

PN Code Acquisition in Multiplicative and Weakly Dependent Non-Gaussian Noises based on Locally Optimum Detection Statistic

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Abstract

In most previously reported studies on pseudonoise (PN) code acquisition, independent additive noises have been assumed in various noise environments. The use of an independent observation model may cause considerable performance degradation in modern high data-rate communication systems. In these studies, only additive noise model is considered. In this paper, a new detector for PN code acquisition in multiplicative and weakly dependent non-Gaussian noises is proposed. Modeling the acquisition problem as a hypothesis testing problem, the test statistical is derived for multiplicative and weakly dependent non-Gaussian noises, based on the locally optimum detection technique. Numerical results show that the proposed detector can offer substantial performance improvement over the conventional schemes in multiplicative and weakly dependent non-Gaussian noises.

Keywords: *direct sequence spread spectrum; locally optimum detection; multiplicative noise; weakly dependent non-Gaussian noises; code acquisition*

1. Introduction

Rapid establishment of code acquisition is an important technical issue in direct-sequence spread-spectrum (DS-SS) systems. The basic unit in an acquisition system is a decision-making device or a detector. Most schemes proposed for rapid code acquisition employ the squared-sum (SS) detector with a noncoherent in-phase/quadrature-phase (I-Q) correlator [1]. This is because the SS detector is optimized for Gaussian noise channels, and the statistics due to fading in acquisition systems can usually be modeled as Gaussian processes by virtue of the central limit theorem. Some schemes are proposed for code acquisition systems in applications with non-Gaussian noises [2-3], such as atmospheric and man-made noise arising in indoor/outdoor mobile communication systems. These acquisition schemes are also proposed for the additive noise model, in which noise is added to a signal to generate observations. In some different types of situations, such as the multipath or reverberation phenomena and the actions of automatic gain control circuits or of nonlinearities acting on additive signal and noise components, the multiplicative noise models have to be considered [4-5].

In a number of the studies [4-8], the signal detection in multiplicative noise has been explored. But those researches are limited only in one-dimensional signal detection. And in these studies, independent noises have been assumed in various noise environments, however, is frequently violated, especially in modern discrete-time signal-detection applications, where the noise components contained in the sampled data tend to have dependence due to a very high sampling rate. As a consequence, a detector optimized for independent noise is often not guaranteed to be optimum in practical signal detection systems, which becomes more critical as the sampling rate increases.

In this paper, we focus on the code acquisition in multiplicative and weakly dependent non-Gaussian noise environments. In particular, we consider the locally optimum detection of DS/SS signal based on the generalized version of the Neyman-Pearson fundamental lemma

[9-11] of statistical hypothesis testing. Locally optimum (LO) detectors are optimum in detecting weak signals, since an LO detector has the maximum slope of its power function when the signal-to-noise ratio (SNR) approaches zero [12-13]. Although much effort has been devoted to the code acquisition using LO detectors in non-Gaussian or Gaussian noise environments [12-14], only the independent and additive noise model has been considered. In the rest of this paper, we will employ locally optimum (LO) detectors for code acquisition systems in multiplicative and weakly dependent non-Gaussian noises.

The remainder of this paper is organized as follows. In section 2 the observation model of the received signal and the hypothesis testing model in multiplicative and weakly dependent non-Gaussian noises are given. Then the LO detector test statistic is derived in section 3. In section 4, based on the new acquisition scheme, the numerical results of detection probability are given.

2. Observation Model

Considering the multiplicative and weakly dependent non-Gaussian noises model for a DS-SS system, the received signal can be expressed as

$$r(t) = s(t) + s(t)n(t) + w(t) \quad (1)$$

where $s(t)$ is the primary direct-path signal; $n(t)$ is the additional multiplicative noise which may be characterizing the fading amplitude characteristics of secondary propagation paths; $w(t)$ is the weakly dependent non-Gaussian noise, $n(t)$ is a random variables independent of $s(t)$ but generally correlated with $w(t)$. And $s(t)$ can be expressed as [1]

$$s(t) = \sqrt{2E}d(t - \tau T_c)c(t - \tau T_c)\cos(\omega_c t + \phi) \quad (2)$$

In equation (2), E is the energy per chip; $d(t)$ is the data sequence waveform, and in this paper it is assumed that there is a preamble for acquisition so that no data modulation is present during acquisition (i.e., $d(t) = 1$); T_c is the chip duration; τ is the time delay normalized by T_c ; $c(t) = \sum_{-\infty}^{\infty} c_i p_{T_c}(t - iT_c)$ where $c_i \in \{-1, +1\}$ is the i th chip of a pseudonoise (PN) code sequence of period L and $p_{T_c}(t)$ is the PN code waveform defined as a unit rectangular pulse over $[0, T_c]$; ω_c is the carrier angular frequency; ϕ is the phase distributed uniformly over $[0, 2\pi)$.

A typical structure of the noncoherent I-Q correlators receiver is shown in Fig.1. We consider the serial search scheme with single dwell [1]. The i th sampled I-Q components X_i^I and X_i^Q can be obtained as, for $i = 1, 2, \dots, M$

$$X_i^I = \int_{t_i - T_c}^{t_i} r(t)c(t - \hat{\tau}T_c)\sqrt{2}\cos(\omega_c t)dt \quad (3)$$

$$X_i^Q = \int_{t_i - T_c}^{t_i} r(t)c(t - \hat{\tau}T_c)\sqrt{2}\sin(\omega_c t)dt \quad (4)$$

respectively, where M is the correlation length, $\hat{\tau}$ is the time delay (normalized by the chip duration T_c) of the locally generated PN code, and $t_i = t_0 + iT_c$. Here, t_0 is an initial time. A test statistic is evaluated with the vectors $\{X_i^I\}_{i=1}^M$ and $\{X_i^Q\}_{i=1}^M$, and then compared with a threshold. In the conventional systems, the test statistic is, for example, the SS statistic $T_{SS}(X^I, X^Q) = (\sum_{i=1}^M X_i^I)^2 + (\sum_{i=1}^M X_i^Q)^2$.

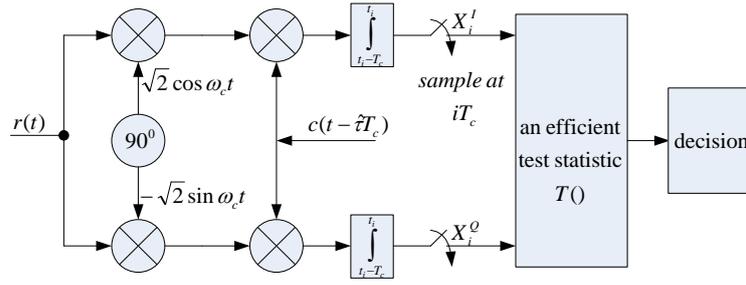


Figure 1. Structure of PN Code Acquisition System

The PN code acquisition problem can be regarded as a hypothesis testing problem: given $\{X_i^I\}_{i=1}^M$ and $\{X_i^Q\}_{i=1}^M$, a decision is to be made between the null hypothesis H_0 and alternative hypothesis H_1 , where $H_0: |\tau - \hat{\tau}| \geq 1$ and $H_1: |\tau - \hat{\tau}| < 1$. Under H_1 , each sampled correlation value between the locally generated and received PN codes is $\sqrt{E}(1 - |\delta|)$, where δ is the residual shift (normalized by T_c) between the two PN codes, with the value ranging in the interval $(-1, +1)$. For simplicity, we assume that the system is chip synchronous (that is, $\delta = 0$) as in most other studies (an investigation of the effect of a nonzero δ has been considered in [15]). Thus, each sampled correlation value is \sqrt{E} . On the other hand, each sampled correlation value is $+1$ or -1 with equal probability, and the mean value of the sampled correlation is 0 under H_0 . From these results and equation (3) and (4), H_0 and H_1 can be alternatively expressed as

$$H_0: (X_i^I = W_i^I, X_i^Q = W_i^Q), \quad i = 1, 2, \dots, M \quad (5)$$

$$H_1: (X_i^I = \theta \cos \varphi + \theta \cos \varphi N_i^I + W_i^I, X_i^Q = \theta \cos \varphi + \theta \cos \varphi N_i^Q + W_i^Q), \quad i = 1, 2, \dots, M \quad (6)$$

Or simply described as

$$H_0: \theta = 0 \quad (7)$$

$$H_1: \theta > 0 \quad (8)$$

In equations (5)-(8), $\theta = \sqrt{E}$ is the signal strength parameter, $\{N_i^b\}_{i=1}^M$ and $\{W_i^b\}_{i=1}^M$ with $b = \{I, Q\}$ are the I-Q multiplicative noise and additive noise respectively. Let f_{N^I} , f_{N^Q} , f_{W^I} and f_{W^Q} be the common probability density function (PDF) of the I-Q multiplicative noise and additive noise, and $f_{N^I W^I}$ and $f_{N^Q W^Q}$ denote the common joint PDF of (N^I, W^I) and (N^Q, W^Q) respectively, which are i.i.d. random variables. Finally, $f_{(N^I W^I, N^Q W^Q)}$ and f_W denote the joint PDF of observation noise of I-Q components under H_1 , and H_0 respectively.

Assuming that the degree of dependence among noise components is weak, the dependent-noise components $\{W_i^I\}_{i=1}^M$ and $\{W_i^Q\}_{i=1}^M$ in this paper are modeled by the first-order moving average (FOMA) of i.i.d. random variables, which has been proved to be a simple and good approximation to the weakly dependent noise environment [16-17]. Specifically, let us assume that

$$W_i^I = \Lambda_i^I + \rho \Lambda_{i-1}^I \quad (9)$$

$$W_i^Q = \Lambda_i^Q + \rho \Lambda_{i-1}^Q \quad (10)$$

where ρ is called the dependence parameter determining the correlation coefficient of W_i^I and W_i^Q especially, and $\{\Lambda_i^I\}_{i=1}^M$ and $\{\Lambda_i^Q\}_{i=1}^M$ are i.i.d. random process with $\Lambda_0^I = \Lambda_0^Q = 0$. In this paper, We model Λ^I and Λ^Q with symmetric α -stable (S α S) distributions, which have been proved to be very useful in modeling non-Gaussian noise [18]. The joint probability density function of Λ_i^I and Λ_i^Q can be most conveniently defined by the inverse Fourier transform (IFT) of their characteristic function [18]

$$f_{\alpha,\gamma,\beta_1,\beta_2}(x_1,x_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[i(\beta_1\omega_1 + \beta_2\omega_2) - \gamma(\omega_1^2 + \omega_2^2)^{\frac{\alpha}{2}}] e^{-i(x_1\omega_1 + x_2\omega_2)} d\omega_1 d\omega_2 \quad (11)$$

where the parameter α and γ are termed the characteristic exponent and dispersion, respectively, and β_1 and β_2 are location parameters. The characteristic exponent α ranges in the interval $0 < \alpha \leq 2$ with a smaller α indication heavier tails (more impulsive behavior). The dispersion γ is a positive constant relation to the spread of the PDF. The location parameters β_1 and β_2 are the symmetry axes of the marginal S α S distributions: we assume $\beta_1 = \beta_2 = 0$, without loss of generality. Unfortunately, no closed-form expression exists for (11), except for the special cases of $\alpha = 1$ and $\alpha = 2$

$$f_{\alpha,\gamma}(x_1,x_2) = \begin{cases} \frac{\gamma}{2\pi(x_1^2 + x_2^2 + \gamma^2)^{\frac{3}{2}}} & \text{for } \alpha = 1 \\ \frac{1}{4\pi\gamma} \exp(-\frac{x_1^2 + x_2^2}{4\gamma}) & \text{for } \alpha = 2 \end{cases} \quad (12)$$

Because of such a lack of closed-form expressions, we concentrate on the case $\alpha = 1$. Nonetheless, we shall see in Section 4 that the system obtained for $\alpha = 1$ is not only robust to the variation of α , but also outperforms the conventional system for most values of α .

Assuming that the samples $\{X_i^I, X_i^Q\}$ of the bivariate noise process form a sequence of independent random vectors for a given ϕ , these joint PDF of $2M$ sampled in-phase and quadrature observations $\{X_i^I, X_i^Q\}$, $i = 1, 2, \dots, M$ is

$$\begin{aligned} f_{X^I, X^Q}(X^I, X^Q) &= E_{\phi} \left\{ \int f_{(N^I W^I, N^Q W^Q)} dN \right\} \\ &= E_{\phi} \left\{ \prod_{i=1}^M \iint f_{(n_i^I, y_i^I(\theta)), (n_i^Q, y_i^Q(\theta))} dn_i^I dn_i^Q \right\} \end{aligned} \quad (13)$$

Where E_{ϕ} denotes the expectation over ϕ , and y_i^I and y_i^Q can be expressed as

$$y_i^I(\theta) = x_i^I - \theta \cos \phi - n_i^I \theta \cos \phi \quad (14)$$

$$y_i^Q(\theta) = x_i^Q - \theta \cos \phi - n_i^Q \theta \cos \phi \quad (15)$$

Substituting equations (9) and (10) into equation (13), the joint PDF of $2M$ sampled in-phase and quadrature observations $\{X_i^I, X_i^Q\}$ can be rewritten as follow

$$\begin{aligned}
 f_{X^I, X^Q}(x^I, x^Q) &= E_\phi \left\{ \prod_{i=1}^M \iint f_{(n_i^I, y_i^I(\theta)), (n_i^Q, y_i^Q(\theta))} dn_i^I dn_i^Q \right\} \\
 &= E_\phi \left\{ \iint f_{(n_1^I, y_1^I(\theta)), (n_1^Q, y_1^Q(\theta))} dn_1^I dn_1^Q \times \iint f_{(n_2^I, y_2^I(\theta) - \rho y_1^I(\theta)), (n_2^Q, y_2^Q(\theta) - \rho y_1^Q(\theta))} dn_2^I dn_2^Q \right. \\
 &\quad \times \iint f_{(n_3^I, y_3^I(\theta) - \rho y_2^I(\theta) + \rho^2 y_1^I(\theta)), (n_3^Q, y_3^Q(\theta) - \rho y_2^Q(\theta) + \rho^2 y_1^Q(\theta))} dn_3^I dn_3^Q \times \dots \times \\
 &\quad \left. \iint f_{(n_M^I, y_M^I(\theta) - \rho y_{M-1}^I(\theta) + \dots + (-\rho)^{M-1} y_1^I(\theta)), (n_M^Q, y_M^Q(\theta) - \rho y_{M-1}^Q(\theta) + \dots + (-\rho)^{M-1} y_1^Q(\theta))} dn_M^I dn_M^Q \right\} \\
 &= E_\phi \left\{ \prod_{i=1}^M \iint f_{(n_i^I, Y_i^I - C_i \theta \cos \phi - U_i^I \theta \cos \phi), (n_i^Q, Y_i^Q - C_i \theta \sin \phi - U_i^Q \theta \sin \phi)} dn_i^I dn_i^Q \right\}
 \end{aligned} \tag{16}$$

where $C_i = \sum_{j=0}^{i-1} (-\rho)^j$, $U_i^I = -\sum_{j=0}^{i-1} (-\rho)^j n_{i-j}^b$ and $Y_i^b = \sum_{j=0}^{i-1} (-\rho)^j X_{i-j}^b$ for $b = \{I, Q\}$. Then the null hypothesis H_0 and alternative hypothesis H_1 can be transformed as for $i = 1, 2, \dots, M$

$$H_0 : (Y_i^I = \Lambda_i^I, Y_i^Q = \Lambda_i^Q) \tag{17}$$

$$H_1 : (Y_i^I = \theta \cos \phi C_i + U_i^I \theta \cos \phi + \Lambda_i^I, Y_i^Q = \theta \sin \phi C_i + U_i^Q \theta \sin \phi + \Lambda_i^Q) \tag{18}$$

And the joint PDF of $2M$ sampled in-phase and quadrature observations $\{Y_i^I, Y_i^Q\}_{i=1}^M$ is

$$\begin{aligned}
 f_{Y^I, Y^Q}(Y^I, Y^Q) &= E_\phi \left\{ \int f_{(U^I \Lambda^I, U^Q \Lambda^Q)} dU \right\} \\
 &= E_\phi \left\{ \prod_{i=1}^M \iint f_{(U_i^I, V_i^I(\theta)), (U_i^Q, V_i^Q(\theta))} dU_i^I dU_i^Q \right\}
 \end{aligned} \tag{19}$$

Where

$$V_i^I(\theta) = Y_i^I - \theta \cos \phi C_i - U_i^I \theta \cos \phi \tag{20}$$

$$V_i^Q(\theta) = Y_i^Q - \theta \sin \phi C_i - U_i^Q \theta \sin \phi \tag{21}$$

3. LO Detector Test Statistic

The test statistic of an LO detector is obtained as [6, 12]

$$T_{LO}(Y^I, Y^Q) = \frac{1}{f_{Y^I, Y^Q}(Y^I, Y^Q)|_{\theta=0}} \times \frac{d^v f_{Y^I, Y^Q}(Y^I, Y^Q)}{d\theta^v} \Big|_{\theta=0} \tag{22}$$

where v is the order of the first nonzero derivative of $f_{Y^I, Y^Q}(Y^I, Y^Q)$ at $\theta = 0$. As shown in the appendix, the test statistic of the LO detector is

$$\begin{aligned}
 T_{P-LO}(X^I, X^Q) &= T_{LO}(Y^I, Y^Q) \Big|_{Y_i^b = \sum_{j=0}^{i-1} (-\rho)^j X_{i-j}^b} \\
 &= \sum_{i=1}^M \{h(Y_i^I) + h(Y_i^Q)\} + \sum_{i=1}^M \sum_{j \neq i, j=1}^M \{g(Y_i^I)g(Y_j^I) + g(Y_i^Q)g(Y_j^Q)\}
 \end{aligned} \tag{23}$$

where

$$h(Y_i^b) = \frac{1}{f_\Lambda(Y_i^I, Y_i^Q)} \times \frac{\partial^2 (f_\Lambda(Y_i^I, Y_i^Q) E\{(C_i + U^b)^2 | \Lambda = Y_i^I, Y_i^Q\})}{\partial(Y_i^b)^2} \tag{24}$$

$$g(Y_i^b) = \frac{1}{f_\Lambda(Y_i^I, Y_i^Q)} \times \frac{\partial (f_\Lambda(Y_i^I, Y_i^Q) E\{(C_i + U^b) | \Lambda = Y_i^I, Y_i^Q\})}{\partial Y_i^b} \tag{25}$$

for $b = \{I, Q\}$.

Compared with the test statistic of the LO detector in the additive and independent noise environments [13], it is noteworthy that the multiplicative noise introduces the extra terms $E\{(C_i + U^b)^2 | \Lambda = Y_i^I, Y_i^Q\}$ and $E\{(C_i + U^b) | \Lambda = Y_i^I, Y_i^Q\}$ for each $h(Y_i^b)$ and $g(Y_i^b)$ in the test statistic, and the dependent noises introduces the new variables Y_i^b, U_i^b and C_i . If there is no multiplicative noise ($N = 0$) and the noise is independent, the proposed LO detector has the same test statistic with the LO detector work in non-Gaussian channels which researched in [13]. Since the FOMA noise model is a simple and good approximation to the weakly dependent noise environment, we can ignore the high-order terms of $\rho (\{\rho^i | i = 2, 3 \dots M\})$ to get the simpler-LO detector. And then the test statistic of simpler-LO detector can be obtained

$$T_{SLO}(X^I, X^Q) = T_{LO}(Y^I, Y^Q) \Big|_{Y_i^b = X_i^b - \rho X_{i-1}^b, C_i = 1 - \rho u(i-2)} \quad i = (1, 2 \dots M) \quad (26)$$

In equation (26) $X_0^b = 0$ and $b = \{I, Q\}$, and $u(i)$ is the unit step sequence, i.e., $u(i) = 0$ when $i < 0$ and $u(i) = 1$ when $i \geq 0$.

4. Simulation Results

In this section, the performance of the proposed simpler-LO (PS-LO) detector and conventional SS detector and the Non-Gaussian Additive and Independent LO (NAI-LO) detector [13] is simulated with a PN code of $L = 1023$ chips, generated from an m sequence with the primitive polynomial $1 + z^3 + z^{10}$ when $M = 50$. Because it is difficult to get explicit closed-form formulations for finite sample-size performance, we resorted to Monte Carlo simulations to show the results.

In our simulations, we will consider a specific example to illustrate the results, we assume $N^b = rS W^b$ for $b = \{I, Q\}$. r is the correlation coefficient between N^b and W^b , and N^I, N^Q have same variance $\sigma_{N^I}^2 = \sigma_{N^Q}^2 = s^2$. Noise samples $\{\Lambda_i^I\}$ and $\{\Lambda_i^Q\}$ are generated from [19]

$$\gamma^\alpha \frac{\sin(\alpha A)}{(\cos A)^\alpha} \frac{1}{\frac{1}{B}} \left(\frac{\cos[(1-\alpha)A]}{B} \right)^{\frac{1-\alpha}{\alpha}} \quad (27)$$

where A is uniform on $(-\pi/2, \pi/2)$ and B is exponential with mean 1. Since $S\alpha S$ noise with $\alpha < 2$ has no finite variance, making the standard SNR measure inconsistent, a new scale parameter is used to indicate the strength of the $S\alpha S$ noise. The new SNR, which provides a mathematically and conceptually valid characterization of the relative strength between the information-bearing signal and channel noise with infinite variance, is defined as

$$\frac{1}{2C_g(1+\rho^2)} \left(\frac{\sqrt{E}}{S_0} \right)^2 \quad (28)$$

where $S_0 = (C_g \gamma)^{1/\alpha} / C_g$ with $C_g \approx 1.78$ the exponential of the Euler's constant $(\lim_{n \rightarrow \infty} (\sum_{k=1}^n (1/k) - \ln n))$. The normalizing constant $2C_g(1+\rho^2)$ ensures that the definition of the SNR coincides with that of the standard SNR in the independent Gaussian case. Because γ can be easily and exactly estimated using only the sample mean and variance of

observations X^I or X^Q [20], it may be regarded as a known value: in our simulations, we assume $\gamma = 1$.

Each point in Figure 2, 3, 4 and 5 was obtained from 100 000 runs to make the relative error less than 1% for a false-alarm probability $P_{fa} = 10^{-2}$. Figure 2 shows the detection probabilities of conventional SS detector, the NAI-LO detector and the proposed Simpler-LO detector for some values of rs and dependent parameter ρ in multiplicative and weakly dependent non-Gaussian noises for $\alpha = 1$. Figure 3 shows the detection probabilities plotted as a function of ρ for some values of SNR/chip and rs in multiplicative and weakly dependent non-Gaussian noises for $\alpha = 1$. Figure 4 shows the detection probabilities plotted as a function of rs for some values of SNR/chip and ρ in multiplicative and weakly dependent non-Gaussian noises for $\alpha = 1$. From these plots, it is observed that the proposed detector significantly outperforms the conventional SS detector and the NAI-LO detector in multiplicative and weakly dependent non-Gaussian noises.

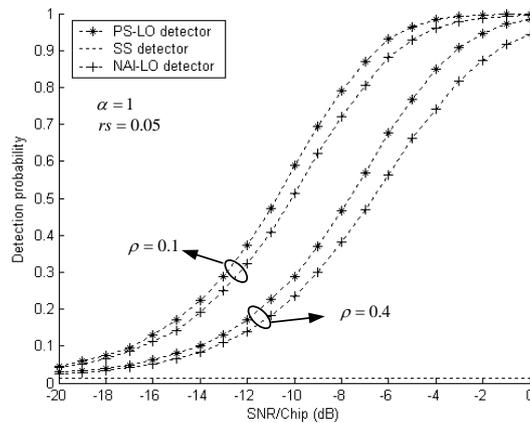


Figure 2. Detection Probabilities of the Conventional SS Detector, the NAI-LO Detector and the Proposed Schemes for Different rs and ρ when $P_{fa} = 10^{-2}$ and $M = 50$ in Multiplicative and Weakly Dependent Non-Gaussian Noises

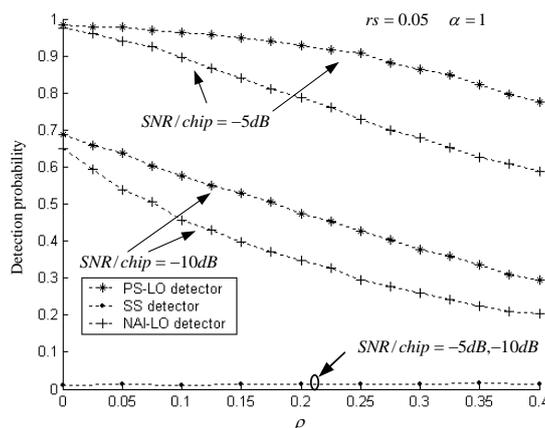


Figure 3. Detection Probabilities Plotted as a Function of ρ for some Values of SNR/chip and rs when $P_{fa} = 10^{-2}$ and $M = 50$ in Multiplicative and Weakly Dependent Non-Gaussian Noises for $\alpha = 1$

Figure 5 shows the detection probabilities of the detectors as a function of α for some values of ρ , rs and SNR/chip. We can clearly see that the proposed simpler-LO detector significantly outperforms the conventional SS detector for most values of α , and performs better than the NAI-LO detector for all values of α . Only when α is close to two, the conventional SS detector performs slightly better. Another important observation is that the performance of proposed simpler-LO detector are robust to the variation of the value of α . In addition, the proposed detector performs better as the impulsiveness becomes higher. This can be explained as follows, Impulses which have large amplitudes are clipped in the proposed detector. Therefore, the effective noise variance at the output of the proposed detector is smaller than the total input noise variance.

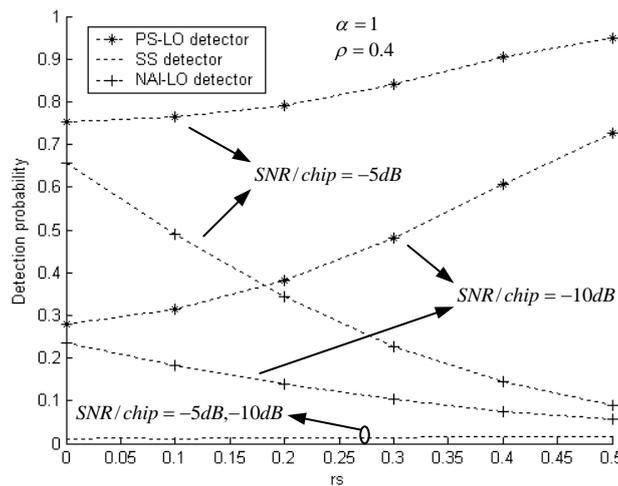


Figure 4. Detection Probabilities Plotted as a Function of rs for Some Values of SNR/chip and ρ when $P_{fa} = 10^{-2}$ and $M = 50$ in Multiplicative and Weakly Dependent Non-Gaussian Noises for $\alpha = 1$

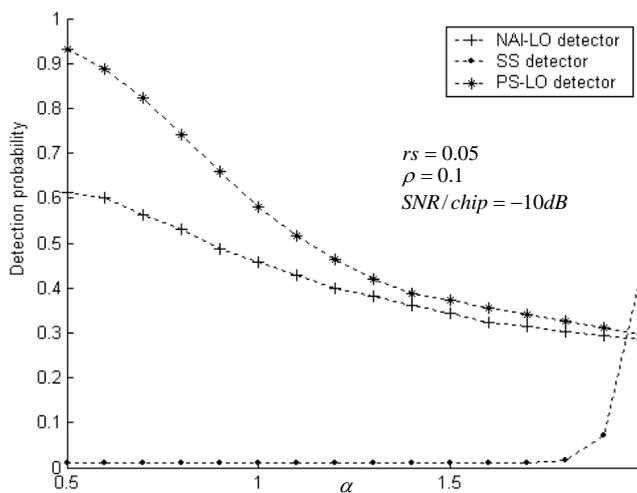


Figure 5. Detection Probabilities of the Detectors as a Function of α for Some Values of ρ , rs and SNR/chip when $P_{fa} = 10^{-2}$ and $M = 50$

5. Conclusions

In this paper, a new detector for DS-SS code acquisition in the environments with multiplicative and weakly dependent non-Gaussian noises has been proposed. Using the locally optimum test statistic, a decision rule of code acquisition has been derived and a new acquisition system has been proposed based on the decision rule. The performance of the proposed scheme has been compared with that of the conventional schemes. From the simulation results, it has been observed that the proposed scheme significantly outperforms the conventional SS detector and the NAI-LO detector in multiplicative and weakly dependent non-Gaussian noises.

Appendix

Derivation of the LO detector test statistic

We have defined

$$V_i^I(\theta) = Y_i^I - \theta \cos \varphi C_i - U_i^I \theta \cos \varphi \quad (29)$$

$$V_i^Q(\theta) = Y_i^Q - \theta \sin \varphi C_i - U_i^Q \theta \sin \varphi \quad (30)$$

so we can get

$$V_i^b(\theta)|_{\theta=0} = V_i^b(0) = Y_i^b \quad (31)$$

for $b = \{I, Q\}$.

We also define

$$f_i(\theta) = f_{(U_i^I, V_i^I(\theta)), (U_i^Q, V_i^Q(\theta))}$$

$$f_i(0) = f_{(U_i^I, V_i^I(0)), (U_i^Q, V_i^Q(0))}$$

$$f_i'(\theta) = f'_{(U_i^I, V_i^I(\theta)), (U_i^Q, V_i^Q(\theta))}$$

and $f_i''(\theta) = f''_{(U_i^I, V_i^I(\theta)), (U_i^Q, V_i^Q(\theta))}$.

The first derivative of $f_{Y^I, Y^Q}(Y^I, Y^Q)$ at $\theta = 0$ is

$$\frac{df_{Y^I, Y^Q}(Y^I, Y^Q)}{d\theta} \Big|_{\theta=0} = E_{\varphi} \left\{ \sum_{i=1}^M \iint f_i'(\theta) dU_i^I dU_i^Q \prod_{j \neq i, j=1}^M \iint f_j(\theta) dU_j^I dU_j^Q \right\} \quad (32)$$

where

$$\frac{df_i(\theta)}{d\theta} = \frac{\partial f_i(\theta)}{\partial V_i^I(\theta)} \times \frac{dV_i^I(\theta)}{d\theta} + \frac{\partial f_i(\theta)}{\partial V_i^Q(\theta)} \times \frac{dV_i^Q(\theta)}{d\theta} \quad (33)$$

where

$$\frac{dV_i^I(\theta)}{d\theta} = \frac{d(Y_i^I - C_i \theta \cos \varphi - U_i^I \theta \cos \varphi)}{d\theta} = -C_i \cos \varphi - U_i^I \cos \varphi \quad (34)$$

$$\frac{dV_i^Q(\theta)}{d\theta} = \frac{d(Y_i^Q - C_i \theta \sin \varphi - U_i^Q \theta \sin \varphi)}{d\theta} = -C_i \sin \varphi - U_i^Q \sin \varphi \quad (35)$$

So

$$\begin{aligned} \frac{df_i(\theta)}{d\theta} \Big|_{\theta=0} &= \{-(C_i + U_i^I) \cos \varphi \times \frac{\partial f_i(\theta)}{\partial V_i^I(\theta)} - (C_i + U_i^Q) \sin \varphi \times \frac{\partial f_i(\theta)}{\partial V_i^Q(\theta)}\} \Big|_{\theta=0} \\ &= -(C_i + U_i^I) \cos \varphi \times \frac{\partial f_i(0)}{\partial Y_i^I} - (C_i + U_i^Q) \sin \varphi \times \frac{\partial f_i(0)}{\partial Y_i^Q} \end{aligned} \quad (36)$$

and

$$\begin{aligned} E_{\varphi} \{f_i'(\theta)\} &= E_{\varphi} \{-(C_i + U_i^I) \cos \varphi \times \frac{\partial f_i(0)}{\partial Y_i^I} - (C_i + U_i^Q) \sin \varphi \times \frac{\partial f_i(0)}{\partial Y_i^Q}\} \\ &= 0 \end{aligned} \quad (37)$$

So the first derivative of $f_{Y^I, Y^Q}(Y^I, Y^Q)$ at $\theta=0$ is vanishes. We thus obtain the second derivative of $f_{Y^I, Y^Q}(Y^I, Y^Q)$ at $\theta=0$

$$\begin{aligned} \frac{d^2 f_{Y^I, Y^Q}(Y^I, Y^Q)}{d\theta^2} \Big|_{\theta=0} &= E_\phi \left\{ \sum_{i=1}^M \left[\int \int f_i''(0) dU_i^I dU_i^Q \prod_{j \neq i, j=1}^M \int \int f_j(0) dU_j^I dU_j^Q \right. \right. \\ &\left. \left. + \int \int f_i'(0) dU_i^I dU_i^Q \sum_{j=1, j \neq i}^M \int \int f_j'(0) dU_j^I dU_j^Q \prod_{k=1, k \neq i, j}^M \int \int f_k(0) dU_k^I dU_k^Q \right] \right\} \end{aligned} \quad (38)$$

where

$$\begin{aligned} \frac{d^2 f_i(\theta)}{d\theta^2} &= (C_i + U_i^I)^2 \cos^2 \phi \times \frac{\partial^2 f_i(\theta)}{\partial (V_i^I(\theta))^2} + (C_i + U_i^Q)^2 \sin^2 \phi \frac{\partial^2 f_i(\theta)}{\partial (V_i^Q(\theta))^2} + \\ &(C_i + U_i^I)(C_i + U_i^Q) \sin \phi \cos \phi \frac{\partial^2 f_i(\theta)}{\partial (V_i^I(\theta)) \partial (V_i^Q(\theta))} + (C_i + U_i^I)(C_i + U_i^Q) \sin \phi \cos \phi \frac{\partial^2 f_i(\theta)}{\partial (V_i^Q(\theta)) \partial (V_i^I(\theta))} \end{aligned} \quad (39)$$

the first term of (38) is

$$E_\phi \left\{ \sum_{i=1}^M \int \int f_i''(0) dU_i^I dU_i^Q \prod_{j \neq i, j=1}^M \int \int f_j(0) dU_j^I dU_j^Q \right\} = \frac{1}{2} \sum_{i=1}^M \{h(Y^I) + h(Y^Q)\} \prod_{j=1}^M \int \int f_j(0) dU_j^I dU_j^Q \quad (40)$$

where

$$\begin{aligned} h(Y^b) &= \left\{ \int \int (C_i + U_i^b)^2 \frac{\partial^2 f_i(\theta)}{\partial (V_i^b(\theta))^2} \times \frac{1}{\int \int f_i(\theta) dU_i^I dU_i^Q} dU_i^I dU_i^Q \right\} \Big|_{\theta=0} \\ &= \left\{ \frac{1}{\int \int f_i(\theta) dU_i^I dU_i^Q} \times \frac{\partial^2 (\int \int (C_i + U_i^b)^2 f_i(\theta) dU_i^I dU_i^Q)}{\partial (V_i^b(\theta))^2} \right\} \Big|_{\theta=0} \\ &= \frac{1}{f_\Lambda(Y^I, Y^Q)} \times \frac{\partial^2 (f_\Lambda(Y^I, Y^Q) E\{(C_i + U^b)^2 | \Lambda = Y^I, Y^Q\})}{\partial (Y^b)^2} \end{aligned} \quad (41)$$

for $b = \{I, Q\}$.

Similarly, noting that

$$\begin{aligned} f_i'(\theta) f_j'(\theta) &= \cos^2 \phi (C_i + U_i^I)(C_j + U_j^I) \frac{\partial f_i(\theta) \partial f_j(\theta)}{\partial V_i^I(\theta) \partial V_j^I(\theta)} + \sin^2 \phi (C_i + U_i^Q)(C_j + U_j^Q) \frac{\partial f_i(\theta) \partial f_j(\theta)}{\partial V_i^Q(\theta) \partial V_j^Q(\theta)} \\ &+ \sin \phi \cos \phi (C_i + U_i^I)(C_j + U_j^Q) \left(\frac{\partial f_i(\theta) \partial f_j(\theta)}{\partial V_i^I(\theta) \partial V_j^Q(\theta)} + \frac{\partial f_i(\theta) \partial f_j(\theta)}{\partial V_i^Q(\theta) \partial V_j^I(\theta)} \right) \end{aligned} \quad (42)$$

the second term of (38) is

$$\begin{aligned} E_\phi \left\{ \sum_{i=1}^M \sum_{j=1, j \neq i}^M \int \int f_i'(0) dU_i^I dU_i^Q \int \int f_j'(0) dU_j^I dU_j^Q \prod_{k=1, k \neq i, j}^M \int \int f_k(0) dU_k^I dU_k^Q \right\} \\ = \frac{1}{2} \sum_{i=1}^M \sum_{j=1, j \neq i}^M \{g(Y^I) g(Y^I) + g(Y^Q) g(Y^Q)\} \times \prod_{k=1}^M \int \int f_k(0) dU_k^I dU_k^Q \end{aligned} \quad (43)$$

where

$$\begin{aligned} g(Y^b) &= \left\{ \int \int (C_i + U_i^b) \frac{\partial f_i(\theta)}{\partial V_i^b(\theta)} \times \frac{1}{\int \int f_i(\theta) dU_i^I dU_i^Q} dU_i^I dU_i^Q \right\} \Big|_{\theta=0} \\ &= \left\{ \frac{1}{\int \int f_i(\theta) dU_i^I dU_i^Q} \times \frac{\partial (\int \int (1 + U_i^b) f_i(\theta) dU_i^I dU_i^Q)}{\partial V_i^b(\theta)} \right\} \Big|_{\theta=0} \\ &= \frac{1}{f_\Lambda(Y^I, Y^Q)} \times \frac{\partial (f_\Lambda(Y^I, Y^Q) E\{(C_i + U^b)\} | \Lambda = Y^I, Y^Q)}{\partial Y^b} \end{aligned} \quad (44)$$

for $b = \{I, Q\}$.

Using (40) and (43) in (38), and then dividing the result by $f_{Y^I, Y^Q}(Y^I, Y^Q)|_{\theta=0} = \prod_{k=1}^M \iint f_k(0) dU_k^I dU_k^Q$, we can get (23).

Acknowledgements

This work was supported by National Natural Science Foundation of China (Grant No. 61102107, 61374208), the Fundamental Research Funds for the Central Universities (HEUCFX41310).

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