a more efficient method than trial and error obtains a quality decision vector, so the optimization techniques are employed [1]. Optimization problems are widely encountered in various fields in science and technology. Sometimes such problems can be very complex due to the actual and practical nature of the objective function or the model constraints [1].

An optimization problem is the problem of finding the *best* solution from all the feasible solutions. Optimization problems can be divided into two categories depending on the nature of variables i.e. whether they are continuous or discrete. Some optimization

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problems include: PSO (Particle Swarm Optimization), ACO (Ant Colony Optimization), and ABC (Artificial Bee Colony Optimization). The algorithms used to optimize the minimization or maximization problems are called Optimization Algorithms. Example: Particle Swarm Optimization (PSO) Algorithm, Genetic Algorithm.

2. Particle Swarm Optimization Technique

2.1. Particle Swarm Optimization

Particle swarm optimization (PSO) is a population based stochastic optimization technique [2] proposed by Kennedy and Eberhart [1]. PSO is a heuristic global optimization method, which is based on the swarm intelligence. It is based on research on the moving behaviour of birds and fish flock. PSO algorithm is used efficiently for solving optimization problems. During a search process, each particle has a tendency to move towards a better search area with a velocity, that is dynamically adjusted according to its own and other particles' its behaviour. Various PSO variants have been proposed to improve the performance of PSO for global optimization [4].

2.2. PSO Algorithm

In an n- dimensional space, each particle's position is updated using particle's new velocity according to equations:

$$v(t+1) = (w * v(t)) + (c_1 * r_1 * (p(t) - x(t)) + (c_2 * r_2 * (g(t) - x(t))$$

$$x(t+1) = x(t) + v(t+1)$$

Here, v (t) is the velocity vector and x(t) is the position vector. Comparison in PSO can be done in three ways:

1. pbest- the best solution achieved by the particle so far.

2. **lbest -** the another best value that is tracked by the particle swarm optimizer. It is the best value,obtained so far by any particle in the neighbors of the particle.

3. gbest - is the globally found best value in the swarm.

2.3. Principles of PSO Algorithm

In Particle Swarm Optimization Algorithm, the particles change their condition according to the following three principles:

- *to keep its inertia.*
- *to change condition according to its most optimal position.*
- *to change condition according to the swarm's most optimal position.*

The position of each particle in the swarm is affected both by the most optimal position during its own movement and the position of the most optimal particle in its surrounding [3].

2.4. Analysis of PSO Algorithm

The PSO algorithm is based on the research of the behaviour of movement of birds and fish flock. These creatures need to search their food in order to survive. While searching for food, the birds are either scattered or they go together before they locate the place where they can find the food. While the birds are searching for the food from one place to another, there can be a bird that can smell the food very well. It means the bird has the idea about the place where the food can be found. It may have the better food resource information. The information is conveyed to other members of the flock so that they can come to that place. This process can be applied to particle swam optimization algorithm as:

- *The solution swam is compared to bird swarm.*

- *The birds which are moving from one place to another are equal to the development of the solution swarm.*
- *good information is equal to most optimist solution*
- *the resource of food is the most optimist solution during the whole course.*
- *The most optimist solution can be found out in particle swarm optimization algorithm by the cooperation of each individual* [3].

3. Test Functions

Test functions are used to evaluate characteristics of optimization algorithms, such as: velocity of convergence, precision, robustness, general performance. Whenever a new algorithm is to be evaluated, the test functions are employed to check its reliability, efficiency and validity. Multimodal test functions show some regularity such as: symmetry with respect to one axis, uniform spacing among optima and exponential increase in the number of global optima with respect to the increase in number of decision variables. Such regularities can be exploited by optimization algorithms, decreasing their degree of difficulty [7]. To overcome these regularities and have better testing environments of optimization algorithms, some test function generators have been employed [8]. Some common characteristics of test functions can be noticed to have a better view about test functions.

4. Generating a Test Function

Homogeneous coordinates can be used in an efficient manner to build a test function generator.A general procedure to generate a new test function using linear transformations and function composition as follows:

1. Begin with a point P in the search space.
2. A point Q in homogeneous coordinates is computed using P.
3. The point Q is multiplied by a matrix representing a sequence of linear transformations to obtain Q0.
4. Using Q0, compute P0, which gives us the point P transformed in the search space.
5. The value of the test function fi is obtained passing the point P0 as the argument to the test function, and f(P0) is computed.
6. Before computing the composite function F, other linear transformations can be applied to fi(P0) and obtain a f0i (P0).
7. Finally, we compute the composite function F by adding the fi(P0) values or by computing the max of them.

A suitable combination of linear transformations can break the regularities of a test function.

The test functions are widely used to compare an algorithm’s performance with other algorithms. To successfully implement optimization algorithms, some parameters must be assigned in advance [6].Some commonly used test functions and their properties like: search range, number of optimums, global minima, optimum point, are presented in a tabular form.

5. Evaluating Test Functions for Assessing the Performance of Particle Swarm Optimization

Multimodal problems are those in which the search space has several local optima and possibly more than one global optimum. A type of optimization problem in which the particle swarm optimization (PSO) algorithm has been only scarcely applied, has been proposed [14]. Common multimodal test functions show regularities like:

- *symmetry with respect to one axis,*
- *uniform spacing among optima*

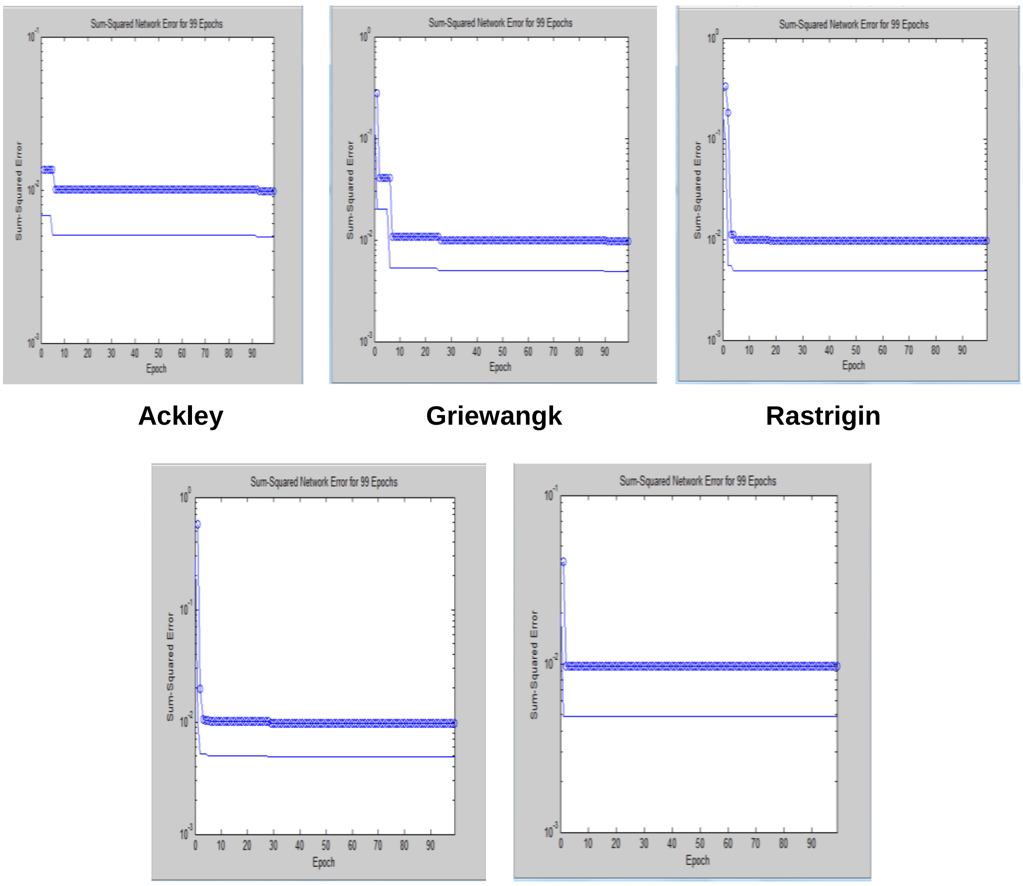
- *exponential increase in the number of global optima with respect to the increase in the number of decision variables*.

Such regularities can be exploited by an optimization algorithm such as PSO, decreasing their degree of difficulty [15]. To overcome these regularities and have a better testing environment of an optimization algorithm, some test functions generators have been developed [17] as well as methodologies to create new test functions by using a composition procedure or through the application of linear transformations on common test functions [18].

As PSO is adopted in many types of application domains, it becomes more important to have well-established methodologies to assess its performance. For that purpose, several test problems have been proposed for assessing the performance of PSO variants. Test problems sometimes have regularities which can be easily exploited by PSO, resulting in an outstanding performance. In order to avoid such regularities, several basic design principles that should be followed when creating a test function generator for single-objective continuous optimization are presented. These test functions can be scaled up to any number of decision variables [13]. To successfully implement PSO algorithm, some of the parameters must be assigned in advance [12].

6. Implementation and Results

The PSO algorithm is implemented on MATLAB (R2011b). The algorithm gives the convergence point at which all the particles of the swarm get accumulated, means the optimal solution is found and the optimum point is achieved.



Ackley Griewangk Rastrigin

Rosenbrock Sphere

Figure 1. PSO with Test Functions

Table 1. Results of Implementing PSO with Test Functions

***Sum- squared network error*- the difference between the solutions of gbest values**
***Epoch*- instant of time**

	Solutions	Optimal Solution	Optimum Point
PSO with Ackley Test function	-0.3253 3.1021 3.1060 -0.3577	3.1060 -0.3577	0.0049
PSO with Griewangk Test function	1.8600 1.7111 2.4333 2.6159 2.6915 6.0814 1.7497 2.5598 2.7451 1.5880 0.8602 3.0072 1.4011 2.8161 2.6248 1.7093 -3.1266 -0.2902	-3.1266 -0.2902	0.0049
PSO with Rastrigin test function	0.2418 2.4453 1.2448 2.2685 1.1245 2.7387 2.2496 2.1439 -1.6927 -2.6277 2.3772 -2.0376	2.3772 -2.0376	0.0049
PSO with Rosenbrock test function	2.0132 0.5546 1.7467 1.4758 2.7884 1.2059 3.0673 0.5203 2.9279 1.1997 2.8929 1.2725 2.5375 1.8767 -0.9611 2.9902 2.9730 1.0022	2.9730 1.0022	0.0049
PSO with Sphere test function	1.6190 2.4799 2.4842 1.7781 2.2334 2.2031 2.9857 0.9701	2.9857 0.9701	0.0049

7. Related Work

1. J. Kennedy and R. C. Eberhart, "Particle swarm optimization," In this paper, a concept for the optimization of nonlinear functions using particle swarm methodology was introduced. The evolution of several paradigms was outlined and applications, including nonlinear function ptimization and neural network training, were proposed. The relationships between the particle swarm optimization and both artificial life and genetic algorithms were described.

2. Wei-Neng Chen , Jun Zhang, Ni Chen, Zhi-Hui Zhan , Henry Shu-Hung Chung , Yun Li , Yu-Hui Shi "Particle Swarm Optimization with an Aging Leader and Challengers" This paper presents the aging mechanism applied to particle swarm optimization (PSO) and proposes a PSO with an aging leader and challengers (ALC-PSO). ALC-PSO is designed to overcome the problem of premature convergence without significantly impairing the fast-converging feature of PSO.

3. Qinghai Bai," Analysis of Particle Swarm Optimization Algorithm " The Particle swarm optimization Algorithm has been analysed thoroughly. The major impairments in the area of convergence has been given. It also presents the advantages and disadvantages of PSO algorithm, giving the new improvements in it to enhance its performance.

4. Julio Barrera and Carlos A.Coello Coello , "Test Function Generators for Assessing the Performance of PSO Algorithms in Multimodal Optimization" As PSO is adopted in more types of application domains, it becomes more important to have well-established methodologies to assess its performance. For that purpose, several test problems have been proposed. Several state-of-the art test function generators that have been used for assessing the performance of PSO variants have been reviewed.

5. Woo Nam Lee and Jong Bae Park ,"Educational Simulator for Particle Swarm Optimization and Economic Dispatch Applications" It presents a simulator for the particle swarm optimization (PSO) and application for solving mathematical test functions. Using this simulator, instructors and students can select the test functions for the simulation and set parameters that have an influence on the PSO performance. Through visualization process of each particle and variation of the value of objective function, the simulator is effective in providing the users an intuitive feel for the PSO algorithm.

8. Conclusion and Future Scope

Test functions are important in testing or evaluating any algorithm. These functions are well-suited to evaluate a new algorithm, by comparing its efficiency with other algorithms and testing its validity using different parameters can be done. The details about the characteristics of these test functions and some features like: search space, global optimum, optimal point, number of optimums etc are presented here. These properties are helpful in differentiating the test functions from each other.The PSO algorithm can be tested for its performance by using several test functions available. Here, five of the test functions are used to present the results which show the performance enhancement of PSO algorithm by using these test functions. These test functions can present the PSO algorithm in a better light. Many other available test functions can be used for testing the performance of the PSO algorithm, in order to have much more results for comparison and to have the improved and better PSO algorithm for solving the optimization problems. These test functions can be applied to check and validate the performance of other optimization problems such as Ant Colony Optimization (ACO), Genetic Algorithms (GA), Artificial Bee Colony Optimization (ABC) and others, as well.

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