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An Efficient Combination Method of Conflict Evidences

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Abstract

Evidence theory is an effective method for uncertainty reasoning, which is widely used in areas like expert system, artificial intelligence, pattern recognition and system decision. But traditional DS combination rule will produce the result contrary to intuition on the condition of high confliction. To solve the problem, this paper proposes a modified method based on the option of distance function and correction of support degree. Firstly, it introduces the Minkowsky distance as distance function of evidences and finds the support degree of each evidence in system, then corrects the support degree on the basic of its distribution, finally, it gets the weighted average of evidence by the normalized evidence credibility, and uses the DS combination rule to synthesis evidences. The simulation results demonstrate the effectiveness and reliability of modified method.

Keywords: *Evidence theory, DS combination rule, Evidence credibility, Conflict evidences*

1. Introduction

By concerning lower and upper probability, evidence theory is initially presented by Dempster in 1967. From these mathematical foundations, Shafer has shown its ability of evidence reasoning in 1976. Therefore, it is also known as D-S theory. In the development of multi-source fusion on intelligent computing and identification theory, evidence theory weights a large proportion. It is a further expansion of probability theory, as well as a kind of inaccurate reasoning theory with the ability of dealing with uncertain information. In addition, evidence theory provides the DS combination rule which can combines evidences from multi-source. Thus it has been widely and successfully applied in the field of information fusion, like spectrum sharing in cognitive radio networks, direction finding in presence of auxiliary sensors. But in some case, combination of evidences may yield conclusions contrary to what we expect or consider reasonable. In this paper, we develop a modified method based on evidence credibility. With the simulation results, we can see that the modified method can solve the problem of high conflict in combination by improving the credibility degree of evidences.

2. Review of Evidence Theory

Evidence theory firstly supposes the definition of a set of hypotheses called frame of discernment, which consists of a set of *N* mutually exclusive and exhaustive hypotheses. The frame of discernment is defined as follows:

$$\Theta = \{ H_1, H_2, \ldots, H_N \} \tag{1}$$

where, H is one hypothesis of the frame of discernment Θ .

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Then let us denote 2^{Θ} as the power set composed with 2^{N} propositions of Θ .The basic probability assignment (called BPA for short) on 2^{Θ} can range from 0 to 1, defined as follows:

$$\begin{cases} m(\Phi) = 0 \\ \sum_{A \subseteq \Theta} m(A) = 1 \end{cases} \tag{2}$$

It represents how strongly evidences support the hypothesis A . Due to lack of further information, the BPA cannot be subdivided to its proper subset. Then, we can call the hypothesis A as focus element if $m(A) > 0 (A \subseteq \Theta)$, and call the set of all focus elements as core.

Evidence theory offers an appropriate combination rule in the case of uncertain and imprecise data fusion. DS combination rule, the so-called orthogonal sum is commutative and associative is defined as follows:

$$\begin{cases} m_{\oplus}(A) = \dfrac{1}{1 - k} \sum_{A_{i1} \cap A_{i2} \cap \cdots \cap A_{in} = A} m_1(A_{i1}) \cdot m_2(A_{i2}) \cdot \cdots \cdot m_n(A_{in}) \quad \forall A \subseteq \Theta \\ m_{\oplus}(\Phi) = 0 \end{cases} \tag{3}$$

where k is the sum conflict probability, which reflects the degree of conflict between all evidences , defined as follows:

$$k = \sum_{A_{i1} \cap A_{i2} \cap \cdots \cap A_{in} = \Phi} m_1(A_{i1}) \cdot m_2(A_{i2}) \cdot \cdots \cdot m_n(A_{in}) \tag{4}$$

k represents how strongly evidences conflict. $1/(1-k)$ is normalization factor which ensures that the sum of BPA can be unit, and the BPA for null set is none.

3. Problems and Classical Solutions

3.1. Problems

When the information sources to be combined are numerous, a conflicting mass can be induced. We cannot use DS combination rule if conflict $k = 1$, and combination result will be contrary to what we expect or consider reasonable if conflict $k \to 1$. It represents that DS combination rule cannot get reasonable conclusions when evidences highly conflict. We give two examples here to demonstrate the situation as described below.

Example 1: Assuming that $\Theta = \{A, B, C\}$, the basic probability assignment of four evidences is as follows:

$$\begin{cases} m_1 : m_1(A) = 0.98, m_1(B) = 0.01, \ m_1(C) = 0.01 \\ m_2 : m_2(A) = 0, \quad m_2(B) = 0.01, m_2(C) = 0.99 \\ m_3 : m_3(A) = 0.9, \ m_3(B) = 0, \quad m_3(C) = 0.1 \\ m_4 : m_4(A) = 0.9, \ m_4(B) = 0.1, \quad m_4(C) = 0 \end{cases}$$

We can obviously see that $m_2(A) = m_3(B) = m_4(C) = 0$. By calculating, we can get the result of sum conflict probability $k = 1$. We can see from the formula (3), when $k = 1$, the denominator is zero, which directly lead to wrong combination. So we cannot use DS combination rule when sum conflict probability $k = 1$.

Example 2: Assuming that $\Theta = \{A, B, C\}$, the basic probability assignment of two evidences is as follows:

$$\begin{cases} m_1 : m_1(A) = 0.9, m_1(B) = 0.1, m_1(C) = 0 \\ m_2 : m_2(A) = 0, \quad m_2(B) = 0.1, m_2(C) = 0.9 \end{cases}$$

By calculating, we can get the result of sum conflict probability $k = 0.99$,which means the two evidences highly conflict. We can see from the formula (3), when $k = 0.99$, the denominator equals to 0.01, we can get the conclusion of combination rule as follows:

$$m_\oplus : m_\oplus(A) = 0, m_\oplus(B) = 1, m_\oplus(C) = 0$$

Although that the BPA of both evidences for proposition B are both low, the conclusion affirms that proposition B is true, which is an unreasonable conclusion. So we cannot use DS combination rule when evidences highly conflict either.

3.2. Classical Solutions

Several researchers have proposed different solutions in order to manage the problem of conflict, which can mainly be divided into two categories: improved combination rule and modified conflict evidences.

Some researchers think that unreasonable conclusion is mainly caused by normalization procedure. Thus it improves the combination result by giving sum conflict probability to certain subset with certain proportion, which called improved combination rule. The general representation is defined by:

$$m(A) = \sum_{\cap A_i = A} \prod_{1<j<n} m_j(A_i) + K \cdot \delta(A, m) \tag{5}$$

Hence, the key of this method is to confirm combination operator $\delta(A, m)$,which means that we should consider assigning conflict mass to which subset with what kind of proportion. Yager [3] postulates that the frame of discernment is exhaustive, and it consists in assigning the sum conflict probability to Θ which includes unknown propositions. But Yager's idea is too conservative to be applied. It increases uncertainty of reasoning without considering practical application, and produces unreasonable result when evidences are more than two. The combination rule proposed by Sun Quan [4] improves Yager's idea by distributing sum conflict probability through weighted average. On these basis, Li Bicheng [5] introduces a further ideal combination rule. It distributes sum conflict probability according to the weighted average of each proposition's support, which changes the view of denying the total conflict evidences, and improves the reliability and rationality of combination result. In addition, Takahiko, Toshiyuki, Lefevre [7], Zhang Shanying [8] also propose different improvement methods.

Others consider that the production of paradox is mainly caused by unreliable evidences. Thus it modifies the expression of evidences before evidences fusion, which called modified conflict evidences. The general representation is defined by:

$$m(A) = \sum_{i=1}^{n} m_i(A) \cdot w_i(A) \tag{6}$$

Hence, the key of this method is to confirm weighted operator $w_i(A)$ .Murphy introduces a combination rule by averaging evidences mass. But the disadvantage is obvious that the idea only average the evidences mass without considering the relationship of them. As distance function of evidences is proposed by Jousselme [9], the combination rule based on evidence credibility becomes a hot spot. In addition, Deng Yong's modified method based on a validity coefficient [12] and modified method based on weighted average evidences synthetically consider the influence of support evidences and conflict evidences. What's more, Zhang Bing regards the eigenvector as credibility according to similarity matrixes, and Guo Huawei [13] considers absolute credibility as weighted factor, which all give great combination results.

4. Modified Combination Rule

This paper improves the combination rule based on the second classical method, so-called modified conflict evidences. In order to get a better similarity measurement between evidences, let us introduce the Minkowsky distance instead of Euclidean Distance.

Let us suppose that m_i, m_j is two different basic probability assignments which are mutually exclusive and exhaustive, defined as follows:

$$m_i(\boldsymbol{A}) = \begin{cases} p_1 & \boldsymbol{A} = H_1 \\ p_2 & \boldsymbol{A} = H_2 \\ p_3 & \boldsymbol{A} = H_3 \\ \dots & \dots \\ p_n & \boldsymbol{A} = H_n \end{cases}, \quad m_j(\boldsymbol{B}) = \begin{cases} q_1 & \boldsymbol{B} = H_1 \\ q_2 & \boldsymbol{B} = H_2 \\ q_3 & \boldsymbol{B} = H_3 \\ \dots & \dots \\ q_n & \boldsymbol{B} = H_n \end{cases} \tag{7}$$

Then, we can get the distance function of evidences m_i, m_j:

$$d_{BPA}(m_i, m_j) = \left(\sum_{l=1}^{n} |p_l - q_l|^m \right)^{1/m} \tag{8}$$

Suppose that the number of evidences collected by system are n, we can define the distance matrix as follows:

$$D = \begin{bmatrix} 0 & d_{12} & \dots & d_{1n} \\ d_{21} & 0 & \dots & d_{2n} \\ \dots & \dots & \dots & \dots \\ d_{n1} & d_{n2} & \dots & 0 \end{bmatrix} \tag{9}$$

Then define the similarity measure between m_i, m_j as

$$sim(m_i, m_j) = 1 - d_{BPA}(m_i, m_j) \tag{10}$$

So, we can get the similarity matrix as

$$S = \begin{bmatrix} 1 & sim(m_1, m_2) \dots sim(m_1, m_n) \\ sim(m_2, m_1) & 1 \quad \dots sim(m_2, m_n) \\ \dots & \dots \quad \dots \quad \dots \\ sim(m_n, m_1) & sim(m_n, m_2) \ \dots \ 1 \end{bmatrix} \tag{11}$$

There, we define the support degree of each evidence in system as

$$\sup(m_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} sim(m_i, m_j) \tag{12}$$

Where $\sup(m_i)$ means the support degree of m_i which represents the similarity degree between m_i and other evidences as m_j. We can see above that, the less distance function of evidences are, the more similarity of evidences are. And the support degree of m_i describes how strongly other evidences support m_i, which represents the similarity degree of m_i and other evidences must be high.

This paper filtrates and corrects the support degree according to its mean value and standard deviation. Firstly let us calculate the average support degree and standard deviation of evidences, as follows:

$$\overline{\sup(m_i)} = \frac{\sum_{i=1}^{n} \sup(m_i)}{n} \tag{13}$$

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{\left(\overline{\sup(m_i)} - \sup(m_i)\right)^2}{n-1}} \tag{14}$$

Then filtrate and correct the support degree, as follows:

$$\sup'(m_i) = \begin{cases} \sup(m_i) & \rho < \sigma \\ 0 & \rho \geq \sigma \end{cases} \tag{15}$$

where, $\rho = \left|\overline{\sup(m_i)} - \sup(m_i)\right|$.

Finally, we can get the evidence credibility by normalized $\sup'(m_i)$, defined as follows:

$$crd(m_i) = \begin{cases} \dfrac{\sup'(m_i)}{\sum_{i=1}^{n} \sup'(m_i)} & \sum_{i=1}^{n} \sup'(m_i) \neq 0 \\ \dfrac{1}{n} & \sum_{i=1}^{n} \sup'(m_i) = 0 \end{cases} \tag{16}$$

where $\sum_{i=1}^{n} crd(m_i) = 1$. So we can regard evidence credibility as weighted factor to average and weight the evidences before combination, defined as follows:

$$m(A) = \sum_{i=1}^{n} m_i(A) \cdot crd(m_i) \tag{17}$$

Follow formula (8)-(17), we can get the modified evidences as formula (17). And the fusion result can be obtained by using DS combination rule to combine modified evidences for $n-1$ times.

5. Simulation

5.1. Simulation 1

Considering the situation of conflict $k = 1$ and $k \to 1$, that is example 1 and example 2. We can get the simulation results by the modified method. Compared to the DS combination rule, the results are respectively shown in Table 1 and Table 2.

Table 1. Combination Result of Example 1

	DS	Modified method
$m_{\oplus}(A)$	NaN	1
$m_{\oplus}(B)$	NaN	0
$m_{\oplus}(C)$	NaN	0

Table 2. Combination Result of Example 2

	DS	Modified method
$m_{\oplus}(A)$	0	0.4880
$m_{\oplus}(B)$	1	0.0240
$m_{\oplus}(C)$	0	0.4880

As said in 3.1, DS combination cannot be used when evidences entirely conflict, while using the modified method, we can get a reasonable result. We can see from Table 1, the modified method completely support hypothesis *A*. the reason is that although m_2, m_3, m_4 highly conflict, m_1, m_3, m_4 all support hypothesis A to a great extent. Also we can see from Table 2, it represents that the modified method can not only combine evidences when they highly conflict, but also improve the rationality and effectiveness.

5.2. Simulation 2

In order to verify the effectiveness and reliability of modified method, we use five classical methods to combine evidences as well. Compared with other classical methods, the result of modified method can be seen as follows:

Suppose that the frame of discernment Θ includes five target A, B, C, D, E , and the basic probability assignment of five evidences are showed in Table 3. The simulation results of classical and modified methods are showed in Table 4.

Table 3. Basic Probability Assignment

	A	B	C	D	E
m_1	0.7	0.1	0.1	0	0.1
m_2	0	0.5	0.2	0.1	0.2
m_3	0.6	0.1	0.15	0	0.15
m_4	0.55	0.1	0.1	0.15	0.1
m_5	0.6	0.1	0.2	0	0.1

We can intuitively estimate in Table 3 that the belief function of target A is the highest, which means that the effective method should recognize target A as the decision. It represents in Table 4: Because that $m_2(A) = 0, m_1(D) = m_3(D) = m_5(D) = 0$, the simulation results cannot recognize target A and D correctly, which means DS combination rule cannot combine when evidences conflict highly. With the increasing number of evidences, the support degree of unknown target is always on the rise, it increases the uncertainty of Sun Quan method to some extent. Although Li Bicheng, Murphy, Deng Yong and the modified method all can recognize target A exactly, only Deng Yong and modified method consider the relationship between evidences, and only modified method that this paper proposes filtrates and corrects the support degree according to the standard deviation, which improves the reliability of combination. In addition, the support degree for A by modified method is the highest, which demonstrates its best astringency.

Table 4. Combination Result

		DS	SunQuan	Li Bicheng	Murphy	Deng Yong	Modified
m_1 m_2	$m_\oplus(A)$	0	0.1282	0.3185	0.4712	0.4712	0.4712
	$m_\oplus(B)$	0.5556	0.1599	0.3230	0.3462	0.3462	0.3462
	$m_\oplus(C)$	0.2222	0.0749	0.1565	0.0865	0.0865	0.0865
	$m_\oplus(D)$	0	0.0183	0.0455	0.0096	0.0096	0.0096
	$m_\oplus(E)$	0.2222	0.0749	0.1565	0.0865	0.0865	0.0865
	$m_\oplus(\Theta)$	0	0.5437	0	0	0	0
m_1	$m_\oplus(A)$	0	0.1027	0.4286	0.6513	0.8930	0.9822

m_2	$m_{\oplus}(B)$	0.4545	0.0603	0.2358	0.1888	0.0608	0.0036
m_3	$m_{\oplus}(C)$	0.2727	0.0386	0.1514	0.0780	0.0230	0.0071
	$m_{\oplus}(D)$	0	0.0079	0.0330	0.0039	0.0001	0
	$m_{\oplus}(E)$	0.2727	0.0386	0.1514	0.0780	0.0230	0.0071
	$m_{\oplus}(\Theta)$	0	0.7519	0	0	0	0
m_1	$m_{\oplus}(A)$	0	0.0652	0.4620	0.7236	0.9790	0.9967
m_2	$m_{\oplus}(B)$	0.4545	0.0287	0.2003	0.1353	0.0121	0.0007
m_3	$m_{\oplus}(C)$	0.2727	0.0197	0.1376	0.0640	0.0044	0.0013
m_4	$m_{\oplus}(D)$	0	0.0088	0.0624	0.0132	0.0002	0
	$m_{\oplus}(E)$	0.2727	0.0197	0.1376	0.0640	0.0044	0.0013
	$m_{\oplus}(\Theta)$	0	0.8579	0	0	0	0
m_1	$m_{\oplus}(A)$	0	0.0443	0.4620	0.7637	0.9959	0.9991
m_2	$m_{\oplus}(B)$	0.3571	0.0163	0.1998	0.1031	0.0019	0.0001
m_3	$m_{\oplus}(C)$	0.4286	0.0136	0.1374	0.0716	0.0016	0.0006
m_4	$m_{\oplus}(D)$	0	0.0045	0.0624	0.0080	0	0
m_5	$m_{\oplus}(E)$	0.2143	0.0118	0.1374	0.0538	0.0007	0.0006
	$m_{\oplus}(\Theta)$	0	0.9094	0	0	0	0

6. Conclusion

Because of complexity of multi-sensor, the evidences that system gets may highly conflict, which will results in wrong combination. This paper proposes a modified combination rule of conflict evidence considering the relationship of evidences. It improves the effectiveness and reliability of results compared with classical methods. At the same time, with the thought of using standard deviation to filtrate evidences, the modified combination rule can work more effective when number of sensor increase, which meets the demand of practical application.

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