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Periodic Non-uniform Sampling for Sparse Signals in shift-invariant Spaces*

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Abstract

Periodic Non-uniform sampling in shift-invariant spaces is an effective method for low-rate sampling of continuous time sparse signals, but, recovery of sampled time sparse signals must be unstable in general. In the paper, the possibility of stable recovery under a combination of sufficient sparsity is proposed. The sampling and reconstruction of sparse signals were transformed into matrix and vector operations by using minimum L1 normal. Necessary condition of reconstruction is analyzed. Finally, we are taking multi-band sinusoidal signals as an example to prove that the method can achieve the sampling and reconstruction of sparse signals from recovery successful rate and integer of system.

Keywords: *minimum L1 normal, periodic non-uniform sampling, sparse signals*

1. Introduction

It is an important purpose of Software Defined Radio (SDR) that to keep AD converter closer to RF antenna [1]. Therefore, AD converter with broadband and high speed is regularly used to sample RF signals. With the development of wireless technology, the carrier frequency of narrowband signals is becoming higher. A higher sampling speed of AD converter is demanded to sample the narrowband signals with the traditional SHANNON theorem, which is neither economical nor practicable [2-3]. Reference [4-5], in its initial proposal of periodic non-uniform sampling theory, states that the periodic non-uniform sampling can be used to sample any signals at a speed twice the greatest carrier frequency of interest without distortion. Generally, periodic non-uniform sampling approach adopts iterative method to reconstruct sampled signal on condition that the frequency band of the signal is given. But traditional periodic non-uniform sampling approach is low in efficiency when those signals have sparse expansion [6]. Reference [7-8] point out that if a signal has a sparse expansion, one can discard the most coefficients without much perceptual loss. Typical reconstruction algorithms consist of multiple measurement vectors approach [9], minimum L1 normal approach [7], orthogonal matching pursuit approach [10], *etc.* The application of compressed sensing in periodic non-uniform sampling can effectively solve problems in sampling and reconstruction of blind and spare analog signals.

This paper expounds the principle of periodic non-uniform sampling approach based on union of subspaces [11], which transitions the sampled signal reconstruction into a vector equation group for solution. The sampling and reconstruction of sparse signal is realized finally through test and simulation verification after the signal's unique

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expression is determined via minimum L1 normal algorithm and the signal's complete reconstruction is finalized through uniform sampling of reconstructed sequence.

2. Principle of Periodic Non-uniform Sampling

The periodic non-uniform sampling and reconstruction model for signal *x(t)* is given in Figure 1. Assuming there are *s* sampling sequences, which are defined as:

$$a_i(t) = \sum_{n=-\infty}^{+\infty} \delta(t - Tn - i\tau) \qquad 0 \leqslant i \leqslant s-1 \tag{1}$$

Where, T is sampling period; τ is time interval between two adjacent sampling channels.

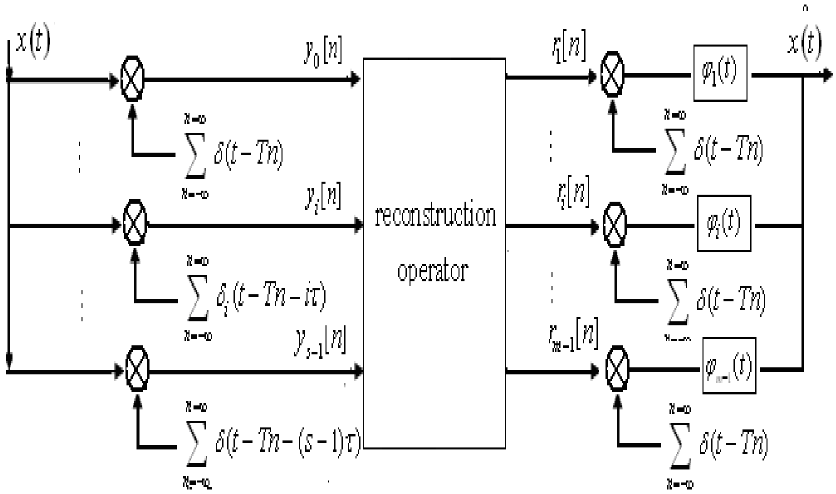

Figure 1. The Structure of Periodic Nonuniform Samplin

Use $a_i(t)$ $(0 \leqslant i \leqslant s)$ to sample $x(t)$:

$$y_i(t) = x(nT + i\tau)\sum_{n=-\infty}^{n=\infty} \delta(t - nT - (s-1)\tau) \tag{2}$$

Applying Fourier transform to $y_i(t)$ will get:

$$Y_i(\omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(\omega - 2\pi n/T)\, e^{-j2\pi ni\tau/T} \tag{3}$$

To reconstruct $x(t)$ from sampling point $Y_i(\omega)$, suppose $x(t)$ is in L_2 's subspace $V(\varphi)$. Suppose $W(t) = [\varphi_0(t), \varphi_1(t), \cdots, \varphi_{m-1}(t)]'$ related to *m* order is generated, and the space is defined as:

$$V(\varphi) = \left\{ \sum_{p=0}^{m-1} \sum_{n \in z} r_p[n]\varphi_p(t - nT) : r_p[n] \in L_2 \right\}$$

Where, *T* is sampling period and $\varphi(t)$ is space generation function.

For any $x(t) \in V(\varphi)$:

$$x(t) = \sum_{p=0}^{m-1} \sum_{n \in z} r_p[n]\, \varphi_p(t - nT) \tag{4}$$

The only restriction on the choice of the function train$\{\varphi_p(t)\}$ is for guaranteeing a unique stable representation of any signal in $V(\varphi)$ by sequence $\{r_p[n]\}$, so the generators $\varphi(t)$ must form a Riesz basis of L_2. In other words, there exist two constants A>0 and B<∞, such that:

$$A\left\|\mathrm{r}[n]\right\|_2^2 \leq \left\|\sum_{p=0}^{m-1}\sum_{n\in z} r_p[n]\varphi_p(t-nT)\right\|_2^2 \leq B\left\|\mathrm{r}[n]\right\|_2^2$$

Where, $\left\|\bullet\right\|_2$ is l_2 normal.

The above-mentioned subspace $V(\varphi)$ is a single space, the more interesting aspect we are considering is that $x(t)$ lies in a union of subspaces $\bigcup V_p(\varphi)$ ($0\leq p\leq m$-1):

$$x(t)\in\bigcup_p V_p(\varphi) \quad 1\leqslant p\leqslant m-1 \tag{5}$$

The union of subspaces is purposed to obtain the value of reconstructed sequence $r[n]$. Applying Fourier transform to formula (4) as:

$$X(\omega)=\sum_{p=0}^{m-1} R_p(\omega)\,\psi_p(\omega) \tag{6}$$

Where, $R(\omega)s \geqslant m\,r[n]=[r_0[n],\ r_1[n],\cdots,\ r_{m\text{-}1}[n]]$ is $r_p[n]$'s discrete Fourier transform, and $\psi_p(\omega)$ is $\varphi_p(t)$'s continuous Fourier transform. Substitute function (6) into (3) to get:

$$\begin{aligned} Y_i(\omega) &= \frac{1}{T}\sum_{n=-\infty}^{+\infty}\sum_{p=0}^{m-1} R_p(\omega-2\pi n/T)\psi_p(\omega-2\pi n/T)e^{-j2\pi ni\tau/T} \\ &= \frac{1}{T}\sum_{p=0}^{m-1} R_p(\omega)\sum_{n=-\infty}^{+\infty}\psi_p(\omega-2\pi n/T)e^{-j2\pi ni\tau/T} \end{aligned} \tag{7}$$

Where, $R_p(\omega)$ is a function with a $2\pi/T$ period. Rewrite function (7) in matrix form:

$$\boldsymbol{Y}(\omega)=\boldsymbol{H}(\omega)\boldsymbol{R}(\omega) \tag{8}$$

Where,

$$\boldsymbol{Y}(\omega)=(Y_0(\omega),Y_1(\omega),\cdots,Y_{s-1}(\omega))'$$

$$\boldsymbol{R}(\omega)=(R_0(\omega),R_1(\omega),\cdots,R_{m-1}(\omega))'$$

$$\boldsymbol{H}(\omega)=\begin{bmatrix} h_{0,0} & h_{0,1} & \cdots & h_{0,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{s-1,0} & h_{s-1,1} & \cdots & h_{s-1,m-1}\end{bmatrix}$$

$$h_{i,p}(\omega)=\frac{1}{T}\sum_{n=-\infty}^{+\infty}\psi_p(\omega-2\pi n/T)\mathrm{e}^{-\mathrm{j}2\pi ni\tau/T}$$

To sum up, the signal sampling and reconstruction is converted to formula (8) for the unique solution of $R(\omega)$. From formula (8), if $s\geq m$, or the number of equations is greater than or equal to the number of unknowns, and $R(\omega)$ can be obtained to solve linear equation group, then the periodic non-uniform sampling approach proposed herein can completely reconstruct signals.

3. Reconstruction Approach

In this paper, we are taking multiband sparse signal as example to analyze the reconstruction approach of the periodic non-uniform sampling system on particular signals. Suppose $x(t)$ is a RF signal with k sparsity, then in the union of subspaces there are k subspaces at most having effective values.; further, according to formula (4), in $r[n]=[r_0[n],r_1[n],\ \cdots,\ r_{m-1}[n]]$, there are k elements at most are nonzero. In order to raise sampling efficiency and completely reconstruct $x(t)$, the number of sampling channel s must be $k < s \ll m$, then formula (8) is undetermined equation and solution of $R(\omega)$ is unbounded. Unique sparse solution cannot be obtained directly than through the minimum L_0 normal:

$$\min \|\boldsymbol{R}(\omega)\|_0 \ \text{s.t}\ \boldsymbol{Y}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{R}(\omega) \quad (9)$$

Unique sparse solution in formula (8) is obtainable from the minimum L_0 normal in formula (9). Theorem 1 gives the unique vector group $\boldsymbol{R}(\omega)$ based on minimum L_0 normal when L_0 normal meets certain conditions.

Theorem **1 :**If $\|\boldsymbol{R}(\omega)\|_0 \leqslant kr(\boldsymbol{H}(\omega))/2$ and meets formula (9), then $\boldsymbol{R}(\omega)$ is the unique sparse solution, and $kr(H(\omega))$ is the Kruskal rank of $H(\omega)$, which is the maximal q such that every set of q columns of $H(\omega)$ is linearly independent.

Proof: Suppose $R_a(\omega)$ and $R_b(\omega)$ meet formula (9), then $\boldsymbol{H}(\omega)(R_b(\omega) - R_a(\omega)) = 0$. Plus, $\boldsymbol{H}(\omega)$ has $kr(\boldsymbol{H}(\omega))$ linearly independent column, so:

$$\|R_b(\omega) - R_a(\omega)\|_0 \geqslant kr(\boldsymbol{H}(\omega)) \quad (10)$$

Plus, $\max\{\|R_b(\omega)\|_0, \|R_a(\omega)\|_0\} \leqslant kr(\boldsymbol{H}(\omega))/2$, so:

$$\|R_b(\omega)\|_0 + \|R_a(\omega)\|_0 \leqslant kr(\boldsymbol{H}(\omega))$$

It is known given the nature of normal that:

$$\|R_b(\omega) - R_a(\omega)\| \leqslant \|R_b(\omega)\|_0 + \|R_a(\omega)\|_0 \leqslant kr(H(\omega))$$

But it is contradictory to formula (10), so $R_b(\omega) = R_a(\omega)$. Q.E.D.

Given the difficulty of solving formula (9) directly based on minimum L_0 normal, we adopt the minimum L_1 normal algorithm to obtain the unique expression of equation group (8).

3.1. Minimum L1 Normal Approach

According to analysis aforesaid, minimum L_1 normal approach is purposed to obtain the unique sparse expression of $R(\omega)$. The solving process is: 1) To obtain the positional parameter set B where the sampled signal's zero element sits through minimum L_1 normal approach, and to take new matrix $\boldsymbol{H}_B(\omega)$ formed by column vectors from sensing matrix $\boldsymbol{H}(\omega)$ that are relative to positional parameter set B; 2) To find solution of the new generalized inverse matrix $\boldsymbol{H}_B^-$, therefore obtain k nonzero $r[n]$ sequences; 3) To obtain all m $r[n]$ sequences through interpolation, the structure of which is shown in Figure 2.

If factor s in the sensing matrix $\boldsymbol{H}(\omega)$ is smaller than m, then formula (8) has unbounded solutions, which sit in a space with (m-s) dimensions:

$$T=N(\boldsymbol{H}(\omega))+\boldsymbol{R}^*(\omega)$$

Where, $N(\boldsymbol{H}(\omega))$ is $\boldsymbol{H}(\omega)$'s null space. Formula (8) can be rewritten as:

$$\boldsymbol{Y}(\omega) = \boldsymbol{H}(\omega)(\boldsymbol{R}^*(\omega) + \boldsymbol{O}(\omega))$$

Where, $\boldsymbol{O}(\omega)$ is the vector in null space $N(\boldsymbol{H}(\omega))$.

Therefore, this paper aims to obtain solution $\boldsymbol{R}^*(\omega)$ in null space $N(\boldsymbol{H}(\omega))$ through minimum L_1 normal.

$$\min \|\boldsymbol{R}(\omega)\|_1 \ \text{s.t}\ \boldsymbol{Y}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{R}(\omega) \quad (11)$$

Suppose $R_{\text{opt}}(\omega)$ meets the optimal solution of (8), then:

$$\boldsymbol{Y}(\omega)=\boldsymbol{H}(\omega)\,\boldsymbol{R}_{\text{opt}}(\omega) \tag{12}$$

Positional parameter set B is the position of nonzero elements in $\boldsymbol{R}_{\text{opt}}(\omega)$, then new matrix $\boldsymbol{H}_B(\omega)$ formed by column vectors corresponding to nonzero elements in $\boldsymbol{H}(\omega)$ and $\boldsymbol{R}_{\text{opt}}(\omega)$, then:

$$\boldsymbol{Y}(\omega)=\boldsymbol{H}_B(\omega)\,\boldsymbol{R}^B(\omega) \tag{13}$$

Where, RB(ω) is a vector formed by nonzero elements in $R_{\text{opt}}(\omega)$.

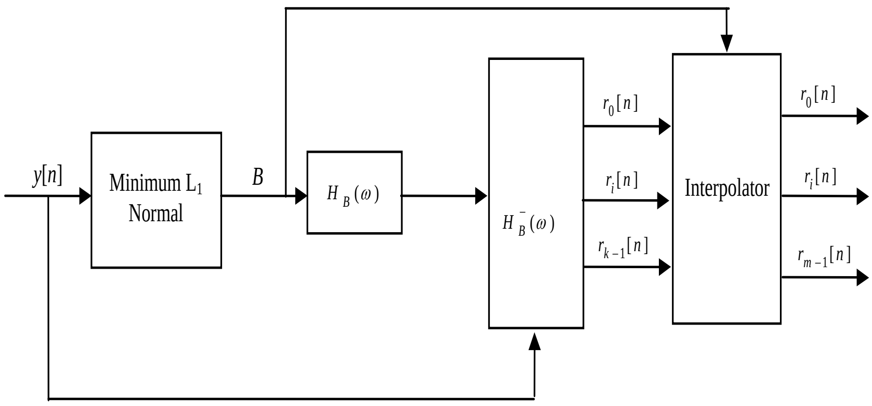

Figure 2. Partial Inner Structure Diagram of L_1 Normal

In $\boldsymbol{R}_{\text{opt}}(\omega)$, there are k nonzero elements at most, then $\boldsymbol{H}_B(\omega)$ has k column vectors at most. According to analysis of Theorem 1, one necessary condition for complete reconstruction of k signal by the periodic non-uniform system is $kr(\boldsymbol{H}(\omega))\geq 2k$, then $\boldsymbol{H}_B(\omega)$ must be column non-singular matrix, otherwise, the number of nonzero elements in $\boldsymbol{R}_{\text{opt}}(\omega)$ will drop, which is contradictory to optimality. Therefore, the Penrose generalized inverse matrix of $\boldsymbol{H}_B(\omega)$ is defined as:

$$\boldsymbol{H}_B^{-}(\omega)=(\boldsymbol{H}_B^{\text{H}}(\omega)\boldsymbol{H}(\omega))^{-1}\boldsymbol{H}_B^{\text{H}}(\omega) \tag{14}$$

Where, $(\cdot)^{\text{H}}$ is conjugate transpose. Solution of formula (13) is thereby obtained as:

$$\boldsymbol{R}^B(\omega)=(\boldsymbol{H}_B^{\text{H}}(\omega)\boldsymbol{H}(\omega))^{-1}\boldsymbol{H}_B^{\text{H}}(\omega)\boldsymbol{Y}(\omega) \tag{15}$$

When nonzero elements in positional parameter set B in $r[n]$ is set, connect the interpolator's input channel with corresponding output channel, and connect the nonzero element's output channel to zero, then all m reconstructed sequences $r[n]$ are obtained, that is:

$$r_B[n]=H_B^{-}y[n]$$

$$r_i[n]=0 \qquad i\notin B$$

Where, $r_B[n]=[r_0[n];\ r_1[n],\cdots,\ r_{k-1}[n]]$.

3.2. Analysis on Reconstruction Condition of Minimum L1 Normal Approach

According to the analysis above, the L_1 normal approach is used to determine the frequency band's positional parameter set B of nonzero elements in sampled signals, and then the unique reconstructed sequence is obtained via generalized inverse matrix, which attributes to the reconstruction of the periodic non-uniform sampling. The minimum L_1 normal approach given in Theorem 2 fully meets complete reconstruction conditions.

Theorem **2** If the number of $\boldsymbol{R}(\omega)$'s nonzero elements is k, positional parameter set is B, then column in $\boldsymbol{H}(\omega)$ corresponding to nonzero elements constitute matrix $\boldsymbol{H}_B(\omega)$. When

$$\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}\left|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)\right|_1\right)<1$$

the complete reconstruction is obtainable through minimum L_1 normal approach. Where, $\boldsymbol{H}_B^{-}(\omega)$ is $\boldsymbol{H}_B(\omega)$'s generalized inverse matrix, $\boldsymbol{h}_i(\omega)$, $i\notin B$ denotes the column vector in $\boldsymbol{H}(\omega)$ without $\boldsymbol{H}_B(\omega)$.

proof: Suppose optimal solution is $\boldsymbol{R}^B(\omega)$, $\boldsymbol{R}_r(\omega)$ can be any solution of $\boldsymbol{H}(\omega)\boldsymbol{R}(\omega)=\boldsymbol{Y}(\omega)$, then:

$$\left\|\boldsymbol{R}^B(\omega)\right\|_1=\left\|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{H}_B(\omega)\boldsymbol{R}^B(\omega)\right\|_1=\left\|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{Y}(\omega)\right\|_1=\left\|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{H}(\omega)\boldsymbol{R}_r(\omega)\right\|_1$$

$$<\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)|_1\right)\left\|\boldsymbol{R}_r(\omega)\right\|_1$$

When $\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)|_1\right)<1$, $\left\|\boldsymbol{R}^B(\omega)\right\|_1<\left\|\boldsymbol{R}_r(\omega)\right\|_1$.

Therefore, when $\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)|_1\right)<1$, signal can be completely reconstructed through minimum L_1 normal approach. Therefore, minimum L_1 normal approach can completely reconstruct signals. Define:

$$|D(\omega)|_1=\sum_{i=0}^{N-1}|d_i(\omega)|$$

$$D(\omega)=(d_0(\omega),d_1(\omega),\cdots,d_{N-1}(\omega)))$$

Where, $\|\square\|_1$ is L_1 normal. Theorem 3 gives conditions for complete reconstruction through minimum L_1 normal approach.

Theorem **3** When $\left\|\boldsymbol{R}(\omega)\right\|_0<\frac{1}{2}(\mu^{-1}+1)$, L_1 normal can realize complete reconstruction. Where, $\mu=\max_{j\neq g}(\max_{-\pi/T\leqslant\omega<\pi/T}|\boldsymbol{h}_j^{\mathrm{H}}(\omega)\boldsymbol{h}_g(\omega)|)$.

proof: According to Theorem 2, when $\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)|_1\right)<1$, the minimum L_1 normal can realize complete reconstruction of signals.

$$\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)|_1\right)=\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega<\pi/T}|(\boldsymbol{H}_B^{-}(\omega)\boldsymbol{H}_B(\omega))^{-1}\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{h}_i(\omega)|_1\right)\leqslant$$

$$\max_{i\notin B}\left(|(\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{H}_B(\omega))^{-1}|_{1,1}\max_{-\pi/T\leqslant\omega<\pi/T}|\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{h}_i(\omega)|_1\right) \tag{16}$$

Suppose $\mu=\max_{j\neq g}\left(\max_{-\pi/T\leqslant\omega<\pi/T}\left|\left|\boldsymbol{h}_j^{\mathrm{H}}(\omega)\boldsymbol{h}_g(\omega)\right|\right.\right)$, then:

$$\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega<\pi/T}\left|\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{h}_i(\omega)\right|_1\right)\leqslant\mu k$$

Where, k is signal sparsity.

Suppose $\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{H}_B(\omega)=I_k+E$, I_k is unit matrix, then: $\left|E\right|_{1,1}<\mu(k-1)$. Then:

$$\left|\left(\boldsymbol{H}_B^{\mathrm{H}}(\omega)\boldsymbol{H}_B(\omega)\right)^{-1}\right|_{1,1}=\left|\left(I_k+E\right)^{-1}\right|_{1,1}\leqslant\frac{1}{1-\left|E\right|_{1,1}}<\frac{1}{1-\mu(k-1)} \tag{17}$$

From the formula above:

$$\max_{i\notin B}\left(\max_{-\pi/T\leqslant\omega\leqslant\pi/T}\left|\boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega)\right|_1\right)<\frac{\mu k}{1-\mu(k-1)}$$

When $\max_{i \notin B}\left(\max_{-\pi/T \leqslant \omega \leqslant \pi/T} \left| \boldsymbol{H}_B^{-}(\omega)\boldsymbol{h}_i(\omega) \right|_1 \right) < 1$, minimum L_1 normal can completely reconstruct signals. Then $k < \frac{1}{2}(\mu^{-1}+1)$. Therefore, when $\|\boldsymbol{R}(\omega)\|_0 < \frac{1}{2}(\mu^{-1}+1)$, minimum L_1 normal can realize complete reconstruction. Definite $\left|\boldsymbol{H}(\omega)\right|_{1,1} = \max_i \left(\max_{-\pi/T \leqslant \omega < \pi/T} \sum_{j=0}^{m-1} \left| \boldsymbol{h}_{i,j}(\omega) \right| \right)$.

4. Experiment and Analysis

The periodic non-uniform sampling system is constructed as model in Figure 1 to sample $x(t)$:

$$x(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) \tag{18}$$

Where, the f_1 and the f_2 are carrier frequency. The value of the f is selected from (–610 MHz, 610 MHz) at random yet meeting $x(t) \in (-610\ \text{MHz}, 610\ \text{MHz})$.

According to analysis in Section 3, the above frequency band can be equally divided into m=61 segments, where 4 segments can have energy at most, or the signal sparsity k≤4. To accomplish effective sampling via the model given as Figure 1, we suppose that the time domain expression of generation function $\varphi_p(t)(0 \le p \le m-1)$ is:

$$\varphi_p(t) = \phi(t)\mathrm{e}^{\mathrm{j}2\pi(p-30)t/T} \tag{19}$$

Where, $\phi(t) = \mathrm{sinc}(t/T)$, T is sampling period., $1/T = 20\ \text{MHz}$. Given the nature of sinc function, its space generation function can constitute Riesz basis, and can ensure signal $x(t)$ is expressed by $r_p[n]$ in $V(\varphi)$.

Therefore, according to formula (4), the frequency domain of $x(t)$ can be rewritten as:

$$X(\omega) = \sum_{p=0}^{m-1} R_p(\omega)\theta(\omega - 2\pi(p-30)t/T) \tag{20}$$

Where, $\theta(\omega)$ is the continuous Fourier transform of $\phi(t)$

Set the sampling interval τ of periodic non-uniform sampling as $T/17$. We can know that matrix $\boldsymbol{H}(\omega)$ ’s Kruskal sequence $kr(\boldsymbol{H}(\omega)) = s$. According to above division method of frequency band and formula (4), we can have:

$$x(t) \in \begin{cases} \left\{ \sum_{n \in Z} r_0[n]\varphi_0(t-nT) : \{r_0[n]\} \in l_2 \right\} \\ f \in [-610\ \mathrm{MHz}, -590\ \mathrm{MHz}) \\ \left\{ \sum_{n \in Z} r_1[n]\varphi_1(t-nT) : \{r_1[n]\} \in l_2 \right\} \\ f \in [-590\ \mathrm{MHz}, -570\ \mathrm{MHz}) \\ \left\{ \sum_{n \in Z} r_2[n]\varphi_2(t-nT) : \{r_2[n]\} \in l_2 \right\} \\ f \in [-570\ \mathrm{MHz}, -550\ \mathrm{MHz}) \\ \vdots \\ \left\{ \sum_{n \in Z} r_{30}[n]\varphi_{30}(t-nT) : \{r_{30}[n]\} \in l_2 \right\} \\ f \in [-10\ \mathrm{MHz}, 10\ \mathrm{MHz}) \\ \vdots \\ \left\{ \sum_{n \in Z} r_{58}[n]\varphi_{58}(t-nT) : \{r_{58}[n]\} \in l_2 \right\} \\ f \in [550\ \mathrm{MHz}, 570\ \mathrm{MHz}) \\ \left\{ \sum_{n \in Z} r_{59}[n]\varphi_{59}(t-nT) : \{r_{59}[n]\} \in l_2 \right\} \\ f \in [570\ \mathrm{MHz}, 590\ \mathrm{MHz}] \\ \left\{ \sum_{n \in Z} r_{60}[n]\varphi_{60}(t-nT) : \{r_{60}[n]\} \in l_2 \right\} \\ f \in [590\ \mathrm{MHz}, 610\ \mathrm{MHz}] \end{cases} \tag{21}$$

4.1. Analysis on Reconstruction Success Rate

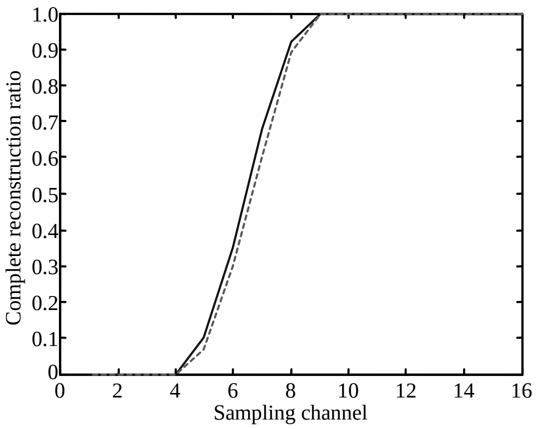

Figure 3. Analysis on Reconstruction Success Rate

Use minimum L_1 normal and orthogonal matching pursuit algorithm separately to verify reconstruction success rate of periodic non-uniform sampling based on union of subspaces herein, after 100 times tests for each, the results are as shown in Figure 3. According to analysis in Section 2 and 3, when minimum L_1 normal is used in periodic non-uniform sampling to reconstruct signal, the signal sparsity is $k < \frac{1}{2}(\mu^{-1}+1)$. For sparsity $k \le 4$ of input signal herein, a system with sampling channel $s > 8$ can realize complete reconstruction of signal with a sparsity smaller than or equal to 4 through minimum L_1 normal approach, yet a system with less than 8 sampling channels cannot realize complete reconstruction. Comparison of orthogonal matching pursuit algorithm and minimum L_1 normal approach is given in Figure 3, where solid line represents minimum L_1 algorithm and dotted line represents orthogonal matching pursuit algorithm, and the success rate of the former is higher than that of the latter.

4.2. Comprehensive Test and Analysis of the System

According to analysis in Section 3.1, number of sampling channel for test s=10.A 10×61 sensing matrix is thereby formed. According to the periodic non-uniform sampling model given in Figure 1, iterative method will not realize complete reconstruction of signals. The idea herein concerning periodic non-uniform sampling and reconstruction is to obtain unique sparse solution of undetermined equation group (8), and, at the final stage, to uniformly sample the reconstructed sequence output to realize complete reconstruction of the signal. The above analysis tells that there are 4 nonzero elements at most in 61 frequency channels. R_a and R_b are used to represent nonzero elements.

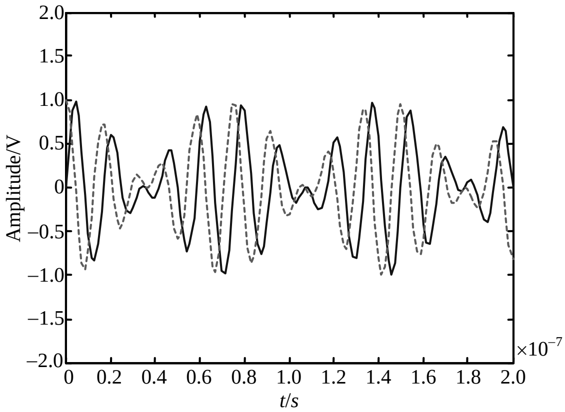

Figure 4. R_a Time Domain Diagram

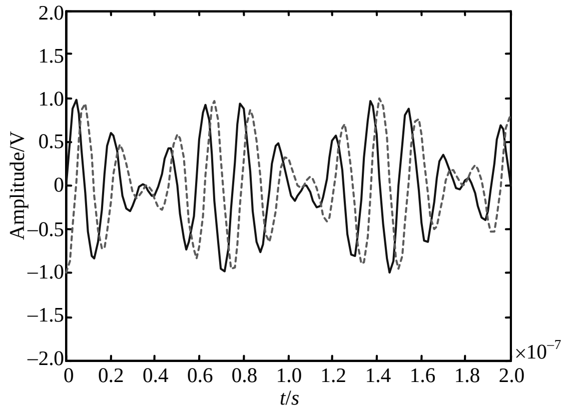

Figure 5. R_b Time Domain Diagram

Figure 4 and 5 separately represent R_a and R_b time domain graphics after interpolation function $\varphi_p(t)$, where solid line represents time graphics of reconstructed signal, and dotted line represents time domain graphics of the mirrored part of reconstructed signal.

The analysis of Figure 4 and 5 tells time domain graphics of positive and negative frequency channels corresponding to R_a and R_b. Figure 6 is the overlapping part of channel R_a and R_b, and further analysis tells that its amplitude doubles, and mirrored signal is restricted. Figure 7 is the time domain diagram of original signal, and its further comparison with Figure 6 tells that Figure 7 should be time domain graphics of reconstructed signal that is overlapped by channel R_a and R_b, thus coming signal sparsity k=2. Figure 8 is the spectrum of original signal after Fourier transform. Based on Figure 8 and formula (21), the 4 frequency spectrums of two signals of $x(t)$ only fall in only 2 frequency channels of 61. The sparsity k of signal $x(t)$ equals 2 in theory, which fully consists with the simulation result. The result of periodic non-uniform sampling is equivalent to the original signal.

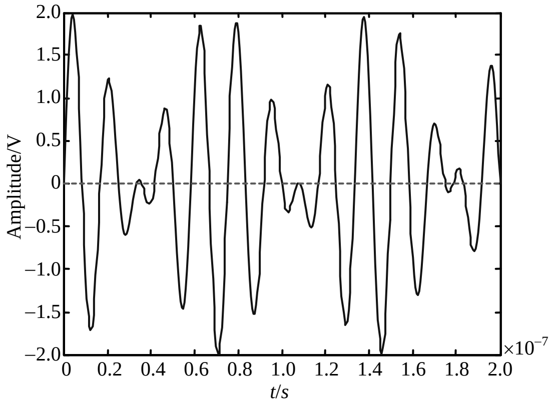

Figure 6. Time Domain Diagram of Reconstructed Signal

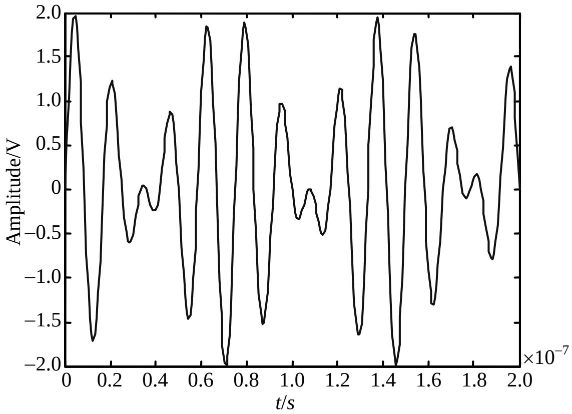

Figure 7. Time Domain Diagram of Original Signal

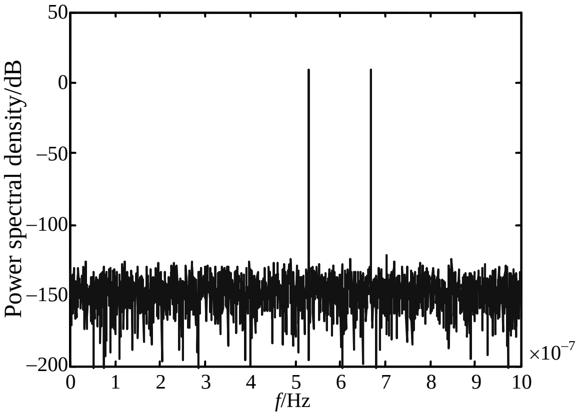

Figure 8. Signal Frequency Spectrum Diagram

5. Conclusion

Considering the low efficiency of traditional ways to sample sparse signals, a periodic non-uniform sampling system using union of subspaces is proposed in this paper, which uses the minimum L_1 normal to obtain the unique solution of undetermined equation. The approach can effectively solve problems in sampling and reconstruction of blind and spare analog signals. The approach can reduce the number of sampling channels, and save system resources and raises sampling efficiency compared with traditional periodic non-uniform sampling system. The simulations demonstrate that the periodic non-uniform sampling proposed here can completely reconstruct the original signal far below Nyquist sampling ratio.

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