International Journal of Hybrid Information Technology
Vol.8, No.12 (2015), pp. 283-298
http://dx.doi.org/10.14257/ijhit.2015.8.12.21

A Hybrid Rough Set Theory-PSO Technique for Solving of Non-convex Economic Load Dispatch

Amin Safari and Davoud Moghaddam Sheibai

Department of Electrical Engineering, Azarbaijan Shahid Madani University, Tabriz, Iran
Engineering Department of Pars Oil and Gas Company (POGC), Iran
safari@azaruniv.edu

Abstract

This paper applies a novel hybrid rough set theory-particle swarm optimizer technique, namely rough particle swarm optimization (RPSO) algorithm, for solving non-convex economic load dispatch (NELD) problem. The RPSO algorithm is based on the notion of rough patterns that uses rough values defined with upper and lower intervals in which represent a set of values. This RPSO method is suggested to deal with the practical constraints such as valve point loading effect, generation limitation, ramp rate limits and prohibited operating zones in the NELD problems. Simulations were performed on four different power systems with 3, 6, 15 and 40 generating units and the results are compared with classical PSO and crazy PSO algorithms. The results of this study reveal that the proposed approach is able to find appreciable NELD solutions than those of previous algorithms.

Keywords: *Rough Particle Swarm Optimization, Non-convex Economic Load Dispatch, Rough Set Theory, Valve Point Loading Effect*

1. Introduction

Economic load dispatch (ELD) is one of the important optimization problems in power system operation, which is used to determine the optimal combination of electrical power outputs of all generating units to minimize the total fuel cost while satisfying various constraints over the entire dispatch periods.

Over the years, various solutions have applied to solve ELD problems by different classic programming methods and optimization techniques in the literatures. Such classical optimization methods [1] are highly sensitive to staring points and often converge to local optimum or diverge altogether. Lately, heuristic search techniques such as genetic algorithm [2], evolutionary programming [3], Tabu search [4], clonal algorithm [5], differential evolution [6], biogeography-based optimization [7], the PSO [8] and the hybrid algorithms [9-15] are being used to find global or near global optimal solution. The PSO method is still one of the best methods, because the rest of the intelligent methods have a long time running problem, the main advantages of PSO are its simple concept, computational efficiency and easy implementation. The PSO has been effectively applied to ELD problems. However, the PSO method has some defects such as premature convergence. Therefore, many variations have been proposed for the classical PSO by various researchers.

In this paper, to enrich the searching behavior and to avoid being trapped into local optimum, Rough set theory [16, 17] in PSO technique is used to speed up population-based optimization problems which is developed to solve the ELD problem. In our study, we compare the performance of the RPSO with the CPSO (Common Particle Swarm Optimization) and crazy PSO methods. For the ELD problem, the numerical results show

ISSN: 1738-9968 IJHIT

that the RPSO method has better convergence property and can get lower generation cost than two other methods.

This paper is organized as follows: Section 2 presents the formulation of ED problems. Section 3 then describes the classical, crazy and rough PSO. Detailed process of using the RPSO method to solve the ELD problems are presented in Section 4, Section 5 shows four application cases using the proposed method to solve the ELD problems and the results have been compared to CPSO and crazy PSO methods and found to be superior.

2. Formulation of ELD Problem

The ELD problem is minimizing the fuel cost of generation units so as to accomplish optimal generation dispatch among operating units and in return satisfying the system load demand and loss, generator operation constraints with ramp rate limits, prohibited operating zones and valve-point loading effects. Hereby, two alternative models for ELD are considered that one with convex cost function and the other with non-convex cost function as detailed below.

2.1. Convex Objective Function

The cost function of ELD corresponding to the total generation cost is approximated by a quadratic function of the power output from the generating units. It can be mathematically described as follows [7, 8]:

$$F_T = \sum_{i=1}^{Ng} F_i(P_i) = \sum_{i=1}^{Ng} (a_i P_i^2 + b_i P_i + c_i) \tag{1}$$

Where F_T is the unit cost rates; $F_i(P_i)$ is the i th unit cost rate; a_i is the incremental heat rate; b_i is the average cost at full load; c_i is the no load cost; P_i is the electric power generation of i th generator; Ng is the number of generators.

2.2. Non-convex Cost Function with Valve-point Loading Effects

Because the fuel cost is the main factor determining in the economic operation, the fuel input-output curves is important. In this curve, the slope of the curve at any point is efficiency of generative unit fuel at that point. Power cost function is not always a convex and due to the effects of some steam valves and it has a non-convex shape and form of this cost function equation, is considered to be in two sentences. The input-output characteristic of large steam turbine generators is not always convex. This type of units have some supply steam valves, upon increase production request the valves open one by one and respectively for increasing output power. When the unit load's increase, the input (fuel) to the unit increases, and between points in each of the two valves open, the incremental heat rate increases, however, when valve is first opened, due to a rapid losses the throttling, the incremental heat rate to be large and suddenly rise, which will rise to discontinuities in the incremental heat rate characteristic. The valve point loading effects introduce ripples in the heat rate curves and make the cost function discontinuous, non-convex and multiple minimum [11].

One of the ways to model the valve effect is adding a second sentence to the sine function, after convex model of the generator cost function. These features are non-convex and could not easily used in optimization methods that need convex characteristics.

$$C = \sum_{i=1}^{Ng} F_i(P_i) = \sum_{i=1}^{Ng} \left(a_i \times P_i^2 + b_i \times P_i + c_i + \left| e_i \times Sin\left(f_i \times (P_i^{\min} - P_i)\right)\right|\right) \tag{2}$$

Where $P_i^{\min}$ is the minimum production of i th generator; e_i and f_i, the fuel cost coefficients of the i th unit with valve point effects or back to i th generator is a cost function coefficients of the knee.

2.3. Equality and Inequality Constraints

Fundamental constraint on the operation of this system is known as the power balance constraint, so that the total power output $(\sum_{i=1}^{Ng} P_i)$ must be equal to the total load (P_D) and total loss of system (P_L). Namely:

$$P_L + P_D = \sum_{i=1}^{Ng} P_i \tag{3}$$

System losses equation is a function of the *B* coefficients and sum of the generators generation. The most famous system losses equation is the formula known as Kron (4). It is noticeable that using any other losses function, (*e.g.,* Jorge equation ($P_L = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_i \times B_{ij} \times P_j$) and …) for ELD will produce the same results and only will change cost [15]. The *B* coefficient matrixes are achieved by using a series of conversions on the total impedance matrixes related to transmission network.

$$P_L = B_{00} + \sum_{i=1}^{Ng} B_{0i} \times P_i + \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_i \times B_{ij} \times P_j \tag{4}$$

Here, P_L is total losses of system and B_{00} , B_{0i} and B_{ij} are *B* coefficient matrixes. Often, it is assumed that the output of the generating unit, soft and instantly adjust to changing times and changes. But in practice, when the load changes, the unit output can't changes, and interval operation of production units within the production is into ramp rate. District operation of all units being produced by the ramp rate limit, *i.e.,* up rate limit UR_i, within the previous down rate limit DR_i and P_i^0 are finite. When the ramp rate of generator is proposed, namely the operation of the i th unit has been changed as follows:

$$Max\left(P_i^{\min}, P_i^0 - DR_i\right) \le P_i \le Min\left(P_i^{\max}, P_i^0 + UR_i\right) \tag{5}$$

Generators are practically discontinuous cost curve, as all units operating range (between maximum production and minimum production) for the work is not always possible. In other words, generating units due to some faults on the shaft bearing or mechanical vibrations or other accessories such as pumps, compressors or boilers, etc., are prohibited operating zone [16]. The prohibited operating zone, loading within a unit is divided into several maximum and minimum generating ranges. Due to the following number subzone convex regions, the total cost curve of the piece-the piece. Best economy is achieved when the operating units don't be in these prohibited zones. A unit with a prohibited operating zone, feature input-output is discontinuous. The forbidden operating zone can be exploited for NELD issues to be formulated this way [13]:

$$Pi \in \begin{cases} P_i^{\min} \le P_i \le P_{i1}^{L} \\ P_{ik-1}^{U} \le P_i \le P_{ik}^{L} \\ P_{izi}^{U} \le P_i \le P_i^{\max} \end{cases} \tag{6}$$

Where z_i are the number of prohibited zones in i th generator curve, k is the index of prohibited zone of i th generator, P_{ik}^{L} is the lower limit of k th prohibited zone, and P_{ik}^{U} is the upper limit of k th prohibited zone of i th generator.

3. Review of CPSO, CRAZY PSO and RPSO

3.1. Classical PSO

In the PSO [21], the particles are in the search space, and change the location of particles in the search space is influenced by experience and knowledge and their neighbors. Therefore, positions of other particle mass effects on how to find a particle. Modeling social behavior is a result of a search process, the particles towards the areas they desire. The PSO is based on the principal that each moment of each particle its location in the search space according to the best place so far it has been and there is the best place in itself whole neighborhood, will set up. This optimizer can be used to solve many of the several of problems as genetic algorithm, artificial bee colony, and does not suffer from some of previous algorithms difficulties. Therefore, it has been found to be robust in solving problem featuring non-linearing, non-continues and high dimensionality. For this purpose, the PSO approach is a high potential in providing an optimal response at the appropriate time to address the ELD problems. The PSO algorithm consists of, at each step, changing the velocity of each particle toward its *pbest* and *gbest* according to Eq (7).

$$V_{id}^{(t+1)} = C \times \left[\omega . V_{id}^{(t)} + \underbrace{C_1 . r_1 \left(P_{id}^{(t)} - x_{id}^{(t)} \right)}_{cognition\ term} + \underbrace{C_2 . r_2 \left(g_{id}^{(t)} - x_{id}^{(t)} \right)}_{Social\ term} \right] \quad (7)$$

Here, C is constriction factor, C_1 and C_2 are acceleration coefficient for cognition sentence and social sentence, respectively. They are random number between 0 and 4, that in this paper $C_1 = C_2 = 2$. The r_1 and r_2 are random number between 0 and 1 whereas w is inertia weight and for each iteration calculated using the following equations:

$$w = (w_{\max} - w_{\min}) \times \frac{(iter_{\max} - iter)}{iter_{\max}} + w_{\min} \quad (8)$$

Here $iter$ is number of iteration and the $iter_{\max}$ is maximum number of iteration. The C is a constant factor from the following relationship is obtained [22].

$$C = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|} \quad (9)$$

Where, φ is a random number between in the $[4.1, 4.2]$, as is observed with increasing φ, the factor C decreases and convergence becomes slower because population diversity reduced [13]. The position of the i th particle is then updated according to:

$$x_{id}^{(t+1)} = x_{id}^{(t)} + V_{id}^{(t+1)} \quad (10)$$

Where, V_i velocity vector between the intervals $[V_i \min, V_i \max]$ are considered. The $V_i \max$ most commonly 10% - 20% of the value is $X_i \max$, or otherwise convergence is achieved immediately or is exit from the search space. The biggest problem with standard PSO, is that sometimes causes problems for multiple peaks with a large state space, trapped in local optimum, or in some cases appear to premature convergence, so try to optimize our algorithms.

3.2. Crazy PSO

To solve the problem of premature convergence on a concept called PSO craziness is introduced. The idea of random particle velocity of the particles are called crazy particles, by a certain probability is selected. In the craziness probability of ρ_{cr} as a function of inertia weight is defined.

$$\rho_{cr} = \omega_{\min} - \exp\left(-\frac{\omega^{(t)}}{\omega_{\max}}\right) \tag{11}$$

Thus, the random particle velocity is obtained from the following relationship:

$$V_j^{(t)} = \begin{cases} rand\ (0, V_{\max}) & if\ \rho_{cr} \geq rand\ (0,1) \\ V_j^{(t)} & otherwise \end{cases} \tag{12}$$

If the CPSO algorithms tend to be saturated in the early stages of a high value of ρ_{cr} crazy as a particle with lower value is used in the search process. The procedure of the crazy PSO is illustrated in Figure 1 [10]:

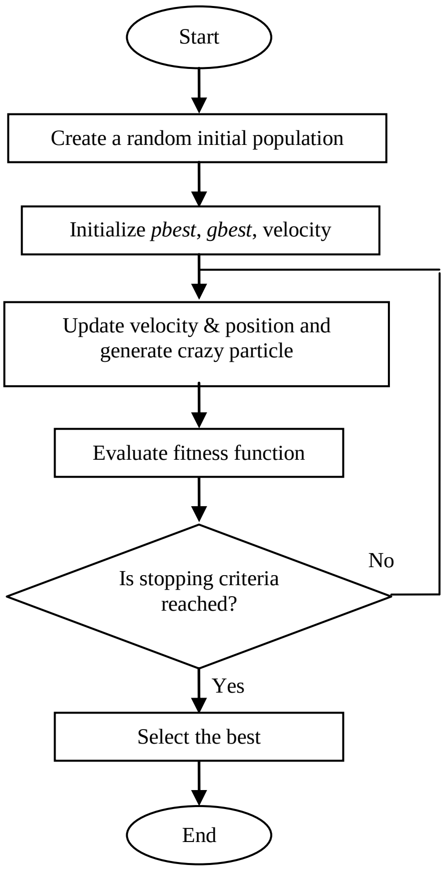

Figure 1. Flow Chart for PSO with Crazy Particle Method

3.3. Rough PSO

Rough set (RS) theory in 1982 by Pawlak [16] was proposed. But in recent years to speed up population-based optimization problems using this theory are realized. The RS extension of classical set theories is based on the logic of three values. This is a new mathematical method for analyzing the data table is working, intelligent data analysis and data mining, loss or reduction of surplus properties and by the most important features is used. The RS philosophy on the assumption that the any object of the world can be as the data (information) considered. The RS theory is a powerful tool for reasoning in that case is unclear and unreliable methods to eliminate and reduce information or irrelevant knowledge database provides surplus to requirements. Objects described by the same data, the information about them are indistinguishable from the point. In this methodology, the basic on RS theory of mathematics is being indistinguishable relationship obtained, namely rough set theory based on the concept of class or category [17].

Let x be an attribute in the description of an object and $\underline{x}$, $\bar{x}$ represent lower and upper bounds of x such that $\underline{x} \leq \bar{x}$ .The lower and upper bounds are endpoint of rough pattern. A rough pattern value of each variable features, including lower and upper bounds can by Equation (13) is shown.

$$x = (\bar{x}, \underline{x}) \tag{13}$$

We can see a rough model with drawing in Figure 2.

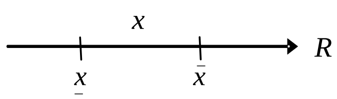

Figure 2. A Rough Value

Rough model is a subset of closed, compact and bounded of the real numbers set ($x \in \Re$). The middle point (*mid*), radios (*rad*) and length of x rough value to be formulated this way [18, 19]:

$$mid\ (x) = \frac{(\bar{x} + \underline{x})}{2} \tag{14}$$

$$rad\ (x) = \frac{(\bar{x} - \underline{x})}{2} \tag{15}$$

$$width\ (x) = (\bar{x} - \underline{x}) = 2 \times rad\ (x) \tag{16}$$

The rough values can representation with middle point and radios instead end points as following:

$$x = (mid\ (x) - rad\ (x)\quad ,\quad mid\ (x) + rad\ (x)) \tag{17}$$

The rough values are useful for indicate to intervals or set of values for an attribute, where only lower and upper bounds are considered relevant in a computation. The rough values may be is vogue for most areas of calculation mathematics. For example, if the calculations are performed by rough values, it is possible to evaluate a function over an entire interval rather than a single value. In other words, if we evaluate a function $f(x)$ over some interval of x (for example $[x \in (\bar{x}, \underline{x})]$), we know what the possibly overestimated bounds of the function are within that interval [19]. Rough values of the application, most real bounds or overestimate offer that is not a critical value of the function [20]. So, to find the robust root and the global minimum, maximum and other optimization problems are also very beneficial. In fact, a conventional pattern can be easily represented as a rough pattern to apply to replace the two lower and upper bounds, to be of equal value. Some of the operating on the rough value can be implanted as follows [20]:

$$x + y = (\underline{x}, \bar{x}) + (\underline{y}, \bar{y}) = (\underline{x} + \underline{y}\ ,\ \bar{x} + \bar{y}) \tag{18}$$

$$x - y = (\underline{x}, \bar{x}) + \left(-(\underline{y}, \bar{y})\right) = (\underline{x} - \bar{y}, \bar{x} - \underline{y}) \tag{19}$$

$$x \times y = \left(\min\ (\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})\ ,\ \max\ (\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})\right) \tag{20}$$

$$\frac{1}{x} = \left(\frac{1}{\overline{x}}, \frac{1}{\underline{x}}\right) \qquad 0 \notin (\underline{x}, \overline{x}) \tag{21}$$

$$\frac{x}{y} = \frac{(\underline{x}, \overline{x})}{(\underline{y}, \overline{y})} = \frac{(\underline{x}, \overline{x})}{\left(\frac{1}{\overline{y}}, \frac{1}{\underline{y}}\right)} \qquad 0 \notin (\underline{x}, \overline{x}) \tag{22}$$

$$c \times x = c \times (\underline{x}, \overline{x}) = (\underline{x}, \overline{x}) \times c = \begin{cases} (c \times \underline{x}, c \times \overline{x}) & if\ c \geq 0 \\ (c \times \overline{x}, c \times \underline{x}) & ifc\ < 0 \end{cases} \tag{23}$$

The algebraic properties of addition and multiplication operations on values in the Table 1 are described.

Table 1. Algebraic Properties

Algebraic properties	Description	Condition
Commutativity	$x + y = y + x$ $x.y = y.x$	No condition
Associativity	$(x + y) + z = x + (y + z)$ $(x.y).z = x.(y.z)$	No condition
Neutral element	$0 + x = x$ $1 \times x = x$	No condition
Distributivity	$x.(y + z) = x.y + x.z$ $x.(y + z) = x.y + x.z$ $x.(y + z) = x.y + x.z$ $x.(y + z) = x.y + x.z$ $x.(y + z) = x.y + x.z$ $x.(y - z) = x.y - x.z$ $x.(y - z) = x.y - x.z$	If $\underline{x} = \overline{x}$ If $y \geq 0$ and $z \geq 0$ (non − negative terms) If $y \leq 0$ and $z \leq 0$ (non − positive terms) If $x \geq 0$, $\underline{y} = 0$ and $\overline{z} = 0$ (positive factor, zero − sraddling terms) If $x \leq 0$, $\underline{y} = 0$ and $\overline{z} = 0$ (negative factor, zero − sraddling terms) If $y \geq 0$ and $z \leq 0$ (non − negative terms variation) If $y \leq 0$ and $z \geq 0$ (non − positive terms variation)

Rough particle X is a string of x_i rough parameters. Namely:

$$x = (x_i \mid 1 \leq i \leq n) \tag{24}$$

Rough parameter x_i is a pair of conventional parameters, the one used for lower bound and the lower parameter is called $(\underline{x})$, and other upper bound use and the upper parameters are known $(\overline{x})$:

$$x_i = (\underline{x_i}, \overline{x}_i) \tag{25}$$

Figure 3 shows an example of rough particles.

x_1																				
	1	3	11	16	24	28	30	40	90	99	108	140	190	191	25	29	40	60	12	19

Figure 3. Rough Particle

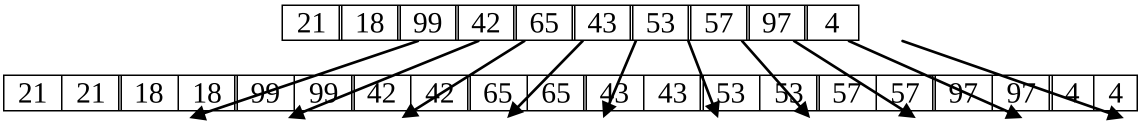

Figure 4. Conventional Particle and its Rough Equivalent

The values of each rough parameter, a range that varies. The use of this range indicates that the data represents a rough particle is not accurate. Hence, a data measure called precision [20], which may be used when evaluating fitness levels. The conventional parameters and particles used at PSO, is a special case of their rough equivalents. This content showed in Figure 4. In the boundary constant problems, that's very important to assured that decision variable values established inside the allowed range, after update the position and velocity equations. This technique can be to make prevalent for RPSO with this constraint that, lower bounds smaller than upper bounds. The particles which are being produced and modify along the evaluation process represent rules. Each of particles involved decision variables that indicate items and intervals. A positional encoding, used where the i th decision variable is encode. Each decision variable consists of the three parts. The first part (AC_i) of decision variable indicate antecedent or consequent of the rule and it can be between $[0, 1]$ and second indicate lower bound (LB_i) and third part indicate upper bound part (UB_i). If AC_i is between $[0, 0.33)$, this item is antecedent of the rules and if the AC_i is between $[0.33, 0.66)$, this item is consequent of the rules and if AC_i is between $[0.66, 1]$, this item is involved of the rules [17]. The structure of a particle is shown in Figure 5.

Variable$_1$			Variable$_2$			…			Variable$_m$		
AC$_1$	LB$_1$	UB$_1$	AC$_2$	LB$_2$	UB$_2$				AC$_m$	LB$_m$	UB$_m$

Figure 5. Particle Representation

4. Implementation of RPSO

Considering the constraints and inequality constraints are very difficult to solve these equations, that exclusively we can solve it with the constraints expressed as a function of output units [13]. To reduce the random motion of particles and the possible diversion that is sometimes causing premature convergence, RPSO method is useful because the data that the deviations are randomly removed and not allowed to enter in the search process shows. The Pseudo Code of the RPSO algorithm for solving ELD problems can be summarized as follows:

Step 1: generate initial population randomly;
Step 2: convert the data table to the decision table;
Step3: calculate the significant of each attribute and assigned to any given feature; *Step 4:* calculate fitness of particles; when optimization algorithm are applied for constrained ELD problems, it is common to handle constraints using concepts of penalty factors. The popular penalty function method employs functions to reduce the merit of the particle in relation to the magnitude of the constraints violation. The penalty factors are carefully chosen to distinguish between feasible and infeasible solution space. The evaluation function is defined as follows [13]:

$$C = \sum_{i=1}^{Ng} F_i(P_i) + \alpha \left[\sum_{i=1}^{Ng} P_i - P_L - P_D \right] + \beta \left[\sum_{k=1}^{n_i} P_i(violation)_k \right] \quad (26)$$

Where α is the penalty parameter for not satisfying load demand and β is the penalty for unit loading falling whit in a prohibited operating zone.
Step 5: Check the stop criterion, the criterion was satisfactorily completed the go to seven step, otherwise, go to the six step;
Step 6: update the particle velocity and position according to equation (7) and (10) and back to the step 4;
Step 7: Finish the RPSO algorithm solving.

5. Test and Results

To validate the performance of the proposed RPSO efficiently, four standard problems have been taken from the literature of multi-minima ELD for thorough analysis. In each case study, 50 independent runs were made for each of the optimization methods. In implementation of proposed algorithm, some RPSO parameters should be predefined. The parameters of RPSO are selected as following: $c_1 = c_2 = 2$, $w_{max} = 0.9$ and $w_{min} = 0.4$. We consider the number of iterations to end criteria, and the number of iterations to solve ELD problems for 3, 6 and 15 units is 200 and 300 generations for 40 units. The initial population was selected 500 and the generator cost range between minimum and maximum and the average generator cost is achieved in 50 iterations and all computation is performed with 32-bit microcomputer with Intel Core 2 Duo P8700@2.53GHz CPU and 4.00GB RAMs. In order to clarify this issue, we compared the RPSO with two previous methods.

5.1 Case Study I: Three Units System

The system contains three thermal units whose characteristics are given in [10]. The load demand of the system is $P_D = 850\ MW$. The network losses are calculated by B matrix loss formula. Figure 6 shows that the convergence characteristics of PSO, crazy PSO and RPSO in the process of searching for the minimum objective function. The performance of RPSO algorithm is better among all algorithms in terms of solution quality and consistency.

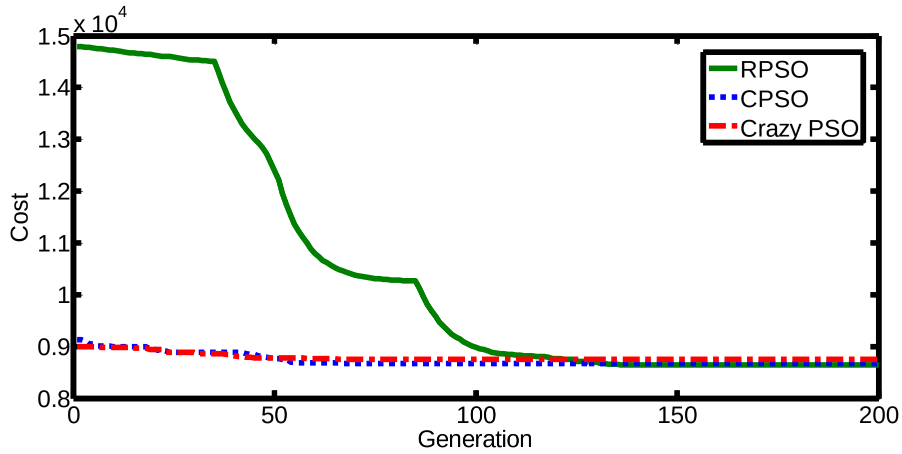

Figure 6. Comparison of Convergence Characteristics (3 Units System)

The best solutions using the proposed RPSO are shown in Table 2 that satisfy the generator constraints to prove its effectiveness. It can be seen from Table 2 that the RPSO provided better results compared with two previous algorithms.

Table 2. Generator Output for Least Cost (3 Units Systems)

Unit power output	CPSO	Crazy PSO	RPSO
P1 (MW)	390.9397675772537	420.6552012922881	383.1388299739129
P2 (MW)	372.5279635144842	339.6326339233074	384.9310144902642
P3 (MW)	112.7714016736586	116.4885411879479	107.7759696899202
Total power output (MW)	850.0650566077397	850.0434511877630	849.9473396496687
Total loss (MW)	26.174076157656856	26.732925215780377	**25.898474504428535**
Total generation cost ($/h)	8847.165865017638	8837.865111003826	**8811.334756267777**
Maximum generator cost ($/h)	9168.986742661490	9103.910813542262	**9081.562447056117**
Average generator cost ($/h)	8941.477241545766	8930.275248769199	**8875.874819397785**
CPU times (s)	0.332436	0.346433	0.015162

5.2. Case Study II: Six Units System

The system contains six thermal generating limits, 26 buses and 46 transmission lines. The load demand is $P_D = 1263 \ MW$. The characteristics of the six thermal units are given in [8]. The network losses are calculated by *B*-matrix loss formula [9]. Figure 7 shows the convergence characteristic of RPSO on a system with 6 generators. It can be seen that the convergence characteristic of proposed method is dramatically improved and the algorithm discovers optimal cost in accepted region by effective control of particles value.

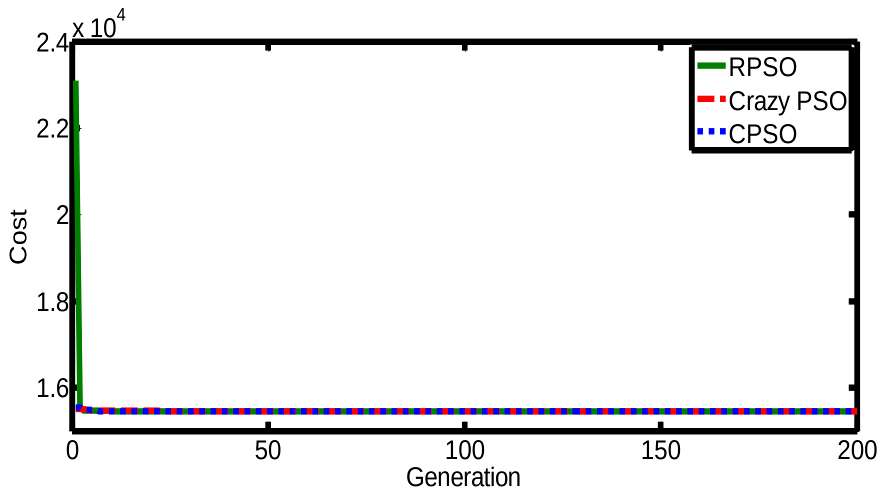

Figure 7. Comparison of Convergence Characteristics (6 Units System)

Table 3 presents the best cost achieved by the different PSO algorithms for the six unit system while satisfying the practical constraints. It can be seen from Table 3 that the RPSO perform better than the PSO and crazy PSO methods in terms of solution quality.

Table 3. Generator Output for Least Cost (6 Units Systems)

Unit power output	CPSO	Crazy PSO	RPSO	RGA
P1 (MW)	440.4483	437.0719	442.8750	420.2342
P2 (MW)	185.7758	181.2549	182.1862	199.4412
P3 (MW)	255.3500	257.5738	258.2577	263.7234
P4 (MW)	128.7508	132.2514	134.1376	120.0030
P5 (MW)	172.3584	172.7676	168.6197	167.2319
P6 (MW)	93.0921	94.8099	89.5751	105.1250
Total power output (MW)	1263.0560	1263.0559	1263.0572	1263.0000
Total loss (MW)	12.7197	12.6737	**12.5942**	12.7588
Total generator cost ($/h)	15449.0684	15448.0255	**15446.3669**	15461..3992
Maximum generator cost ($/h)	15467.3420	15457.9599	**15455.0348**	
Average generator cost ($/h)	15453.6934	15451.4541	**15450.5327**	
CPU times (s)	0.381320	0.385840	0.032851	

5.3. Case Study III: Fifteen Units System

The system contains fifteen thermal units whose characteristics are given in [8]. The load demand of the system is $P_D = 1630 \quad MW$. The convergence of optimal solution using RPSO is shown in Figure 8, where only about 18 iterations were needed to find the optimal solution.

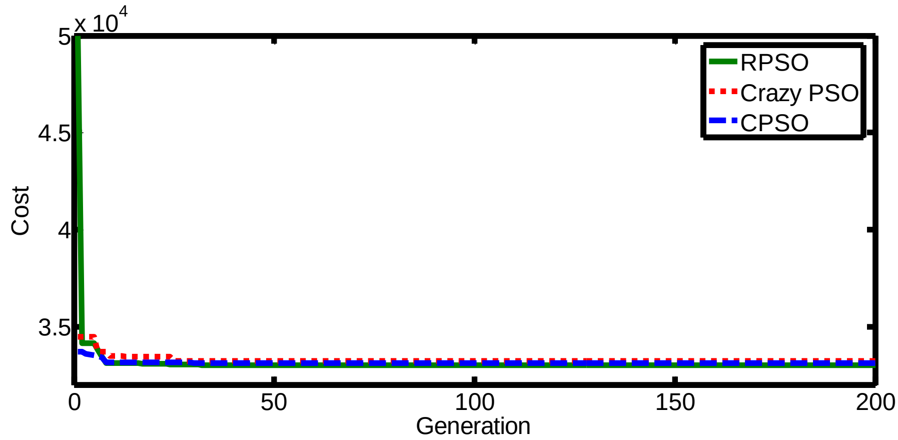

Figure 8. Comparison of Convergence Characteristics (15 Units System)

Table 4 shows the optimal solutions with execution time determined by RPSO for the fifteen units. It can be seen from Table 4 that the RPSO algorithm is significantly faster than two other methods.

5.4. Case Study IV: Forty Units System

The system contains forty thermal units whose characteristics are given in [6]. The load demand of the system is $P_D = 10500 \quad MW$. Figure 9 represents the evolution of the PSO, crazy PSO and RPSO for the forty units test system. The figure shows the better convergence behavior of the proposed methodology than the conventional and crazy PSO.

Table 4. Generator Output for Least Cost (15 Units Systems)

Unit power output	CPSO	Crazy PSO	RPSO
P1 (MW)	4.10.8461946934712	377.4523936890492	398.5432065445076
P2 (MW)	386.2983704353361	399.8338774687803	402.7462375890957
P3 (MW)	101.1302347483946	100.6339497402430	122.9324597602036
P4 (MW)	66.6837406961973	109.9108526146100	113.9268021586653
P5 (MW)	157.1709338373313	357.5860293844644	172.0303076648710
P6 (MW)	456.9796788696853	347.4015467281061	455.9744022897995
P7 (MW)	420.1021447748283	410.1302126109625	420.6227941610609
P8 (MW)	254.6588541369302	145.7218921352832	191.3202894363353
P9 (MW)	48.8947158168000	90.0993371190863	108.3324844685564
P10 (MW)	114.3650035471438	118.5710529247734	33.0232058499311
P11 (MW)	52.0302818485325	62.4511553922839	51.5435668699465
P12 (MW)	69.9116961831379	28.9446794361923	69.2989878428894
P13 (MW)	46.8647227180579	56.6521342977031	50.7399074988663
P14 (MW)	39.4508508772880	34.9782330901829	26.6673446594840
P15 (MW)	44.4040670146800	29.0591590998666	46.5571406991935
Total power output (MW)	2630.004998337718	2630.005596997874	2629.989812881391
Total loss (MW)	39.786491860095850	39.420908733713986	**34.269324612014664**
Total generator cost ($/h)	33217.64787870983	33106.65995045885	**33014.70712087628**
Maximum generator cost ($/h)	33986.33299854604	33778.22421018622	**33691.82509371925**
Average generator cost ($/h)	33316.90151330197	33229.70200374485	**33178.91035815945**
CPU times (s)	0.501199	0.588769	0.079831

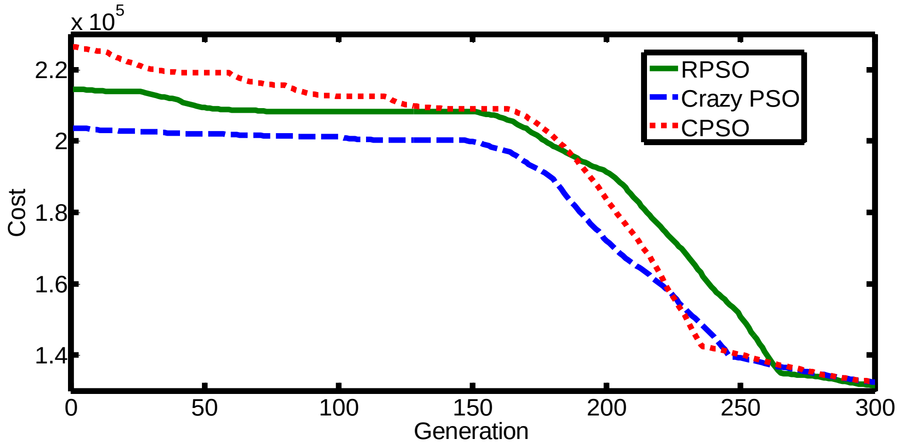

Figure 9. Comparison of Convergence Characteristics (40 Units System)

Through the evolutionary process of the proposed method, it best solutions are shown in Table 5. All of the constraints mentioned before are all satisfied. Similarly, it is obvious that the proposed approach can has a better solution quality than the CPSO and crazy PSO methods in terms of solution quality.

Table 5. Generator Output for Least Cost (40 Units Systems)

Unit power output	CPSO	Crazy PSO	RPSO
P1 (MW)	96.1210654716844	109.2221290841517	85.8414288165953
P2 (MW)	112.1993995997332	105.1666677211546	107.6570721999196
P3 (MW)	104.6155564021428	110.0075545625287	117.6982564532653
P4 (MW)	113.4358634401608	183.6598674883210	153.0278787725914
P5 (MW)	90.2273513452734	96.6069847064799	92.3629977968542
P6 (MW)	111.2696533925485	130.1454464800015	100.2976534976840
P7 (MW)	249.8530563205373	259.1442923590874	280.7498088649456
P8 (MW)	266.1691971231929	295.5584542417683	257.6751915170932
P9 (MW)	235.1331954204524	239.5671452473287	289.4098285100608
P10 (MW)	181.4963941776537	192.8913274869628	183.8360638010822
P11 (MW)	224.2004447334717	226.0726964374880	212.0797324886661
P12 (MW)	211.4881543746019	112.3900897821331	153.3636252486208
P13 (MW)	381.9768034664493	353.1289607585590	371.6295945004467
P14 (MW)	444.9729056615891	440.0257892032752	444.7309825559050
P15 (MW)	310.0346843006126	453.2918677671727	396.4380688757888
P16 (MW)	447.6873976491254	486.5820808758448	369.0696432652105
P17 (MW)	419.3965822171387	470.8430404479381	467.1504303089193
P18 (MW)	494.0843605802577	488.0020537166629	477.5309704739569
P19 (MW)	526.0846400537616	434.6932174725525	403.0097102795208
P20 (MW)	483.5959457933753	513.2611562853271	546.9591076166763
P21 (MW)	525.8587675901597	546.7310732770799	451.4803070280900
P22 (MW)	493.0184744514004	456.1264957503767	403.6662148743168
P23 (MW)	542.6042190618319	517.7665171812440	428.2291763489933
P24 (MW)	528.0123189797246	476.1497676919369	539.0653593647378
P25 (MW)	453.0017238796072	400.4109694899233	516.3489417351497
P26 (MW)	452.1792795717104	420.8996634891330	529.7568299977232
P27 (MW)	37.2099477275351	73.7169005516534	44.6623729945533
P28 (MW)	62.0288419701471	40.4764033289619	57.9007183351797
P29 (MW)	77.9845283958896	43.5559726131503	69.6038416211700
P30 (MW)	93.6084632026037	64.6007418720537	79.3180093066862
P31 (MW)	155.9630736299436	168.4901407361259	185.8949222629833
P32 (MW)	136.0608952063332	129.3237265553761	189.1215827594523
P33 (MW)	169.4389884779662	157.0576165559041	172.6710453465058
P34 (MW)	148.7160144154168	196.3228591923808	190.9494646176008
P35 (MW)	171.9658586538157	186.7608554600954	179.2431341534190
P36 (MW)	196.1249120553391	121.8759540619925	166.4187027689855
P37 (MW)	106.3392178911445	46.2057015629702	106.6119495294630
P38 (MW)	95.0416833841993	107.0733454387672	87.2274242020469
P39 (MW)	99.9909577745334	100.7807782633237	100.2733496790664
P40 (MW)	450.6238219765394	545.3411059563623	491.7958150268424
Total power output (MW)	10499.81463981961	10499.92741115355	10500.75720779677
Total loss (MW)	0	0	0
Total generation cost ($/h)	132450.0772633460	132259.1811758676	**131432.4708821575**
Maximum generator cost ($/h)	139766.1430087937	140208.4557898509	**139661.0966101166**
Average generator cost ($/h)	133698.9061081834	135677.5542665015	**133161.1222973071**
CPU times (s)	1.427584	1.638923	0.209725

6. Conclusions

In this work, to enrich the searching behavior and to avoid being trapped into local optimum, using rough set theory, RPSO approach that uses rough decision variables and rough particles that are based on notion of rough patterns, is proposed. The application feasibility of Rough PSO for solving economic load dispatch with smooth and non-smooth cost function by taking into account of various systems constraints like valve point loading effect, prohibited operating zones, ramp rate limits have been investigated successfully. The proposed approach has produced results comparable or better than those generated by crazy PSO and CPSO algorithms and the solutions obtained have superior solution quality, the CPU time and good convergence characteristics. The numerical results obtained for four cases clearly demonstrated that proposed algorithm which is capable of achieving global solutions is simple, computationally efficient and has better and stable dynamic convergence characteristics.

References

[1] A. J. Wood and B. F. Wollenberg, Power Generation, Operation, and Control. New York: Wiley, **(1984)**.

[2] D. C. Walters and G. B. Sheble, “Genetic algorithm solution of economic dispatch with the valve point loading”, IEEE Trans Power Syst., vol. 8, no. 3, **(1993)**, pp. 1325-1332.

[3] H. T. Yang, P. C. Yang and L. C. Huang, “Evolutionary programming based economic dispatch for units with non-smooth fuel cost functions”, IEEE Trans. Power Syst., vol. 11, no. 1, **(1996)**, pp. 112-118.

[4] W. M. Lin, F. S. Cheng and M. T. Tsay, “An improved Tabu search for economic dispatch with multiple minima”, IEEE Trans Power Syst., vol. 17, no. 1, **(2002)**, pp. 108-112.

[5] B. K. Panigrahi, S. R. Yadav, S. Agrawal and M. K. Tiwari, “A clonal algorithm to solve economic load dispatch”, Electrical Power System Research, vol. 77, **(2007)**, pp. 1381-1389.

[6] N. A. Rahmat and I. Musirin, “Differential Evolution Ant Colony Optimization (DEACO) technique in solving Economic Load Dispatch problem”, Power Engineering and Optimization Conference (PEDCO) Melaka, IEEE International, **(2012)**, pp. 263-268.

[7] A. Bhattachaya and P. Kumar, “Biogeography-Based Optimization for Different Economic Load Dispatch Problems”, IEEE Trans Power Syst., vol. 25, no. 2, **(2010)**, pp. 1064-1077.

[8] Z. L. Gaing, “Particle swarm optimization to solving the economic dispatch considering the generator constraints”, IEEE Trans Power Syst., vol. 18, no. 3, **(2003)**, pp. 1187-1195.

[9] J. S. Alsumait, J. K. Sykulski and A. K. Al-Othman, “A hybrid GA-PS-SQP method to solve power system valve-point economic dispatch problems”, Appl. Energy, vol. 87, no. 5, **(2010)**, pp. 1773-1781.

[10] T. C. Krishna, P. Manjaree and S. Laxmi, “Particle swarm optimization with crazy particles for non-convex economic dispatch”, Applied Soft Computing, vol. 9, **(2009)**, pp. 962-969.

[11] X. Yuan, A. Su, Y. Yuan, H. Nie and L. Wang, “An improved PSO for dynamic load dispatch of generators with valve-point effects”, Energy, vol. 34, no. 1, **(2009)**, pp. 67-74.

[12] N. Singh and Y. Kumar, “Economic Load Dispatch Using MRPSO with Generator Ramp Rate Limits Constraint”, Computational Intelligence and Communication Networks (CICN), **(2012)**, pp. 632-637.

[13] A. Safari and H. Shayeghi, “Iteration particle swarm optimization procedure for economic load dispatch with generator constraints”, Expert Systems with Applications, vol. 38, no. 5, **(2011)**, pp. 6043-6048.

[14] T. M. Subhani, C. S. Babu and A. S. Reddy, “Particle swarm optimization with time varying acceleration coefficients for economic dispatch considering valve point loading effects”, Computing Communication & Networking Technologies (ICCCNT), **(2012)**, pp. 1-8.

[15] T. C. Krishna, P. Manjaree and S. Laxmi, Particle swarm optimization with time varying acceleration coefficients for non-convex economic power dispatch”, Electrical Power and Energy Systems, vol. 31, no. 6, **(2009)**, pp. 249-257.

[16] Z. Pawlak, “Rough sets”, International Journal of Computer and Information Sciences, vol. 11, **(1982)**, pp. 341-356**.**

[17] A. P. Engelbrecht, “Computational intelligence an introduction”, John Wiley & Sons, 2nd Ed, **(2007)**.

[18] W. Ding, J. Wang and Z. Guan, “Cooperative extended rough attribute reduction algorithm based on improved PSO”, Systems Engineering and Electronics Journal, vol. 23, no. 1, **(2012)**, pp. 160-166.

[19] P. Lingras and C. Davies, “Applications of Rough Genetic Algorithms”, Computational Intelligence: An International Journal, vol. 3, no. 17, **(2001)**, pp. 435-445.

[20] B. Alatas and E. Akin, “Rough particle swarm optimization and its applications in data mining”, published online: Springer-Verilog, **(2008)**.

[21] J. Kennedy and R. Eberhart, “Particle swarm optimization”, in: Proceedings of the IEEE Conference on Neural Networks (ICNN’95), vol. IV, Perth, Australia, **(1995)**, pp. 1942-1948.

[22] R. C. Eberhart and Y. Shi, "Comparing inertia weights and constriction factors in particle swarm optimization", in Proceedings of the Congress on Evolutionary Computation, vol. 1, **(2000)**, pp. 84-88.

Author

Amin Safari, he received the B.Sc. and M.Sc. degrees in Electrical Engineering in 2007 and 2009, respectively. He received his Ph.D. degree in Power Electrical Engineering, Iran University of Science and Technology, Tehran, Iran. Currently, he is an Assistant Professor in Department of Electrical Engineering, Azarbaijan Shahid Madani University, Tabriz, Iran. His areas of interest in research are Application of artificial intelligence to power system control design, FACTS device and fuzzy sets and systems. He has published more than 70 papers in international journals and conference proceedings.