

Integral Criterion-Based Adaptation Control to Vibration Reduction in Sensitive Actuators

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Abstract

Refer to this research, an integral criterion-based hybrid fuzzy controller is proposed for vibration reduction in sensitive actuators. The first problem of the fuzzy logic controller was stability in certain and uncertain systems. This problem can be reduced in certain system by using hybrid fuzzy control law. However fuzzy hybrid controller based on PID methodology has advantage in certain system, but it has challenge in presence of uncertainty and motor vibration. To solve this challenge direct adaptive methodology applied to hybrid fuzzy controller. This type of adaptive technique is model-reference variable structure control. Regarding to proposed method, chattering eliminate as well improve the stability and robustness. The simulation results exhibit that the proposed method works well in certain system.

Keywords: *Inverse dynamic control, fuzzy control, Multi degrees of freedom joints, Model reference-based adaptive control, hybrid control, direct adaptive control*

1. Introduction

Multi degrees of freedom actuator (MDFA) is a type of nonlinear joints. These actuators are finding wide use in a number of Industries such as aerospace, Industrial medical and automotive. For high precession trajectory planning and control, it is necessary to replace the actuator system made up of several single-DOF motors connected in series and/or parallel with a single multi-DOF actuator [1-3]. The spherical motor (MDFA) have potential contributions to a wide range of applications such as coordinate measuring, object tracking, material handling, automated assembling, welding, and laser cutting [4]. All these applications require high precision motion and fast dynamic response, which the spherical motor is capable of delivering [5-6]. The spherical motor exhibits coupled, nonlinear and very complex dynamics. The design and implementation of feedback controllers for the motor are complicated. The controller design is further complicated by the orientation-varying torque generated by the spherical motor [7]. One of the significant challenges in control algorithms is a linear behavior controller design for nonlinear systems. Some of MDFA which work in industrial processes are controlled by linear PID controllers, but the design of linear controller for MDFA is extremely difficult in presence of hand tremors. To solve this challenge, nonlinear model-free controller is a good candidate. Fuzzy logic controller is used as a main controller.

Although the fuzzy-logic control is not a new technique, its application in this current research is considered to be novel since it aimed for an automated dynamic-less response rather than for the traditional objective of uncertainties compensation [8-9]. The intelligent tracking control using the fuzzy-logic technique provides a cost-and-time efficient control implementation due to the automated dynamic-less input. This in turn would further inspire multi-uncertainties testing for continuum robot [10]. In project we can use fuzzy logic theory when a plant can be considered as a black box with outputs available for measurement and a possibility of changing inputs. The plant is supposed to be observable and controllable. Some information about the plant operation or plant control is available, which can or cannot be of a quantitative nature, but it can be formulated as a set of rules (maybe after some processing). An acceptable fuzzy control solution is possible, which should satisfy design specifications. It must not be optimal in regard to some criteria as it is hard to prove that a fuzzy control system is optimal and even stable. However, a fuzzy controller is able to provide a stable and 'good' solution. However, a fuzzy logic controller is used in many applications but it has an important challenge namely; stability. To improve stability in certain conditions, a hybrid fuzzy controller is introduced [11-12].

Linear control systems use linear negative feedback to produce a control signal mathematically based on other variables, with a view to maintain the controlled process within an acceptable operating range. The output from a linear control system into the controlled process may be in the form of a directly variable signal, such as a valve that may be 0 or 100% open or anywhere in between. Sometimes this is not feasible and so, after calculating the current required corrective signal, a linear control system may repeatedly switch an actuator, such as a pump, motor or heater, fully on and then fully off again, regulating the duty cycle using pulse-width modulation. When controlling the temperature of an industrial furnace, it is usually better to control the opening of the fuel valve *in proportion to* the current needs of the furnace. This helps avoid thermal shocks and applies heat more effectively. Proportional negative-feedback systems are based on the difference between the required set point (SP) and process value (PV). This difference is called the *error*. Power is applied in direct proportion to the current measured error, in the correct sense so as to tend to reduce the error (and so avoid positive feedback). The amount of corrective action that is applied for a given error is set by the gain or sensitivity of the control system. At low gains, only a small corrective action is applied when errors are detected: the system may be safe and stable, but may be sluggish in response to changing conditions; errors will remain uncorrected for relatively long periods of time: it is over-damped. If the proportional gain is increased, such systems become more responsive and errors are dealt with more quickly. There is an optimal value for the gain setting when the overall system is said to be critically damped. Increases in loop gain beyond this point will lead to oscillations in the PV; such a system is under-damped. In real systems, there are practical limits to the range of the manipulated variable (MV) [13-14]. For example, a heater can be off or fully on, or a valve can be closed or fully open. Adjustments to the gain simultaneously alter the range of error values over which the MV is between these limits. The width of this range, in units of the error variable and therefore of the PV, is called the *proportional band* (PB) [17-18]. While the gain is useful in mathematical treatments, the proportional band is often used in practical situations. They both refer to the same thing, but the PB has an inverse relationship to gain – higher gains result in narrower PBs, and *vice versa* [12-14]. In this research a PID controller is applied to a fuzzy controller as a hybrid technique to improve stability. However, this technique is good enough to improve stability but it has a new challenge, namely; robustness. To improve robustness, direct adaptive control is used in this research [15-17].

In various dynamic parameters systems that need to be training on-line adaptive control methodology is used. Adaptive control methodology can be classified into two main groups, namely, traditional adaptive method and fuzzy adaptive method. Model reference variable structure adaptive method is used in systems which want to tuning parameters by system's dynamic knowledge. This paper is organized as follows; section 2, is served as an introduction to the dynamic of three degrees of freedom spherical motor. Part 3, introduces and describes the methodology algorithm. Section 4 presents the simulation results and discussion of this algorithm applied to three degrees of freedom spherical motor and the final section describe the conclusion.

2. Theory

Dynamic and Kinematics Formulation of Spherical Motor: Dynamic modeling of spherical motors is used to describe the behavior of spherical motor such as linear or nonlinear dynamic behavior, design of model based controller such as pure sliding mode controller which design this controller is based on nonlinear dynamic equations, and for simulation. The dynamic modeling describes the relationship between motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (*e.g.*, inertia, coriolios, centrifugal, and the other parameters) to behavior of system. Spherical motor has nonlinear and uncertain dynamic parameters 3 degrees of freedom (DOF) motor.

The equation of a spherical motor governed by the following equation:

$$H(q) \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{\gamma} \end{bmatrix} + B(q) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + C(q) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \quad (1)$$

Where τ is actuation torque, $H(q)$ is a symmetric and positive define inertia matrix, $B(q)$ is the matrix of coriolios torques, $C(q)$ is the matrix of centrifugal torques.

This is a decoupled system with simple second order linear differential dynamics. In other words, the component \ddot{q}_i influences, with a double integrator relationship, only the variable q_i , independently of the motion of the other parts. Therefore, the angular acceleration is found as to be:

$$\ddot{q} = H^{-1}(q) \cdot \{\tau - \{B + C\}\} \quad (2)$$

This technique is very attractive from a control point of view.

Study of spherical motor is classified into two main groups: kinematics and dynamics. Calculate the relationship between rigid bodies and final part without any forces is called Kinematics. Study of this part is pivotal to design with an acceptable performance controller, and in real situations and practical applications. As expected the study of kinematics is divided into two main parts: forward and inverse kinematics. Forward kinematics has been used to find the position and orientation of task frame when angles of joints are known. Inverse kinematics has been used to find possible joints variable (angles) when all position and orientation of task frame be active.

The main target in forward kinematics is calculating the following function:

$$\Psi(X, q) = 0 \quad (3)$$

Where $\Psi(.) \in R^n$ is a nonlinear vector function, $X = [X_1, X_2, \dots, X_l]^T$ is the vector of task space variables which generally task frame has three task space variables, three orientation, $q = [q_1, q_2, \dots, q_n]^T$ is a vector of angles or displacement, and finally n is the number of actuated joints. The Denavit-Hartenberg (D-H) convention is a method of drawing spherical motor free body diagrams. Denvit-Hartenberg (D-H) convention study is necessary to calculate forward kinematics in this motor.

A systematic Forward Kinematics solution is the main target of this part. The first step to compute Forward Kinematics (F.K) is finding the standard D-H parameters. The following steps show the systematic derivation of the standard D-H parameters.

1. Locate the spherical motor
2. Label joints
3. Determine joint rotation (θ)
4. Setup base coordinate frames.
5. Setup joints coordinate frames.
6. Determine α_i , that α_i , link twist, is the angle between Z_i and Z_{i+1} about an X_i .
7. Determine d_i and a_i , that a_i , link length, is the distance between Z_i and Z_{i+1} along X_i . d_i , offset, is the distance between X_{i-1} and X_i along Z_i axis.
8. Fill up the D-H parameters table. The second step to compute Forward kinematics is finding the rotation matrix (R_n^0). The rotation matrix from $\{F_i\}$ to $\{F_{i-1}\}$ is given by the following equation;

$$R_i^{i-1} = U_{i(\theta_i)} V_{i(\alpha_i)} \tag{4}$$

Where $U_{i(\theta_i)}$ is given by the following equation;

$$U_{i(\theta_i)} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

and $V_{i(\alpha_i)}$ is given by the following equation;

$$V_{i(\alpha_i)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \tag{6}$$

So (R_n^0) is given by

$$R_n^0 = (U_1 V_1)(U_2 V_2) \dots \dots \dots (U_n V_n) \tag{7}$$

The final step to compute the forward kinematics is calculate the transformation ${}^0_n T$ by the following formulation [3]

$${}^0_n T = {}^0_1 T \cdot {}^1_2 T \cdot {}^2_3 T \dots \dots \dots {}^{n-1}_n T = \begin{bmatrix} R_n^0 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

3. Methodology

Controller Design:

Adaptive control has an important role in modern control systems. During operation, many controlled processes experience abrupt or continuous parameter variations, varying external conditions and, in some occasions, alternations of operating modes. When all aforementioned causes of non-uniform system behavior are not excessive, then they are usually well handled with standard feedback controllers. But, when that is not the case, then standard feedback controllers cannot maintain the desired control quality and some sort of adaptation to the new situation in the process are needed. Adaptation may be used for the purpose of improving system dynamics or to reduce system sensitivity to parameter variations. Figure 1 shows the general structure of adaptive controller.

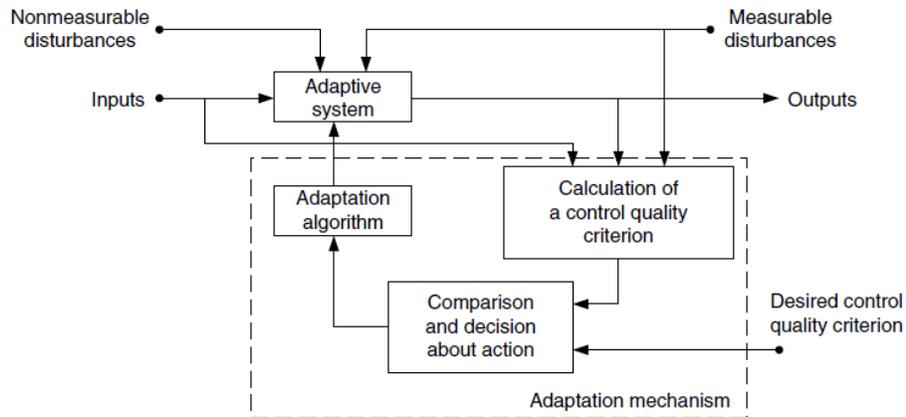


Figure 1. A General Structure of an Adaptive Control System

According to Figure 1, in this research sliding mode control is used as a model-reference control theory. This type of controller is robust, stable and reliable. Hybrid fuzzy controller is used as a main controller. This type of controller is nonlinear as well model-free and integral part of PID controller is used to improve the error. The goal of adaptation is to adjust system parameters so that the selected integral criterion depending on the model tracking error reaches a minimum value. In such case, an adaptive system is considered an optimal control system. Instead of the above commonly used integral criteria, let us observe a performance criterion calculated as the ratio of integrals (areas) determined by the reference model output response y_M and the system output response y_f reaching a specified portion of a complete transient response (we assume that system noise has characteristics of white noise):

$$P_y = \frac{\int_0^{t_{y_f}} y_f(t) \cdot dt}{\int_0^{t_{y_M}} y_M(t) \cdot dt} = \frac{I_{y_f}(t_{y_f})}{I_{y_M}(t_{y_M})} \quad (9)$$

In order to get a broader range of values of y_f , a good choice is to calculate y_f during the rise time interval, when the picture of system dynamics is very clear (Figure 2).

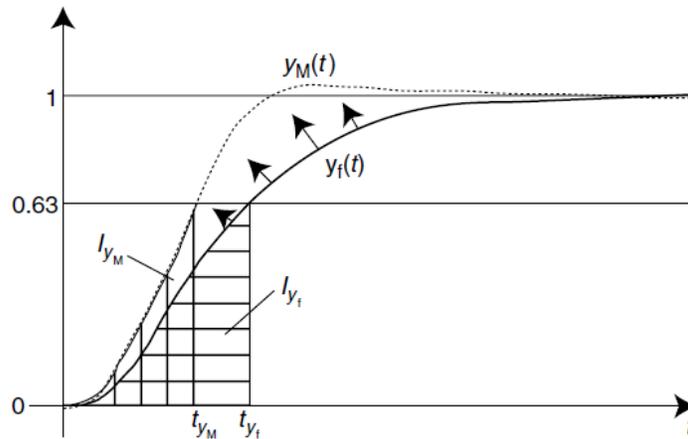


Figure 2. A Reference Model Output Integral (I_{y_M}) and a Closed-loop System Integral (I_{y_f}) for $a = 0.63$

For tuning law (10) we must find an adaptation algorithm, which will change K_f and T_f with respect to changes of γ_T .

$$\begin{aligned} K_f^{K+1} &= K_f^K + \gamma_K \cdot \Delta K_f^{K+1} \\ T_f^{K+1} &= T_f^K + \gamma_T \cdot \Delta T_f^{K+1} \end{aligned} \tag{10}$$

Figure 3 shows the proposed method to control of motor vibration reduction.

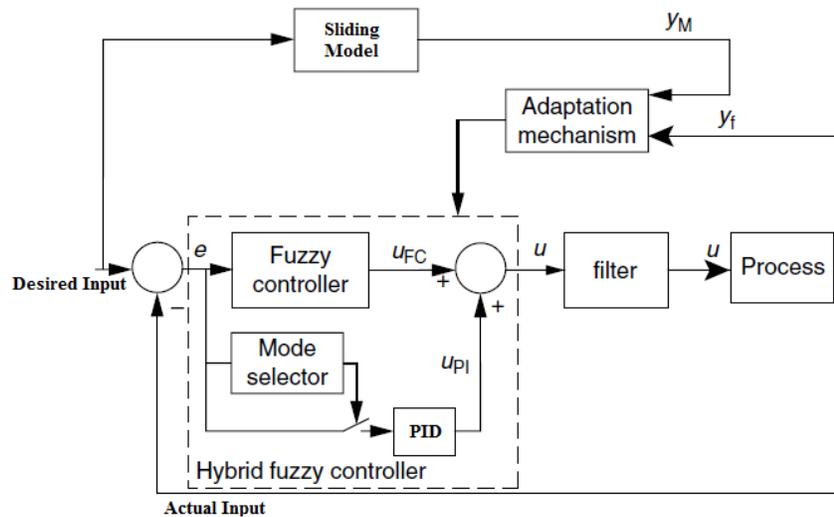


Figure 3. The Structure of an Adaptive Fuzzy Control System

Hybrid Fuzzy Controller divided into two main parts: fuzzy controller and linear PID controller.

Design of a linear methodology to control of nonlinear system was very straight forward. Since there was an output from the torque model, this means that there would be two inputs into the PID controller. Similarly, the outputs of the controller result from the two control inputs of the torque signal. In a typical PID method, the controller corrects the error between the desired input value and the measured value. Since the actual position is the measured signal.

$$e(t) = \theta_a(t) - \theta_d(t) \quad (11)$$

$$U_{PID} = K_{p_a} e + K_{v_a} \dot{e} + K_I \int e \quad (12)$$

The model-free control strategy is based on the assumption that the joints of the motors are all independent and the system can be decoupled into a group of single-axis control systems. Therefore, the kinematic control method always results in a group of individual controllers, each for an active joint of the motor. With the independent joint assumption, no a priori knowledge of spherical motor dynamics is needed in the kinematic controller design, so the complex computation of its dynamics can be avoided and the controller design can be greatly simplified. This is suitable for real-time control applications when powerful processors, which can execute complex algorithms rapidly, are not accessible. However, since joints coupling is neglected, control performance degrades as operating speed increases and a motor controlled in this way is only appropriate for relatively slow motion. The fast motion requirement results in even higher dynamic coupling between the various spherical motor joints, which cannot be compensated for by a standard motor controller such as PID, and hence model-based control becomes the alternative.

Fuzzy inference systems (FISs) are also known as fuzzy rule-based systems, fuzzy model, fuzzy expert system, and fuzzy associative memory. This is a major unit of a fuzzy logic system. The decision-making is an important part in the entire system. The FIS formulates suitable rules and based upon the rules the decision is made. This is mainly based on the concepts of the fuzzy set theory, fuzzy IF–THEN rules, and fuzzy reasoning. FIS uses “IF. . . THEN. . .” statements, and the connectors present in the rule statement are “OR” or “AND” to make the necessary decision rules. The basic FIS can take either fuzzy inputs or crisp inputs, but the outputs it produces are almost always fuzzy sets. When the FIS is used as a controller, it is necessary to have a crisp output. Therefore in this case defuzzification method is adopted to best extract a crisp value that best represents a fuzzy set.

Fuzzy inference system consists of a fuzzification interface, a rule base, a database, a decision-making unit, and finally a defuzzification interface. A FIS with five functional block described in Figure 4. The function of each block is as follows:

- a *rule base* containing a number of fuzzy IF–THEN rules;
- a *database* which defines the membership functions of the fuzzy sets used in the fuzzy rules;
- a *decision-making unit* which performs the inference operations on the rules;
- a *fuzzification interface* which transforms the crisp inputs into degrees of match with linguistic values; and
- a *defuzzification interface* which transforms the fuzzy results of the inference into a crisp output.

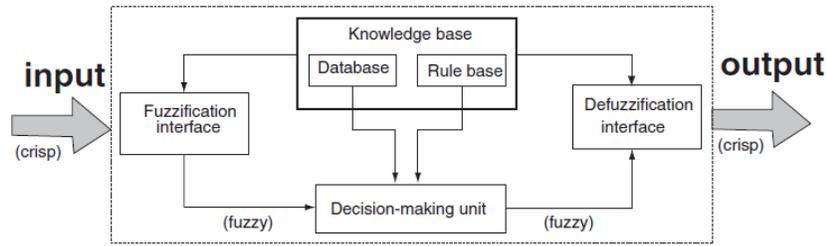


Figure 4. Fuzzy Controller Operation

The working of FIS is as follows. The crisp input is converted in to fuzzy by using fuzzification method. After fuzzification the rule base is formed. The rule base and the database are jointly referred to as the *knowledge base*. Defuzzification is used to convert fuzzy value to the real world value which is the output. The steps of *fuzzy reasoning* (inference operations upon fuzzy IF–THEN rules) performed by FISs are:

1. Compare the input variables with the membership functions on the antecedent part to obtain the membership values of each linguistic label. (this step is often called *fuzzification*.)
2. Combine (through a specific *t*-norm operator, usually multiplication or min) the membership values on the premise part to get *firing strength (weight)* of each rule.
3. Generate the qualified consequents (either fuzzy or crisp) or each rule depending on the firing strength.
4. Aggregate the qualified consequents to produce a crisp output. (This step is called *defuzzification*.)

The most important two types of fuzzy inference method are Mamdani's fuzzy inference method, which is the most commonly seen inference method. This method was introduced by Mamdani and Assilian (1975). Another well-known inference method is the so-called Sugeno or Takagi–Sugeno–Kang method of fuzzy inference process. This method was introduced by Sugeno (1985).

This method is also called as TS method. The main difference between the two methods lies in the consequent of fuzzy rules. Mamdani fuzzy systems use fuzzy sets as rule consequent whereas TS fuzzy systems employ linear functions of input variables as rule consequent. All the existing results on fuzzy systems as universal approximators deal with Mamdani fuzzy systems only and no result is available for TS fuzzy systems with linear rule consequent. Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed by Mamdani (1975) as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani's effort was based on Zadeh's (1973) paper on fuzzy algorithms for complex systems and decision processes. Mamdani type inference, as defined it for the Fuzzy Logic Toolbox, expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. It is possible, and in many cases much more efficient, to use a single spike as the output membership functions rather than a distributed fuzzy set. This is sometimes known as a *singleton* output membership function, and it can be thought of as a pre-defuzzified fuzzy set. It enhances the efficiency of the defuzzification process because it greatly simplifies the computation required by the more general Mamdani method, which finds the centroid of a two-dimensional function. Rather than integrating across the two-dimensional function to find the centroid, the weighted average of a few data points. Sugeno type systems support this type

of model. In general, Sugeno type systems can be used to model any inference system in which the output membership functions are either linear or constant.

Mamdani's fuzzy inference system is divided into four major steps: fuzzification, rule evaluation, aggregation of the rule outputs and defuzzification. Michio Sugeno use a singleton as a membership function of the rule consequent part. The following definition shows the Mamdani and Sugeno fuzzy rule base

$$\begin{aligned} & \text{if } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z \text{ is } C \text{ 'mamdani'} \\ & \text{if } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z \text{ is } f(x, y) \text{ 'sugeno'} \end{aligned} \quad (13)$$

When x and y have crisp values fuzzification calculates the membership degrees for antecedent part. Rule evaluation focuses on fuzzy operation (AND/OR) in the antecedent of the fuzzy rules. The aggregation is used to calculate the output fuzzy set and several methodologies can be used in fuzzy logic controller aggregation, namely, Max-Min aggregation, Sum-Min aggregation, Max-bounded product, Max-drastic product, Max-bounded sum, Max-algebraic sum and Min-max. Two most common methods that used in fuzzy logic controllers are Max-min aggregation and Sum-min aggregation. Max-min aggregation defined as below;

$$\mu_U(x_k, y_k, U) = \mu_{\cup_{i=1}^r FR^i}(x_k, y_k, U) = \max \left\{ \min_{i=1}^r \left[\mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U) \right] \right\} \quad (14)$$

The Sum-min aggregation defined as below

$$\mu_U(x_k, y_k, U) = \mu_{\cup_{i=1}^r FR^i}(x_k, y_k, U) = \sum \min_{i=1}^r \left[\mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U) \right] \quad (15)$$

where r is the number of fuzzy rules activated by x_k and y_k and also $\mu_{\cup_{i=1}^r FR^i}(x_k, y_k, U)$ is a fuzzy interpretation of $i - th$ rule. Defuzzification is the last step in the fuzzy inference system which it is used to transform fuzzy set to crisp set. Consequently defuzzification's input is the aggregate output and the defuzzification's output is a crisp number. Centre of gravity method (COG) and Centre of area method (COA) are two most common defuzzification methods, which COG method used the following equation to calculate the defuzzification

$$COG(x_k, y_k) = \frac{\sum_i U_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)}{\sum_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)} \quad (16)$$

and COA method used the following equation to calculate the defuzzification

$$COA(x_k, y_k) = \frac{\sum_i U_i \cdot \mu_u(x_k, y_k, U_i)}{\sum_i \mu_u(x_k, y_k, U_i)} \quad (17)$$

Where $COG(x_k, y_k)$ and $COA(x_k, y_k)$ illustrates the crisp value of defuzzification output, $U_i \in U$ is discrete element of an output of the fuzzy set, $\mu_u(x_k, y_k, U_i)$ is the fuzzy set membership function, and r is the number of fuzzy rules.

According to above, model free fuzzy controller has many challenges to control of nonlinear systems. However fuzzy controller has many applications to design nonlinear model-free controller but it has many challenges to high quality performance. To reduce

fuzzy controller challenges, PID controller is used as hybrid technique to design parallel hybrid fuzzy controller in this research. This research covers the design of fuzzy control schemes that contain, besides a fuzzy controller, other control elements known from the classical control practice. We discuss typical problems that may occur in cases of a parallel and multimode operation, such as the chattering problem and the problem of providing bumpless transitions among controller operating modes.

In order to get control schemes that would be less sensitive to parameter variations than traditional linear PID controllers, let us analyze the hybrid controller structure shown in Figure 3. As can be seen, it is a controller that contains a PD-type fuzzy and a linear PID control algorithm. It has a single input, error signal $e(k)$, which internally yields another fuzzy controller input, change in error signal $\Delta e(k)$.

This controller is meant as a multimode controller, which has two modes of operation dictated by the mode of operation selector (Figure 3). The change of modes depends on the magnitudes of fuzzy controller inputs according to the following set of relations:

$$\begin{aligned} 1. e(k) \in ZE \text{ and } \Delta e(k) \in ZDE &\rightarrow S_1 = OFF, \\ 2. e(k) \notin ZE \text{ and } \Delta e(k) \in ZDE &\rightarrow S_1 = ON, \end{aligned} \quad (18)$$

where ZE and ZDE are zero fuzzy subsets of the fuzzy controller inputs. The fuzzy control algorithm, activated when switch $S = \text{“ON,”}$ acts in the case of sufficiently large reference input changes, while the PID control algorithm, activated, mainly supports steady-state accuracy and cancels disturbance effects. Both controllers operate together in the case of moderate control error values (usually due to the impact of disturbances). Therefore, PID controller parameters K_P, K_v, K_i may be specified. In a variant of the discussed hybrid fuzzy controller, a PD-type fuzzy controller can take the role of the PD controller. Then the PD-type fuzzy controller would act only in the case of sufficiently large reference input changes, while the PID-type fuzzy controller would overtake the control in other cases. In this way, there would be only two basic operating modes. Regarding the software implementation, special attention must be paid to the switching of operating modes, as the hybrid fuzzy controller contains two control algorithms, which may work either separately or together. In order to avoid the chattering problem of two control algorithms, switching from the integral (PID) to the non-integral (PD-fuzzy) mode of operation should be made in such a way that the controller output value in the previous mode becomes the initial value for the controller output in the current mode. The fuzzy control algorithm belongs to the group of nonlinear PD-type control algorithms. Seven linguistic subsets have been defined for both inputs: NL, NM, NS, Z, PS, PM, and PL. Based on knowledge about the characteristics of the position control loop, the maximum values for both inputs and the output of the fuzzy position controller can be estimated. The distribution of membership functions related to normalized e and Δe subsets is shown in Figure 5. Different forms of membership functions can be used, but experiments have proved that trapezoidal forms contribute the most to achieving lower sensitivity to parameter variations in the designed fuzzy controller.

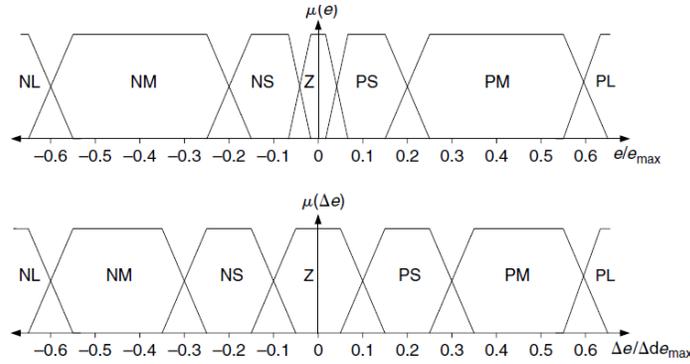


Figure 5. Input Membership Functions of a Hybrid Fuzzy Controller

The universe of discourse of the fuzzy controller output is discrete and contains 15 uniform fuzzy subsets. The distribution of accompanying membership functions is symmetrical and slightly nonlinear because of one extra subset added next to the zero subset to ensure a smooth change of operating modes defined by relations (Table. 1). The corresponding centroids have the following values: $-1, -0.667, -0.5, -0.333, -0.167, -0.0083, -0.0042, 0, 0.0042, 0.0083, 0.167, 0.333, 0.5, 0.667,$ and 1 . The fuzzy rule table of the hybrid controller fuzzy controller is shown in Table 1.

Table 1. The Fuzzy Rule Table of a Hybrid Fuzzy Controller

	NLE	NME	NSE	ZE	PSE	PME	PLE
NLDE	1	1	0.667	0.5	0.333	0.0083	0
NMDE	1	1	0.667	0.167	0.0083	-0.0042	-0.0083
NSDE	1	0.667	0.167			-0.333	-0.5
ZDE	0.833	0.333	0.0083	0	0	-0.333	-0.833
PSDE	0.667	0.333	0	-0.0083	-0.167	-0.667	-1
PMDE	0.0083	-0.0083	-0.167	-0.333	-0.667	-1	-1
PLDE	-0.0083	-0.167	-0.333	-0.5	-0.667	-1	-1

To improve the robustness and stability, sliding mode controller is used as reference model controller for adaptive method. Sliding mode controller is an influential nonlinear controller to certain and uncertain systems which it is based on system's dynamic model. The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness. Sliding mode control theory for control joint of robot manipulator was first proposed in 1978 by Young to solve the set point problem ($\dot{q}_d = \mathbf{0}$) by discontinuous method in the following form;

$$\tau_{(q,t)} = \begin{cases} \tau_i^+(q,t) & \text{if } S_i > 0 \\ \tau_i^-(q,t) & \text{if } S_i < 0 \end{cases} \quad (19)$$

where S_i is sliding surface (switching surface), $i = 1, 2, \dots, n$ for n -DOF joint, $\tau_i(q,t)$ is the i^{th} torque of joint. Sliding mode controller is divided into two main sub controllers:

- Corrective control(U_c)
- Equivalent controller(U_{eq}).

Discontinues controller causes an acceptable tracking performance at the expense of very fast switching. Conversely in this theory good trajectory following is based on fast switching, fast switching is caused to have system instability and chattering phenomenon. Fine tuning the sliding surface slope is based on nonlinear equivalent part. Design a robust controller for multi-DOF-joints is essential because these joints have highly nonlinear dynamic parameters. Consider a nonlinear single input dynamic system is defined by:

$$\mathbf{x}^{(n)} = \mathbf{f}(\vec{\mathbf{x}}) + \mathbf{b}(\vec{\mathbf{x}})\mathbf{u} \quad (20)$$

Where \mathbf{u} is the vector of control input, $\mathbf{x}^{(n)}$ is the n^{th} derivation of \mathbf{x} , $\mathbf{x} = [\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \dots, \mathbf{x}^{(n-1)}]^T$ is the state vector, $\mathbf{f}(\mathbf{x})$ is unknown or uncertainty, and $\mathbf{b}(\mathbf{x})$ is of known *sign* function. The main goal to design this controller is train to the desired state; $\mathbf{x}_d = [\mathbf{x}_d, \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d, \dots, \mathbf{x}_d^{(n-1)}]^T$, and trucking error vector is defined by:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{\mathbf{x}}, \dots, \tilde{\mathbf{x}}^{(n-1)}]^T \quad (21)$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\mathbf{U}_c = \hat{\mathbf{U}} - \mathbf{K}(\vec{\mathbf{x}}, t) \cdot \mathbf{sgn}(\mathbf{s}) \quad (22)$$

where the switching function $\mathbf{sgn}(\mathbf{S})$ is defined as

$$\mathbf{sgn}(s) = \begin{cases} \mathbf{1} & s > 0 \\ -\mathbf{1} & s < 0 \\ \mathbf{0} & s = 0 \end{cases} \quad (23)$$

and the $\mathbf{K}(\vec{\mathbf{x}}, t)$ is the positive constant.

Based on above discussion, the sliding mode control law for multi-DOF-joints is written as:

$$\mathbf{U} = \mathbf{U}_{eq} + \mathbf{U}_c \quad (24)$$

where, the model-based component \mathbf{U}_{eq} is the nominal dynamics of systems and calculated as follows:

$$\mathbf{U}_{eq} = \left[\mathbf{H}^{-1}(q) \left(\mathbf{B}(q) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + \mathbf{C}(q) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{\mathbf{S}} \right] \mathbf{H}(q) \quad (25)$$

and \mathbf{U}_c is computed as;

$$\mathbf{U}_c = \mathbf{K} \cdot \mathbf{sgn}(\mathbf{S}) \quad (26)$$

The sliding mode control of multi-DOF-joint is calculated as;

$$\begin{bmatrix} \widehat{\tau}_\alpha \\ \widehat{\tau}_\beta \\ \widehat{\tau}_\gamma \end{bmatrix} = \left[\mathbf{H}^{-1}(\mathbf{q}) \left(\mathbf{B}(\mathbf{q}) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + \mathbf{C}(\mathbf{q}) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{\mathbf{S}} \right] \mathbf{H}(\mathbf{q}) + \mathbf{K} \cdot \text{sgn}(\mathbf{S}) \quad (27)$$

The Lyapunov formulation can be written as follows [22],

$$\mathbf{V} = \frac{1}{2} \mathbf{S}^T \cdot \mathbf{H} \cdot \mathbf{S} \quad (28)$$

the derivation of V can be determined as,

$$\dot{\mathbf{V}} = \frac{1}{2} \mathbf{S}^T \cdot \dot{\mathbf{H}} \cdot \mathbf{S} + \mathbf{S}^T \mathbf{H} \dot{\mathbf{S}} \quad (29)$$

the dynamic equation of multi-DOF actuator can be written based on the sliding surface as

$$\mathbf{H} \dot{\mathbf{S}} = -\mathbf{V} \mathbf{S} + \mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} \quad (30)$$

it is assumed that

$$\mathbf{S}^T (\dot{\mathbf{H}} - 2\mathbf{B} + \mathbf{C}) \mathbf{S} = \mathbf{0} \quad (31)$$

by substituting (30) in (31)

$$\dot{\mathbf{V}} = \frac{1}{2} \mathbf{S}^T \dot{\mathbf{H}} \mathbf{S} - \mathbf{S}^T \mathbf{B} + \mathbf{C} \mathbf{S} + \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} \mathbf{S}) = \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} \mathbf{S}) \quad (32)$$

suppose the control input is written as follows

$$\widehat{\mathbf{U}} = \mathbf{U}_{\text{Nonlinear}} + \widehat{\mathbf{U}}_c = [\widehat{\mathbf{H}}^{-1}(\mathbf{B} + \mathbf{C}) + \dot{\mathbf{S}}] \widehat{\mathbf{H}} + \mathbf{K} \cdot \text{sgn}(\mathbf{S}) + \mathbf{B} + \mathbf{C} \mathbf{S} \quad (33)$$

by replacing

$$\dot{\mathbf{V}} = \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} - \widehat{\mathbf{H}} \dot{\mathbf{S}} - \widehat{\mathbf{B}} + \mathbf{C} \mathbf{S} - \mathbf{K} \text{sgn}(\mathbf{S})) = \mathbf{S}^T (\widetilde{\mathbf{H}} \dot{\mathbf{S}} + \widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S} - \mathbf{K} \text{sgn}(\mathbf{S})) \quad (34)$$

and

$$|\widetilde{\mathbf{H}} \dot{\mathbf{S}} + \widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S}| \leq |\widetilde{\mathbf{H}} \dot{\mathbf{S}}| + |\widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S}| \quad (35)$$

the Lemma equation in multi-DOF actuator can be written as follows

$$\mathbf{K}_u = [|\widetilde{\mathbf{H}} \dot{\mathbf{S}}| + |\mathbf{B} + \mathbf{C} \mathbf{S}| + \eta]_i, i = 1, 2, 3, 4, \dots \quad (36)$$

and finally;

$$\dot{\mathbf{V}} \leq - \sum_{i=1}^n \eta_i |\mathbf{S}_i| \quad (37)$$

The next step to design proposed method is design a filter to reduce or eliminate the chattering. The main disadvantage of Passive Filters is that the amplitude of the output signal is less than that of the input signal, ie, the gain is never greater than unity and that the load impedance affects the filters characteristics. With passive filter circuits containing multiple stages, this loss in signal amplitude called “Attenuation” can become quiet severe. One way of restoring or controlling this loss of signal is by using amplification through the use of **Active Filters**. As their name implies, **Active Filters** contain active components such as operational amplifiers, transistors or FET’s within their circuit design. They draw their power from an external power source and use it to boost or amplify the output signal. Filter amplification can also be used to either shape or alter the frequency response of the filter circuit by producing a more selective output response, making the output bandwidth of the filter narrower or even wider. Then the main difference between a “passive filter” and an “active filter” is amplification. An active filter generally uses an operational amplifier (op-amp) within its design and an Op-amp has high input impedance, low output impedance and a voltage gain determined by the resistor network within its feedback loop. Unlike a passive high pass filter which has in theory an infinite high frequency response, the maximum frequency response of an active filter is limited to the Gain/Bandwidth product (or open loop gain) of the operational amplifier being used. Still, active filters are generally easier to design than passive filters; they produce good performance characteristics, very good accuracy with a steep roll-off and low noise when used with a good circuit design. The most common and easily understood active filter is the **Active Low Pass Filter**. Its principle of operation and frequency response is exactly the same as those for the previously seen passive filter, the only difference this time is that it uses an op-amp for amplification and gain control. The simplest form of a low pass active filter is to connect an inverting or non-inverting amplifier, the same as those discussed in the Op-amp tutorial, to the basic RC low pass filter circuit as shown in Figure 6.

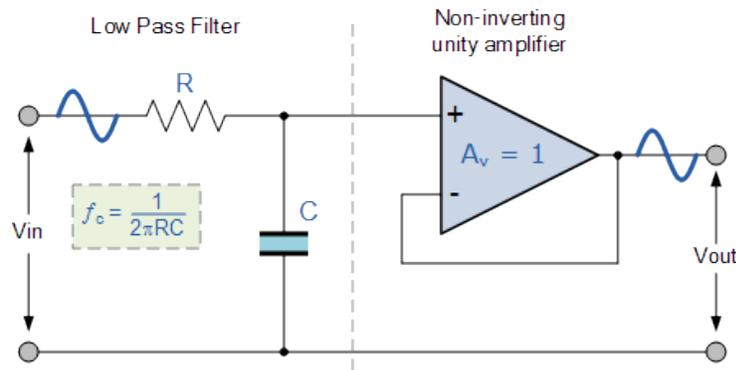


Figure 6. Ideal Low Pass Filter

This first-order low pass active filter, consists simply of a passive RC filter stage providing a low frequency path to the input of a non-inverting operational amplifier. The amplifier is configured as a voltage-follower (Buffer) giving it a DC gain of one, $A_v = +1$ or unity gain as opposed to the previous passive RC filter which has a DC gain of less than unity. The advantage of this configuration is that the op-amps high input impedance prevents excessive loading on the filters output while its low output impedance prevents the filters cut-off frequency point from being affected by changes in the impedance of the load. While this configuration provides good stability to the filter, its main disadvantage is that it has no

voltage gain above one. However, although the voltage gain is unity the power gain is very high as its output impedance is much lower than its input impedance. If a voltage gain greater than one is required we can use the following filter circuit. As with the passive filter, a first-order Low Pass Active Filter can be converted into a second-order low pass filter simply by using an additional RC network in the input path. The frequency response of the second-order low pass filter is identical to that of the first-order type except that the stop band roll-off will be twice the first-order filters at 40dB/decade (12dB/octave). Therefore, the design steps required of the second-order active low pass filter are the same. Figure 7 shows the second order nonlinear filter.

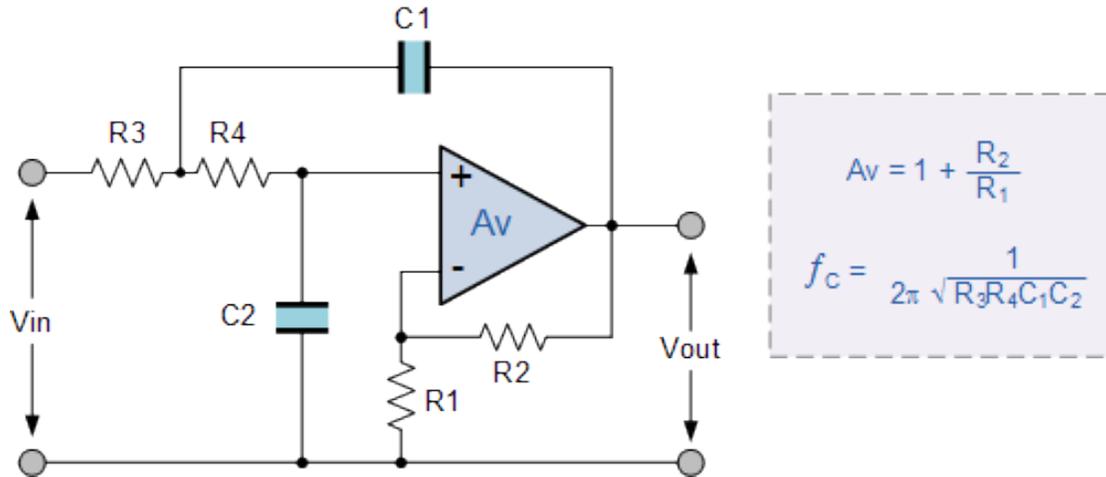


Figure 7. Second Order Nonlinear Filter

4. Results and Discussion

Proposed adaptive controller, conventional linear PID controller and fuzzy hybrid controller are compared. This simulation is used to control position of three degrees of freedom spherical motor without and with external disturbance. The simulation was implemented in MATLAB/SIMULINK environment. These systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems and applied to nonlinear dynamic of these controllers.

Tracking performances: To evidence the power of proposed method; PID controller, hybrid fuzzy controller and adaptive proposed methodology are compared. Regarding to Figure 8; PID controller has 40% under shoot besides about 4 second rise time but the steady state error is near to zero. Hybrid fuzzy logic controller has 4% overshoot besides 0.8 second rise time and the steady state error in this methodology is about 7×10^{-3} . However the rise time in proposed methodology is bigger than hybrid methodology (about 1.1 second) but this method can eliminate the overshoot and undershoot and the steady state error in this method is equal to the zero. Regarding to these three methodology hybrid fuzzy controller is faster than PID and proposed method but it has overshoot and steady state error.

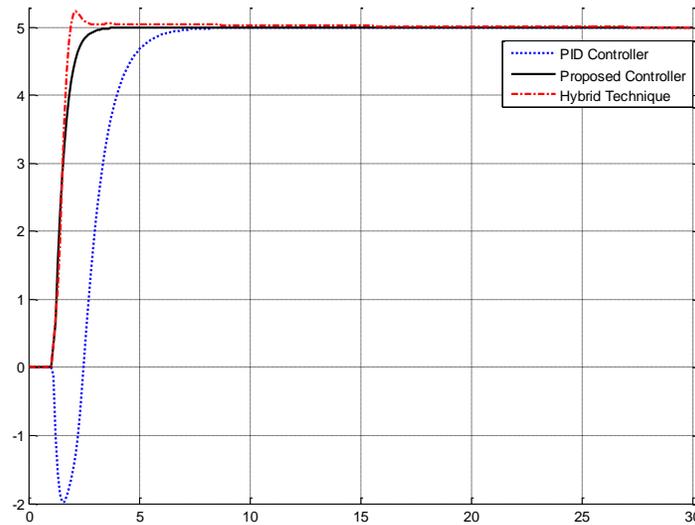


Figure 8. Comparison between Three Types Controller

Disturbance Rejection: Figure 9 shows the power disturbance elimination in proposed method, fuzzy hybrid controller and linear controller in presence of external disturbance and uncertainty parameters. Regarding to Figure 8, PID controller has moderate fluctuations. It has undershoot and steady state error. According to following Figure, PID controller has challenge to robustness. Hybrid fuzzy controller has oscillation in presence of uncertainty. However this controller has a good rise time but the main challenge of this controller is stability in presence of uncertainty. Proposed controller eliminates the fluctuation in presence of uncertainty. This type of controller is more robust than PID and hybrid fuzzy controller.

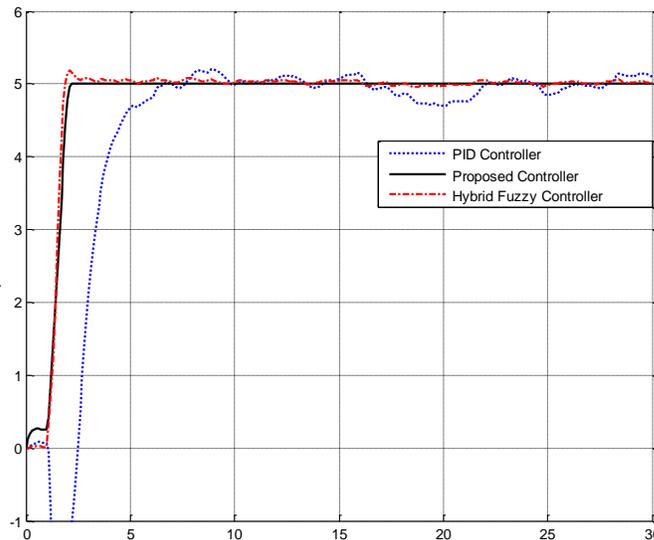


Figure 9. Comparison between Three Types Controller in Presence of External Disturbance

5. Conclusion

Multi degree of freedom actuators are highly nonlinear, MIMO and time variant system. Control these types of systems is the main challenge in this research. In this research hybrid fuzzy controller is used to improve the systems performance compare to pure PID controller and fuzzy logic controller. To improve the robust factors as well stability, model reference adaptive methodology based on variable structure methodology is used. Regarding to test and result part, proposed adaptive control is more robust than PID and hybrid fuzzy controller. This method has a power of overshoot and undershoot eliminate as well reduce the fluctuation in presence of uncertainty.

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Iranian center of Advance Science and Technology (IRAN SSP) is one of the independent research centers specializing in research and training across of Control and Automation, Electrical and Electronic Engineering, and Mechatronics & Robotics in Iran. At IRAN SSP research center, we are united and energized by one mission to discover and develop innovative engineering methodology that solve the most important challenges in field of advance science and technology. The IRAN SSP Center is instead to fill a long standing void in applied engineering by linking the training a development function one side and policy research on the other. This center divided into two main units:

- Education unit
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