International Journal of Hybrid Information Technology
Vol.8, No.3 (2015), pp.287-296
http://dx.doi.org/10.14257/ijhit.2015.8.3.25

A Group Decision Making Method Based on Dempster-Shafer Fuzzy Soft Sets Under Incomplete Information

Yuanxiang Dong[a] and Zhi Xiao[a,b]

[a]School of Economics and Business Administration, Chongqing University, Chongqing 400044, PR China
[b]Chongqing Key Laboratory of Logistics, Chongqing University, Chongqing 400044, PR China
xiaozhicqu@163.com

Abstract

Soft set theory, proposed by Molodtsov, has been regarded as a generic mathematical tool for dealing with uncertainty. Recently, researches of decision making based on soft sets have got some progress, but few people consider both incomplete information and group decision making. This paper introduces the concept of Dempster-Shafer fuzzy soft sets combined Dempster-Shafer theory and fuzzy soft sets. We study the FUSE operation on both Dempster-Shafer fuzzy soft sets, and the relationship between incomplete fuzzy soft sets and D–S fuzzy soft sets. At last, we present a new method of evaluation based on Dempster-Shafer fuzzy soft sets and apply it into the information systems quality evaluating to illuminate the practicability and validity.

Keywords: *Incomplete fuzzy soft sets, Dempster–Shafer theory, unknown information, Dempster-Shafer fuzzy soft sets, group decision making*

1. Introduction

Human activities and natural phenomena are full of uncertainty, including subjective and objective uncertainty. Soft set theory [1] is a newly emerging mathematical tool to deal with uncertain problems. It is free from the inherent limitations of inadequate parameterization tool in classical methods, such as theory of probability, fuzzy set [2], vague sets [3], theory of interval mathematics [4], and rough set theory [5]. An important superiority of soft sets is describing the set of initial objects as a mathematical tool. In classical mathematics, a mathematical model is constructed and the notion of its solution is precisely defined. Sometimes, the mathematical model is too complicated to find out an exact solution, so we have to introduce the notion of approximate solution. On the contrary, in soft set theory, we don't need to introduce the notion of exact solution in the first place because the initial description of the object has an approximate nature in character. This makes soft set theory much more convenient and applicable in practice. Soft sets have been extensively and successfully applied to combined forecasts [6], normal parameter reduction [7], demand analysis [8], data mining [9], and decision-making [10-12].

Recently, soft set theory has been rapidly enriched, including the properties, operations and algebraic structures. Classic uncertain theories, such as fuzzy set theory [13], vague set theory [14], interval mathematics theory [15], rough set theory [16], algebras theory [17, 18], and description logics [19] are gradually initiated and extended in the frame of soft sets.

As an important extension of soft sets, fuzzy soft sets have been increasingly used in the decision making. Xiao and Chen [20] put forward a framework of the interval-valued fuzzy soft sets to tackle multi-attribute group decision-making problems under uncertain environment. Zhang Z. and Zhang S. [21] proposed an approach to multi-attribute group

ISSN: 1738-9968 IJHIT

decision making under interval type-2 fuzzy environment. Zhang, *et al.,* [22] defined the concept of the interval-valued intuitionistic fuzzy soft set and the weighted interval-valued intuitionistic fuzzy soft set and apply them to decision making.

In decision making process, we usually have to face information absence because of the data losses or no data. Such fuzzy soft sets with some unknown data are named as incomplete fuzzy soft sets [23]. As far as we know, almost all applications of fuzzy soft sets ignore the information absence. Up to the present, there have been only two methods to deal with the unknown data of decision making in fuzzy soft sets: the average-probability method proposed by Zou and Xiao in [24] and the object-parameter method proposed by Deng and Wang in [23]. The former has one limitation that decision results are unbelievable when there are a large number of unknown values in corresponding to a parameter. It can be seen in a counterexample given by [23]. The latter in [23] has avoided these disadvantages above by making full use of known data, including the information from the relationship between known values of all objects on a certain parameter on all parameters. But decision-making way of the object-parameter method, transforming incomplete fuzzy soft sets to precise fuzzy complete soft sets and then making decision integrating with some other methods, has disobeyed the feature of basic ideas of soft sets theory on describing objects approximately. Therefore, these two methods are not suitable to solve decision-making problems when under incomplete fuzzy soft sets.

The socio-economic environments are becoming more and more complex, which makes it difficult that a single expert to consider all relevant aspects of a problem increasingly difficult. The application of fuzzy soft sets in group decision making cannot be neglected. Some literatures [20, 21, 25] have proposed some methods related to fuzzy soft sets, and applied these methods in group decision making. However, these existing methods are unavailable when there is the unknown information in group decision making. Hence, it is necessary to extend fuzzy soft set theory to solve the complex decision-making problems.

Dempster–Shafer theory (D–S) of evidence, encompassing both the probability and fuzzy set theories, has advantages to synthesize information of subjectivity and uncertainty. It was initiated in 1967 by Dempster [26] and further developed by Shafer in 1976 in his seminal work [27]. D–S theory of evidence is a powerful method for combining accumulative evidence of changing prior opinions in the light of new evidences [27].

This paper proposes a new type of fuzzy soft sets, D-S fuzzy soft sets which combine D–S theory of evidence and fuzzy soft sets. The FUSE operation between D-S fuzzy soft sets is defined, and the relationship of D-S fuzzy soft sets and incomplete fuzzy soft sets is studied. By using D-S fuzzy soft sets and D–S theory, we introduce a novel group decision-making method and algorithm on incomplete fuzzy soft sets. The primary characteristic of the method is that it not only makes a decision analysis in the form of incomplete fuzzy sets but also incorporates opinions regarding the multi-experts. An evaluation problem of information systems with incomplete information based on multi-experts is analyzed by the proposed method.

The remainder of this paper is organized as follows. Section 2 recalls the basic concepts of fuzzy soft sets, incomplete fuzzy soft sets and D–S theory of evidence. Section 3 defines the notion of D–S fuzzy soft sets, and studies some of its operations and properties. The relationship between incomplete fuzzy soft sets and D–S fuzzy soft sets is also studied. In Section 4, we present a group decision-making method of incomplete fuzzy soft sets based on D-S theory, and specific algorithm is also presented. Section 5 is devoted to proposed decision-making method with applications. Finally, conclusions are given in Section 6.

2. Preliminaries

In this section, we shall briefly recall some fundamental notions of fuzzy soft sets and D-S theory of evidence.

2.1 Fuzzy Soft Sets

Definition 2.1.([1]) A pair (F,E) is called soft set over U, where F is a mapping given by $F: E \to P(U)$.

Definition 2.2.([28]) A pair (F,E) is called a fuzzy soft set over U, where $\tilde{F}$ is mapping given by $\tilde{F}: E \to \mathscr{F}(U)$, where $\mathscr{F}(U)$ is the set of all fuzzy subsets in universe U.

Let $U=\{u_1,u_2,u_3\cdots,u_m\}$ be a set of m objects, which may be characterized by a set of parameters $E=\{e_1,e_2,e_3\cdots,e_n\}$. The entry in tabular representation of the fuzzy soft set $f(u_i,e_j)$, a quantity in the unit interval [0,1], represents the membership degree of the object u_i belonging to the parameter e_j or referring to as the membership degree or the membership probability of the object possessing the related parameter.

Definition 2.3.([24]). A quaternion $S=(U,AT,V,f)$ is called an information system where U is a nonempty finite set of objects, AT is a nonempty finite set of attributes, $V=\cup v_r$ where v_r is called the value domain of attribute r, and f is an information function specifying the attributes-value for each object and denoted by $f: U \times A \to V$.

When an information system has some unknown or missing values of some attributes, it is called an incomplete information system. In such a system, “unknown values” are represented by*in tabular representation.

Each fuzzy soft set can be considered as an fuzzy information system, in which each value domain of attribute v_r is a quantity in the unit interval [0, 1]. If there are unknown values of the elements in a fuzzy soft sets, then the fuzzy soft set are incomplete. And an incomplete fuzzy soft set can be considered as an incomplete information system.

Example1 Let U be the set of information systems. Suppose there are four systems in the universe U given by

$$U=(u_1,u_2,u_3,u_4)$$

E is a family of sets of parameter, and each parameter is a word or sentence. $E=\{e_1,e_2,e_3,e_4\}$, where e_1 stands for ‘‘authenticity”; e_2 stands for ‘‘in time”; e_3 stands for ‘‘share”, and e_4 stands for ‘integrality”. The quality of systems is evaluated from different characteristics by an expert. Because the difficulty of access to information and limitation of knowledge, the experts cannot evaluate the values of parameters on every object. In this case to define an incomplete fuzzy soft set (F,E). The mapping of (F,E) is given as below:

$$F(e_1)=\{u_1/.4,\ u_2/*, u_3/.4, u_4/.3\}$$
$$F(e_2)=\{u_1/.9,\ u_2/.8, u_3/.1, u_4/*\}$$
$$F(e_3)=\{u_1/.8,\ u_2/*, u_3/.8, u_4/.2\}$$
$$F(e_4)=\{u_1/*,\ u_2/.2, u_3/.7, u_4/.4\}$$

where * stands for unknown values of parameters.

We can represent the incomplete fuzzy soft set (F,E) in the form of Table 1

Table 1. Tabular Representation of the Incomplete Fuzzy Soft Set (F,E)

	e_1	e_2	e_3	e_4
u_1	0.4	0.9	0.8	*
u_2	*	0.8	*	0.2
u_3	0.4	0.1	0.8	0.7
u_4	0.3	*	0.2	0.4

2.2 Basic Concept of Dempster–Shafer Theory

Definition 2.4 [27]Let Θ be a finite nonempty set of mutually exclusive alternatives, and be called the frame of discernment. For any proposition A in any problem domain, they all belong to the power set 2^{Θ}. On 2^{Θ} we can define the basic probability assignment function (BPAF), $m:2^{\Theta}\rightarrow[0,1]$, m meets

$$m(\Phi)=0\ ,\ \sum_{A\subset\Theta}m(\mathrm{A})=1$$

$m(\mathrm{A})$ says evidence in support of proposition A, and $m(\Theta)$ is called the degree of ignorance. Each subset $A\subseteq\Theta$ such that $m(\mathrm{A})>0$ is called a focal element of m.

Given an incomplete fuzzy soft sets (F,E), $f(u_i,e_j)$ is membership degree of the object u_i belonging to the parameter e_j. A focal element can be defined from the incomplete fuzzy soft sets.

Definition 2.5 [29] Let (F,E) be an incomplete fuzzy soft sets on the universal set U. For $\forall u_i,u_k\in U$ and $u_i\neq u_k$, if $f(u_i,e_j)=f(u_k,e_j)$, then u_i and u_k belong to the same focal element. And if $f(u_i,e_j)=*$, then u_i belong to the focal element Θ.

Example 2 For Example1, from the incomplete fuzzy soft set (F,E), the focal elements of different parameters are given as follows:

The focal element of e_1 are $\{u_1,u_3\}$, $\{u_4\}$ and Θ; the focal element of e_2 are $\{u_1\}$, $\{u_2\}$, $\{u_3\}$ and Θ; the focal element of e_3 are $\{u_1,u_3\}$, $\{u_4\}$ and Θ; the focal element of e_4 are $\{u_2\}$, $\{u_3\}$, $\{u_4\}$ and Θ.

Definition 2.6 [27] For evidences $A_1,A_2,A_3,\cdots A_s$ and $B_1,B_2,B_3,\cdots B_t$, the corresponding basic probability assignment function are m_1 and m_2 respectively. If $\sum_{A_i\cap B_j=\Phi}m_1(A_i)m_2(B_j)<1$, the rule of evidence combination for the theory is as follow:

$$m(A)=m_1\oplus m_2(A)=\frac{1}{1-K}\sum_{A_i\cap B_j=A}m_1(A_i)m_2(B_j),\qquad \forall A\subset\Theta,\qquad A\neq\Phi$$

$$m(A)=m_1\oplus m_2(A)=0,\qquad \mathrm{A}=\Phi$$

For the above formula: $K=\sum_{A_i\cap B_j=\Phi}m_1(A_i)m_2(B_j)$, which reflects the extent of the conflict between the evidence, is called the conflict probability. Coefficient $\frac{1}{1-K}$ is called normalized factor, its role is to avoid the probability of assigning non-0 to empty set Φ in the combination.

Definition 2.7 [27]Let Θ be the frame of discernment, each BPA is a belief measure (Bel), which is a function: $m:2^{\Theta}\rightarrow[0,1]$, defined by the following equations:

$$Bel(A)=\sum_{B\subseteq A}m(B)\qquad \forall A\in 2^{\Theta}$$

where A and B are subsets of Θ, $Bel(A)$ represents the exact support to A.

By definition 2.7, an object $u_i \in U$ in complete fuzzy soft sets can be determined as follows:

$$Bel(\{u_i\}) = \sum_{u_i \in A} m(A) \qquad \forall u_i \in U$$

where A are subsets of Θ.

3. D-S fuzzy Soft Sets

3.1. Concept of D–S fuzzy Soft Sets

Definition 3.1. Let $U = \{u_1, u_2, u_3 \cdots, u_m\}$ be the universe set of elements and $E = \{e_1, e_2, e_3, \cdots, e_n\}$ be the universe set of parameters. Let $F : E \to P(U)$, where $P(U)$ is the power set of U. Let m be the basic probability assignment function as the universe U,

$$m : P(U) \to [0,1].$$

We say F_D is a D–S fuzzy soft sets, and $F_D(e) = (F_f(e), m(e))$ and $\sum_{A_t \subset U} m^{A_t}(e) = 1$, $\forall e \in E$.

Where $F_f(e)$ is the set of focal elements of e, $A_t \in F_f(e)$ a focal element of $m(e)$, $m(e)$ gives the belief exactly assigned to each focal element in the corresponding $F_f(e)$. And for each parameter e_j, $F_D(e_j) = (F_f(e_j), m(e_j))$, $m(e_j) = \{m^{A_1}(e_j), m^{A_2}(e_j), \cdots, m^{A_t}(e_j), \cdots, m^{A_k}(e_j)\}$ and $\sum_{A_t \subset U} m^{A_t}(e_j) = 1$.

Example 3 For Example 1 and Example 2, let $U = (u_1, u_2, u_3, u_4)$ and $E = \{e_1, e_2, e_3, e_4\}$, let $F_f(e)$ be the set of focal elements of e, $m(e)$ gives the belief exactly assigned to each focal element in the corresponding $F_f(e)$.

The D–S fuzzy soft sets F_D can be defined as follow:

$$F_D(e_1) = \{F_f(e_1), m(e_1)\} = \{\frac{u_1, u_3}{0.23}, \frac{u_4}{0.18}, \frac{\Theta}{0.59}\}$$

where $F_f(e_1) = \{ \{u_1, u_3\}, \{u_4\}, \Theta\}$, and $m^{\{u_1,u_3\}}(e_1) = 0.23$, $m^{\{u_4\}}(e_1) = 0.18$, $m^{\Theta}(e_1) = 0.59$

$$F_D(e_2) = \{F_f(e_2), m(e_2)\} = \{\frac{u_1}{0.32}, \frac{u_2}{0.28} \frac{u_3}{0.04}, \frac{\Theta}{0.36}\}$$

where $F_f(e_2) = \{ \{u_1\}, \{u_2\}, \{u_3\}, \Theta\}$, and $m^{\{u_1\}}(e_2) = 0.32$, $m^{\{u_2\}}(e_2) = 0.28$, $m^{\{u_3\}}(e_2) = 0.04$, $m^{\Theta}(e_2) = 0.36$

$$F_D(e_3) = \{F_f(e_3), m(e_3)\} = \{\frac{u_1, u_3}{0.40}, \frac{u_4}{0.10}, \frac{\Theta}{0.50}\}$$

where $F_f(e_3) = \{ \{u_1, u_3\}, \{u_4\}, \Theta\}$, and $m^{\{u_1,u_3\}}(e_3) = 0.40$, $m^{\{u_4\}}(e_3) = 0.10$, $m^{\Theta}(e_3) = 0.59$

$$F_D(e_4) = \{F_f(e_4), m(e_4)\} = \{\frac{u_2}{0.09}, \frac{u_3}{0.30} \frac{u_4}{0.17}, \frac{\Theta}{0.44}\}$$

where $F_f(e_4) = \{ \{u_2\}, \{u_3\}, \{u_4\}, \Theta\}$, and $m^{\{u_2\}}(e_4) = 0.09$, $m^{\{u_3\}}(e_4) = 0.30$, $m^{\{u_4\}}(e_4) = 0.17$, $m^{\Theta}(e_4) = 0.44$

We can also represent the D–S fuzzy soft sets F_D in the form of Table 2

Table 2. Tabular Representation of the D–S Fuzzy Soft Sets F_D

$F_D(e_1)$		$F_D(e_2)$		$F_D(e_3)$		$F_D(e_4)$	
$F_f(e_1)$	$m(e_1)$	$F_f(e_2)$	$m(e_2)$	$F_f(e_3)$	$m(e_3)$	$F_f(e_4)$	$m(e_4)$
$\{u_1,u_3\}$	0.23	$\{u_1\}$	0.32	$\{u_1,u_3\}$	0.40	$\{u_2\}$	0.09
$\{u_4\}$	0.18	$\{u_2\}$	0.28	$\{u_4\}$	0.10	$\{u_3\}$	0.30
Θ	0.59	$\{u_3\}$	0.04	Θ	0.50	$\{u_4\}$	0.17
		Θ	0.36			Θ	0.44

Definition 3.2 FUSE operation on two D-S fuzzy soft sets $F_D(e)=(F_f(e),m_F(e))$ and $G_D(e)=(G_f(e),m_G(e))$ denoted by $F_D\text{FUSE}G_D$ is defined by

$$H_D(e)=(H_f(e),m_H(e))$$

where $H_f(e)=F_f(e)\cup G_f(e)$ and $m_H(e)=m_F(e)\oplus m_G(e)$

3.2 Relationship between Generalized Fuzzy Soft Sets and D–S Generalized Fuzzy Soft Sets

The key of D–S fuzzy soft sets is constructing the basic utility assignment of each focal element under different parameters e_j. In order to construct the basic utility assignment, we first determine the set of focal elements $F_f(e_j)$ from each e_j of an incomplete soft set by definition 2.5. And the basic probability assignment function corresponding with $f(u_i,e_j)$ can be obtained according to the following function.

Definition 3.3 Let $A_t \in F_f(e_j)$ is a focal element of e_j, we can define a transition function:

$M: f(u_i,e_j)\to m^{A_t}(e_j)$, $u_i\in A_t$

The mapping M can be constructed differently according to the different background. For an example, let Θ be the universal set, the set of focal elements $F_f(e)=\{A_1,A_2,\cdots,A_t,\cdots,A_k\}$, we can define the basic probability assignment function m as follows according to [29]:

$$m^{A_t}(e_j)=\begin{cases}\dfrac{f(u_i,e_j)}{\sum_{t=1}^{k} f(u_i,e_j)+1} & u_i\in A_t, A_t\neq\Theta\\[2ex] \dfrac{1}{\sum_{t=1}^{k} f(u_i,e_j)+1} & A_t=\Theta\end{cases}$$

Proposition 3.1 Every incomplete fuzzy soft set can be transformed into a D–S fuzzy soft set

Proof. Suppose that (F,E) is an incomplete fuzzy soft set, $f(u_i,e_j)$ is the membership degree of the object u_i belonging to the parameter e_j.

The set of focal elements $F_f(e)$ and the basic probability assignment function $m(e)$ of each e of (F,E) can be obtained, according to definition 2.6 and definition 3.3 respectively. Then we can get the D–S fuzzy soft set $F_D(e)=(F_f(e),m(e))$.

4. AGroup Decision-making Method of Incomplete Fuzzy Soft Sets Based on D-S Theory

Based on definitions above, we develop a new analytical method of incomplete fuzzy soft sets for solving multiple attribute group decision-making problems with unknown values. The method involves the following steps:

Step 1. For a group decision-making problem, let $U=\{u_1,u_2,u_3\cdots,u_m\}$ be a finite set of alternatives, and $E=\{e_1,e_2,e_3,\cdots,e_n\}$ be the universe set of parameters. Let $D=\{d_1,d_2,d_3,\cdots,d_p\}$ be the set of experts, and the experts $d_l(l=1,2,\cdots,p)$ provide their fuzzy preferences for each alternative, some of which are unknown because of a lack of expertise or insufficient knowledge. According to the evaluation results of experts, p incomplete fuzzy soft sets can be constructed $(F_l,E),l=1,2,\cdots,p$.

Step 2. Confirm the set of focal elements $F_f(e)=\{A_1,A_2,\cdots,A_t,\cdots,A_k\}$ on different parameters e of incomplete fuzzy soft sets by definition 2.5.

Step 3. Construct transition function by definition 3.3, and transfer incomplete fuzzy soft sets $(F_l,E),l=1,2,\cdots,p$ into D-S fuzzy soft sets $F_{Dl},l=1,2,\cdots,p$ for each expert.

Step 4. Compute the corresponding resultant D-S fuzzy soft set $H_D(e)=(H_f(e),m(e))$ by the operation FUSE of p D-S fuzzy soft sets $F_{Dl}(e)=(F_{fl}(e),m_l(e)),l=1,2,\cdots,p$ by definition 3.2.

Step 5. Combine the BPA values of all intersections $m^{A_t}(e)$ under all parameters e_j of $H_D(e)$ by definition 2.6.

Step 6. Compute $Bel(\{u_i\})$, $u_i\in U$ by definition 2.7 .

Step 7. Determine the ranking of $u_i\in U$ according to the value of $Bel(\{u_i\})$.

5. Illustrative Examples

Suppose three experts conduct an evaluation to the quality of four information systems, respectively. $U=(u_1,u_2,u_3,u_4)$ is the set of alternatives. The set of attributes $E=\{e_1,e_2,e_3,e_4\}$ represents the four factors, that is ''authenticity", ''in time", ''share" and ''integrality". But in the evaluation process, because of a lack of expertise or insufficient knowledge, the result of evaluation is incomplete. According to the evaluation result of three experts, we obtain three incomplete fuzzy soft sets $(F_i,E),i=1,2,3$, as shown in Table 3.

Table 3. Tabular Representation of the Three Incomplete Fuzzy Soft Sets $(F_i,E),i=1,2,3$

	(F_1,E)				(F_2,E)				(F_3,E)			
	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
u_1	0.4	0.9	0.8	0.7	0.2	0.8	0.7	0.8	0.4	0.9	0.8	*
u_2	0.3	0.7	0.2	*	0.3	*	0.5	0.2	*	0.8	*	0.2
u_3	0.6	0.2	0.8	0.6	0.8	0.3	*	0.8	0.4	0.1	0.8	0.7
u_4	*	0.9	0.3	0.2	0.4	0.7	0.3	0.5	0.3	*	0.2	0.4

By definition 2.5 and definition 3.3 respectively, the set of focal elements $F_{fl}(e_j)$ and the basic probability assignment function $m_l(e_j)$ of each e_j of $(F_l,E),l=1,2,3$ are obtained. And we get three D-S fuzzy soft sets $F_{Dl}(e)=(F_{fl}(e),m_l(e)),l=1,2,3$, as shown in Table 4.

According to definition 3.2, we perform FUSE operation of D-S fuzzy soft sets $F_{Dl}(e)=(F_{fl}(e), m_l(e)), l=1,2,3$, and get the $H_D = F_{D1}$ FUSE F_{D2} FUSE F_{D3}, as shown in Table 5.

By applying the rule of evidence combination by definition 2.6, we obtain the BPA values of all focal elements under all parameters of H_D as follows.

$m(u_1)=0.6217$, $m(u_2)=0.0310$, $m(u_3)=0.2541$, $m(u_4)=0.0704$, $m(u_1,u_3)=0.0179$, $m(u_1,u_4)=0.0023$, $m(\Theta)=0.025$

According to definition 2.7, we calculate the $Bel(\{u_i\}), i=1,2,3,4$ represented exact support to u_i .

$Bel(\{u_1\})=0.6419$, $Bel(\{u_2\})=0.0310$, $Bel(\{u_3\})=0.2721$, $Bel(\{u_4\})=0.0727$.

Therefore, the ranking order of the information systems is $u_1 \succ u_3 \succ u_4 \succ u_2$.

Table 4. Tabular Representation of the D–S Fuzzy Soft Sets $F_{Dl}, l=1,2,3$

	$F_D(e_1)$		$F_D(e_2)$		$F_D(e_3)$		$F_D(e_4)$	
	$F_f(e_1)$	$m(e_1)$	$F_f(e_2)$	$m(e_2)$	$F_f(e_3)$	$m(e_3)$	$F_f(e_4)$	$m(e_4)$
F_{D1}	$\{u_1\}$	0.1739	$\{u_1,u_4\}$	0.3214	$\{u_1,u_3\}$	0.3478	$\{u_1\}$	0.2800
	$\{u_2\}$	0.1304	$\{u_2\}$	0.2500	$\{u_2\}$	0.0870	$\{u_3\}$	0.2400
	$\{u_3\}$	0.2609	$\{u_3\}$	0.0714	$\{u_4\}$	0.1304	$\{u_4\}$	0.0800
	Θ	0.4348	Θ	0.3571	Θ	0.4348	Θ	0.4000
F_{D2}	$\{u_1\}$	0.0741	$\{u_1\}$	0.2857	$\{u_1\}$	0.2800	$\{u_1,u_3\}$	0.3200
	$\{u_2\}$	0.1111	$\{u_3\}$	0.1071	$\{u_2\}$	0.2000	$\{u_2\}$	0.0800
	$\{u_3\}$	0.2963	$\{u_4\}$	0.2500	$\{u_4\}$	0.1200	$\{u_4\}$	0.2000
	$\{u_4\}$	0.1481	Θ	0.3571	Θ	0.4000	Θ	0.4000
	Θ	0.3704						
F_{D3}	$\{u_1,u_3\}$	0.2353	$\{u_1\}$	0.3214	$\{u_1,u_3\}$	0.4000	$\{u_2\}$	0.0870
	$\{u_4\}$	0.1765	$\{u_2\}$	0.2857	$\{u_4\}$	0.1000	$\{u_3\}$	0.3043
	Θ	0.5882	$\{u_3\}$	0.0357	Θ	0.5000	$\{u_4\}$	0.1739
			Θ	0.3571			Θ	0.4348

Table 5. The Table Representation $H_D = F_{D1}$ FUSE F_{D2} FUSE F_{D3}

$H_D(e_1)$		$H_D(e_2)$		$H_D(e_3)$		$H_D(e_4)$	
$H_f(e_1)$	$m(e_1)$	$H_f(e_2)$	$m(e_2)$	$H_f(e_3)$	$m(e_3)$	$H_f(e_4)$	$m(e_4)$
$\{u_1\}$	0.1466	$\{u_1\}$	0.4336	$\{u_1\}$	0.3092	$\{u_1\}$	0.1552
$\{u_2\}$	0.1063	$\{u_2\}$	0.1943	$\{u_2\}$	0.1091	$\{u_2\}$	0.0542
$\{u_3\}$	0.4053	$\{u_3\}$	0.0675	$\{u_4\}$	0.1401	$\{u_3\}$	0.3815
$\{u_4\}$	0.1263	$\{u_4\}$	0.1254	$\{u_1,u_3\}$	0.3053	$\{u_4\}$	0.1873
$\{u_1,u_3\}$	0.0616	$\{u_1,u_4\}$	0.0849	Θ	0.1363	$\{u_1,u_3\}$	0.0986
Θ	0.1540	Θ	0.0943			Θ	0.1232

6. Conclusions

In group decision making, we often encounter situations in which experts cannot give an available values when evaluating an alternative with respect to an attribute, owing to a lack of expertise or insufficient knowledge. Incomplete fuzzy soft set describe the situations in which the information about decision alternatives may be unknown. This paper proposed the D-S fuzzy soft sets combining D–S theory and fuzzy soft sets to cope with the group decision making on incomplete fuzzy soft sets. FUSE operation between two D-S fuzzy soft sets is defined. Every incomplete fuzzy soft set can be transformed into D–S fuzzy soft sets by using a transition function, which can be constructed differently according to the different background. Then we brought up the decision algorithm based on D–S fuzzy soft sets and D-S theory. Finally, we applied proposed algorithm into the information systems quality evaluating. In the future work, considering the potential conflict among experts in group decision, we will present a series of effective operations for D–S fuzzy soft sets and use them to develop methods for conflict group decision making problems.

Acknowledgements

This paper was supported by the National Natural Science Foundation of China (Grant No. 71171209) and Major Consulting Research Project of Chinese Academy of Engineering (Grant No. 2012-ZD-15).

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Authors

Yuan-Xiang Dong, she received BS degree in Information management from Harbin University of Commerce in 2008. Currently, she is a Ph.D. candidate in the School of Economics and Business Administration at Chongqing University. Her research interests include intelligent decision-making, information intelligent analysis and business Intelligence.

Zhi Xiao, he received BS degree in Department of Mathematics from Southwest Normal University in 1982. And he received his Ph.D. degree in Technology Economy and Management from Chongqing University in 2003. He is currently a Professor in the School of Economics and Business Administration at Chongqing University. His research interests are in the areas of information intelligent analysis, data mining, and business Intelligence.

