International Journal of Hybrid Information Technology
Vol.8, No.12 (2015), pp. 1-10
http://dx.doi.org/10.14257/ijhit.2015.8.12.01

Nonlinear Error Compensation of Complex Surface for Five-axis Machining

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Abstract

In order to solve non-linear errors caused by rotational movement of the rotary axis, firstly, it analyzes how non-linear errors are generated in the five-axis machining, and establishes the non-linear error model of complex surface in 3D space. Then it proposes a method of controlling and compensating for nonlinearity errors by introducing the interpolate point based on smoothing tool axis vector. In the method of surface interpolation, it is suggested to add an interpolation point, keep the tool axis vector on sector surface boundary within the start and end tool axis vector of the program block, and maintain the tool axis vector velocity and acceleration continuity. Simulation results show the nonlinearity errors of the fan blade machining are effectively controlled.

Keywords: *Five-axis machining, Nonlinearity error, Surface interpolation, Compensation point*

1. Introduction

Five-axis CNC technology is an integration of precision machining, computer control and high-performance servo drive technology. Five-axis CNC machine tools can flexibly control the tool geometry and the workpiece surface to achieve a better match. It can improve the mechanical parts of complex shape machining quality and efficiency. However, because of the introduction of two rotary axes, the five-axis CNC machine tool structure becomes more complex and the posture of the tools becomes more difficult to control. In actual machining process, CNC system drives rotary axis interpolation motion according to the tool axis vector generated by the CAM system. CAM system programmed path is a straight line. For three-axis machining without rotation axis, tool does linear machining motion along the planning tool path. Two rotary axes of the five-axis machines do rotational movement, and machine tool motion axes do linear interpolation movement, thus, the actual cutter after motion synthesis will deviate from linear motion path. This deviation is called non-linear error. Due to the rotation axis movement, non-linear error of five-axis machining is inevitable.

Many scholars have conducted a number of studies on analysis and control of non-linear errors of five-axis machining. Literature [1-3] have already analyzed and researched the nonlinear error of five-axis machining, and clarified the principle of nonlinearity error generation, and clarified the adaptive linearization method to control error. Geng Cong, *et al.*, [4] proposed a cutter path control algorithm by controlling the

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ISSN: 1738-9968 IJHIT

movement of the first tool axis vector and the end of tool axis vector on the plane defined, in order to reduce the non-linear error. Fan Liuqun, *et al.*, [5] studied linear interpolation principle of the angle of rotation axis and what causes nonlinear machining errors, and proposed specific algorithm of tool axis vector interpolation based on tool axis vector plane, thus avoided the non-linear errors. However his method cannot guarantee the continuity of the speed and acceleration of rotating axis. Zhao Wei [6] studied the NC system interpreter and RTCP function, in an effort to achieve interpolation and post-processing in the NC system and solve the non-linear error. Zhang Jian, *et al.*, [7] proposed to compare the three-dimensional Euclidean distance between the actual adjacent cutter location point and the three-dimensional Euclidean distance between corresponding adjacent cutter location point within allowable error range. If it exceeds the error range, a new cutter location point is inserted to reduce the nonlinearity error.

These methods are all analysis of piecewise linear cutting point location. They are the approximate nonlinear error mathematical models based on the analysis of single tool path data. It is simple and feasible in engineering. It can reduce errors and improve the accuracy of certain machining. But the cutting location points are not directly related to the theoretical curved surface and practical machined curved surface, and did not consider the information of the complex theory of curved surface in three-dimensional space, which will certainly affect the establishment of the model of non-linear error and the controlling calculation accuracy. Therefore, it is necessary to further study nonlinear errors modeling and error controlling.

2. Causes of Non-linear Error

Five-axis CNC system is mostly a linear interpolation system and discrete point level control system. Its theoretical interpolation locus is linear. In the actual machining, due to rotational motion of the rotation axis of the five-axis machine, the actual locus of tools is inconsistent with the the linear interpolation locus of the NC system and it will generate deviation. The deviation is called non-linear error. Figure 1 is a schematic diagram of five-axis interpolation locus Straight Line $P_{WL}(t)$ is the ideal programming curve in the workpiece coordinate system. $P_W(t)$ is the actual machining path of the five-axis CNC system in interpolation processing after synthesis movement of each axis. It is a complex space curve. P_{W0} and P_{W1} are the tool axis position vector of adjacent cutter location point. U_{W0} and U_{W1} are the tool axis direction vector of adjacent cutter location point. The maximum deviation value from the deviation value of $P_W(t)$ and $P_{WL}(t)$ can be approximated as an estimate value of the non-linear error.

3. Establish a Nonlinear Error Mathematical Model of Space Mappings

In 3D space, the nonlinear error of complex curved surface can be theoretically expressed as:

In the interpolation program block, machine axes do linear interpolation movement, and it makes tools do machining envelope movement along the surface of the workpiece. The formed envelope surface is the actual machining path. The normal direction distance between this envelope surface and the workpiece machining surface is called non-linear error [3]. Generally, the maximum distance is taken as the value of nonlinearity error.

In the interpolation program block, in Three-dimensional space the non-linear error model can be established, as in Figure 2.

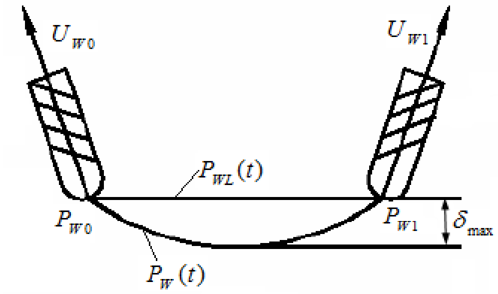


Figure 1. Five-axis Interpolation Locus Schematic

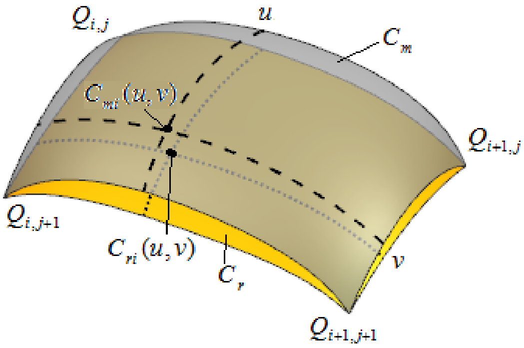


Figure 2. Nonlinear Error Model Diagram

According to the actual path of tool cutting contacts point, the actual cutting surface can be found. It is denoted as S_r. Workpiece geometry surface is denoted as S_m. In the interpolation program block, take any four known adjacent tool cutting contact points, as $Q_{i,j}$, $Q_{i+1,j}$, $Q_{i,j+1}$ and $Q_{i+1,j+1}$ .Using this four points as vertices, the surface region corresponding to the workpiece geometric model surface is denoted as C_m, and the surface region corresponding to the actual cutting surface as C_r. $Q_{i,j}$ represents the I-th toolpath the J-th tool cutting contacts point. $Q_{i,j} = (P_{i,j}, U_{i,j}) = (x_{i,j}, y_{i,j}, z_{i,j}, i_{i,j}, j_{i,j}, k_{i,j})$ In this equation, $P_{i,j}$ and $U_{i,j}$ are tool axis position and direction vector.

$C_{ri}(u,v)$ is any point on the C_r. Under the five-axis machine tool coordinate system, corresponding NC node $N_r(u,v) = [T_1, T_2, T_3, R_1, R_2]^T$ is five-dimensional column vector. T represents translational movement of the translation axis, and R represents rotational movement of the rotation axis. Five-axis machine tool has three translational axes, two rotation axes.

Machine tool motion axes do linear interpolation movement. After quadratic interpolation, motion vector of axes can be gotten at any NC node as follows:

$$N_m(u,v) = \begin{bmatrix} u & 1-u \end{bmatrix} \begin{bmatrix} N_{i,j} & N_{i,j+1} \\ N_{i+1,j} & N_{i+1,j+1} \end{bmatrix} \begin{bmatrix} v \\ 1-v \end{bmatrix} \qquad (0 \le u \le 1, 0 \le v \le 1) \tag{1}$$

In the formula, $N_{i,j}$, $N_{i,j+1}$, $N_{i+1,j}$ and $N_{i+1,j+1}$ are five-dimensional column vector in tool coordinate system.

They are obtained according to $C_{i,j}$, $C_{i,j+1}$, $C_{i+1,j}$ and $C_{i+1,j+1}$ by the machine tool coordinate system and the workpiece coordinate system coordinate reverse transformation [8].

According to the machine kinematics analysis, a forward or reverse coordinate transformation relation between the machine tool coordinate system and the workpiece coordinate system can be established. Between tool coordinate system and the workpiece coordinate system, the mapping relationship is formula (2).

$$\left.\begin{aligned} T_i &= \Phi_i(P,U) \qquad i=1,2,3 \\ R_j &= \Psi_j(U) \qquad j=1,2 \end{aligned}\right\} \tag{2}$$

In the formula, Φ_i and Ψ_j are reverse coordinate transformation mapping function of the machine tool coordinate system and the workpiece coordinate system.

Between tool coordinate system and the workpiece coordinate system, the forward coordinate transformation mapping relations is formula (3).

$$\left.\begin{aligned} P &= \varphi(T_1,T_2,T_3,R_1,R_2) \\ U &= \psi(R_1,R_2) \end{aligned}\right\} \tag{3}$$

In the formula, φ and ψ are reverse coordinate transformation mapping function.

Therefore, insert u and v parameter values for the formula (1), NC nodes-set can be calculated in the actual machining, then use formula (3) to give the corresponding $C_{ri}(u,v)$ set.

All of any point $C_{ri}(u,v)$ are under masked operation, then the actual machining locus envelope surface C_r can be obtained (Figure 1). Assume that the point $C_{ri}(u,v)$ is vertically projected onto the point $C_{mi}(u,v)$ of the surface C_m, then the nonlinearity error is formula (4).

$$\delta(u,v) = \left|C_{ri}(u,v) - C_{mi}(u,v)\right| \tag{4}$$

$$\delta_{\max}(u,v) = \left|C_{ri}(u,v) - C_{mi}(u,v)\right|_{\max} \tag{5}$$

4. Compensation Strategy Based on Smooth Tool Axis Vector

CAM software system generates toolpath data that only have information of tool axis direction vector and the cutter position, but without information of the tool shapes, sizes and the actual cutting point. Because of complex rotary movement of the machine tool rotary axis, as a result, the position of cutting contacts point is difficult to be accurately estimated between tool and the workpiece surface model. Therefore, in five-axis NC machining, it is difficult to accurately calculate which tool position point has the largest non-linearity error. At present, there are mainly three kinds of methods to control nonlinear errors. They are tool cutting contacts offset, linear encryption method and adaptive linearization method. Adaptive linearization method is the more optimized control method and is widely used. The basic idea is that if the non-linear error exceeds the allowable range of values, the original cutting line can be divided linearly, and then divide in half cutting line in order to reduce the tool axis vector change until the actual error is less than allowable error [3, 7]. Literature [1-3] determined the location of the maximum non-linearity error with the adaptive linearization method. It is the position of the insertion point compensation. But the insertion point is often not located on the surface, over-cutting or less-cutting could easily be resulted. These methods are all analyzed for piecewise linear cutting location point. It is the approximate nonlinear error mathematical model based on analysis of single tool path data. But the cutting location points are not directly related to the information of the complex theory of curved surface in three-dimensional space, which will certainly affect on establishing the model of non-linear error and controlling calculation accuracy. Therefore, this paper proposed a method of controlling and reducing the nonlinearity error ,and the method is to insert tool axis vector compensation point based on the smooth tool axis vector.

4.1. Error Compensation Ideas

Using surface interpolation method, compensation point is inserted to the theoretical curve surface's u and v-parameter direction. Change the line spacing and step during CNC machining, and interpolate compensation point on the curve surface. In the running of the tool axis vector, it will be always kept on sector surface boundary within the start and end tool axis vector of the program block, and maintain the tool axis vector velocity and acceleration continuity. Ultimately, control smooth tools machining to achieve the purpose of reducing the nonlinearity error.

4.2. Error Compensation Implementation

4.2.1. Interpolation Point

As is shown in Figure 2, the theoretical surface $S_m(u,v)$ opens along its' u and v-parameters direction. In reference to NURBS surface interpolation thought, which is, a series of values of one of the parameter directions are fixed, such as the value of the U parameter direction, surface is a discrete of a series of curves of V parameter direction. Figure 3 shows discrete surfaces by using the equal parameter method.

The spacing value between the curves is 0.2mm. If any one of the two V direction's boundary curves is discrete surface along U parameter direction, it is a series of discrete curves. Figure 4 is the results of the discrete surface region, and the surface region's vertexes are $Q_{i,j}$, $Q_{i+1,j}$, $Q_{i,j+1}$ and $Q_{i+1,j+1}$.

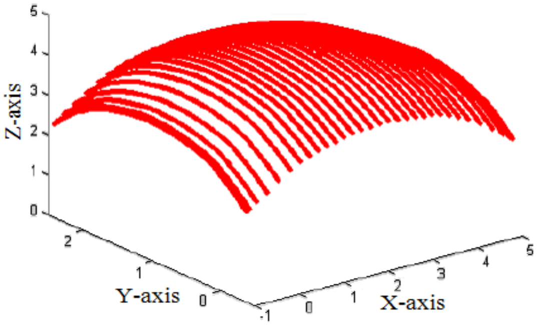


Figure 3. Discrete Surfaces by Equal Parameters Method

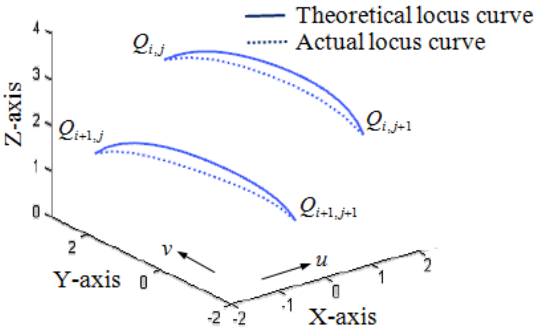
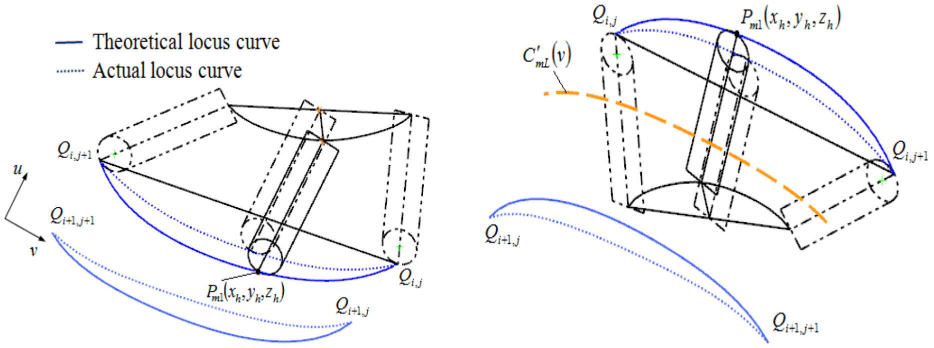


Figure 4. Discrete Results of $C_m(u,v)$ and $C_r(u,v)$

Figure 4 shows, locus curves $C_{mL}(v)$ and $C_{rL}(v)$ are the theoretical and the actual locus along the v direction. Its vertexes $Q_{i,j}$ and $Q_{i,j+1}$ are adjacent to tool cutting contacts point. As the literature [3] shows that the position of the maximum non-linearity error is close to the midpoint of the two tool cutting contacts points. Therefore, compensation point is Inserted in the midpoint of theoretical locus $C_{mL}(v)$. It is point $P_{m1}(x_h, y_h, z_h)$. As shown in Figure 5. After inserting the compensation point, once again calculate the corresponding non-linear error value. If the errors are still beyond error range, continue to insert compensation point in this way, until they meet the specified non-linear error range. If using this method to control and compensate error, there may be multiple compensation points between the tool cutting contacts points. If you insert compensation points too much, it will greatly increase the storage capacity and reduce processing operating speed.

In addition to increasing the compensation point, it is also worth considering reducing the spacing value in the U parameter direction. That is to say, compensation points are inserted in the U parameter direction. Theoretical locus curves are formed through the compensation point, as curve $C'_{mL}(v)$ shown in Figure 5. Then work out the nonlinear error of the curve $C'_{mL}(v)$ and insert the compensation point for the curve $C'_{mL}(v)$. Obtain coordinates position of the compensation points.



(a) V direction Interpolation Points (b) U-direction Interpolation Curve

Figure 5. Non-linear Error Compensation Schematic

4.2.2. Determine the Tool Axis Direction Vector of Compensation Points

To achieve five-axis machining, it is necessary to calculate the position of the two rotary axes or rotation angle. In addition, it is also needed to obtain the position of interpolation point. Here in A, C double turntable rotation axis is an example. With the change of the curved shape, it is difficult to achieve the goal of controlling the cutter shaft along adaptively adjusting. In the five-axis surface machining process, there are three states between the tool axis and the surface: vertical, parallel, and at an angle. In either states, it is angled between the tool axis direction and the surface normal vector. Therefore, before controlling the tool axis direction, the normal processing of surfaces must be determined [9]. In reference to the NURBS surface theory, the unit normal vector of the complex surface can be obtained after the tangential vectors in the U-direction and in the V-direction are worked out.

For the theoretical surfaces $C_m(u,v)$, if the fixed parameters u equals to u_0, this surface becomes a unary function $C_m(u_0,v)$ about parameters v, which is called v-line. Similarly, if the fixed parameters v equals to v_0, the surface $C_m(u,v)$ becomes a unary function $C_m(u,v_0)$ about parameters u, is called u-line. Therefore, at any point $C_m(u_0,v_0)$ on the surface, there must be one u-line passing through this point and one v-line. At this point, the partial derivative vector $C_{mu}(u,v_0)\big|_{u=u_0} = \frac{\partial C(u,v_0)}{\partial u}\big|_{u=u_0}$ of u-line about parameters u is called tangent vector in the U-direction at the point. If the fixed parameters v equals to v_0, the partial derivative vector $C_{mu}(u,v_0) = \frac{\partial C(u,v_0)}{\partial u}$ of u-line about parameters u is called tangent vector in the U-direction of u-line. Therefore, if the fixed parameters u equals u_0, the partial derivative vector $C_{mv}(u_0,v) = \frac{\partial C(u_0,v)}{\partial v}$ of v-line about parameters v is called tangent vector in the V-direction of v-line. If tangent vector in the U-direction and tangent vector in the V-direction are not parallel, that is $C_{mu}(u_0,v_0) \times C_{mv}(u_0,v_0) \neq 0$, it can deduce the unit normal vector $N_m(u_0,v_0)$ from the tangent plane at the point $C_m(u_0,v_0)$ of the surfaces $C_m(u,v)$. It can be illustrated in the following formula:

$$N_m(u_0,v_0) = \frac{C_{mu}(u_0,v_0) \times C_{mv}(u_0,v_0)}{\left|C_{mu}(u_0,v_0) \times C_{mv}(u_0,v_0)\right|}$$

Therefore, the unit normal vector of tool cutting contacts on the surface $C_m(u,v)$ is as follows:

$$N_{m\,ij}(u_i,v_j) = \frac{C_{mu}(u_i,v_j) \times C_{mv}(u_i,v_j)}{\left|C_{mu}(u_i,v_j) \times C_{mv}(u_i,v_j)\right|} \tag{12}$$

$N_{m\,ij}(u_i,v_j)$ can also be expressed as $N_{m\,ij}(N_x,N_y,N_z)$.

In the tool coordinate system, assuming the A-axis rotational motion is represented by $\theta_A(u,v)$; and the C-axis rotational motion represented by $\theta_C(u,v)$. Homogeneous coordinate transformation matrix of A-axis rotary motion represented by $R_x(\theta_A(u,v))$. Homogeneous coordinate transformation matrix of C-axis rotary motion represented by $R_z(\theta_C(u,v))$.

According to reverse coordinate transformation relations between the machine tool coordinate system and the workpiece coordinate system, the angle of tool axis vector interpolation rotation can be obtained. As shown in formula (13) and (14).

$$\theta_A(u,v) = \pm\arccos(N_{mz}) \qquad (-\pi \le \theta_A(u,v) \le \pi) \tag{13}$$

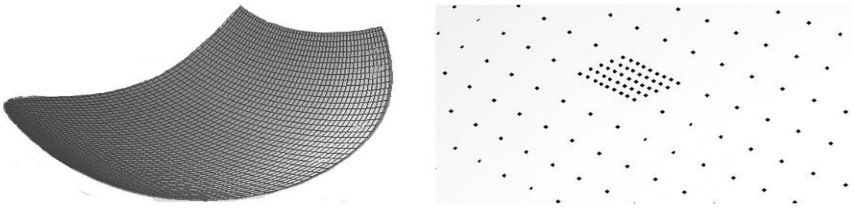
$$\theta_C(u,v) = \arctan(N_{mx}/N_{my}) \text{ or } \theta_C(u,v) = \arctan(N_{mx}/N_{my}) - \pi \quad (0 \le \theta_C(u,v) \le 2\pi) \tag{14}$$

It can be known, θ_A and θ_C may have two solutions. The specific value should be determined according to the continuity of the movement

5. Simulation Analysis and Conclusions

For fan blades, simulation experiments get 50 trajectories of the tool cutting contacts points by using the compensation strategy of inserting compensation points based on smoothing tool axis vector, as shown in Figure 6. And by using the surface interpolation method, non-linear error can be solved. The results showed, the more the number of compensation points inserted, the smaller the error is.

For the machining of the fan blades, the compensation strategy proposed in this paper is compared with traditional compensation strategies, as shown in Figure 7. The results showed the compensation strategy proposed in this paper is more optimized.



(a) Discrete Curved Grid of Fan Blades

(b) Tool Cutting Contacts Locus of Fan Blades

Figure 6. Discrete Curved Grid and Tool Cutting Contacts Locus

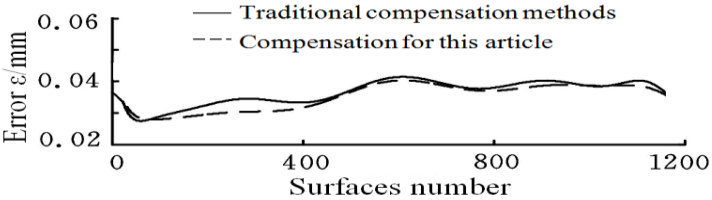
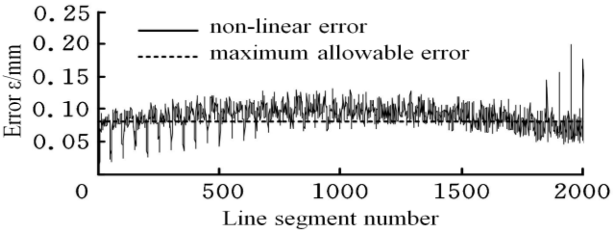


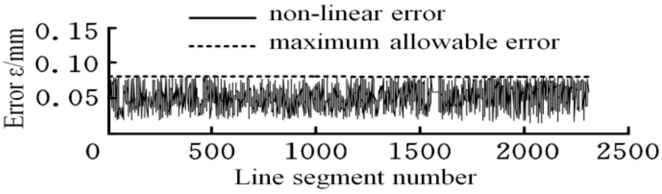
Figure 7. The Results Compared with Different Compensation Methods

Set the maximum allowable error, as shown in Figure 8, horizontal dashed line represents the maximum setpoint. The maximum error is 0.2mm. It exceeds the maximum allowable error range. After the control compensation strategy is implemented, nonlinear error values are within allowable error range. Thus ensure the machining accuracy.

From the simulation experimental results, we can know, this algorithm can effectively reduce the nonlinearity error and achieve a smooth change of tool axis vector, and can improve the accuracy of five-axis NC machining. This verifies the effectiveness and practicality of the algorithm.



(a) Before Error Control



(b) After Error Control

Figure 8. Error Control Results

Acknowledgments

This research was supported by foundation of Chongqing municipal education commission of science and technology research project (No. KJ091063 and No. KJ1360612).

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