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Combined Forecasting Mode of Subgrade Settlement Based on Support Vector Machine and Real-coded quantum Evolutionary Algorithm

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Abstract

Due to the normal forecasting methods for subgrade settlement using observation data have different applicability and disadvantages, The Combined forecasting model is put forward based on support vector machine (SVM) and real-coded quantum evolutionary algorithm (RQEA) in this paper. Its core is that, according to the basic settlement law of subgrade and characteristics of settlement curve, the growth curve which has S-type characteristic are chosen as single forecasting model, then support vector machine is used to combine the predicting results of each single forecasting model, at the same time, RQEA is adopted to optimize support vector machine parameter to improve the SVM's performance. The analytical result of engineering practice indicates that the proposed combined forecasting model of subgrade settlement base on SVM and RQEA can not only improve the predicting accuracy, but also reduce the predicting risk, and can meet engineering demand.

Keywords: *Subgrade Settlement prediction, Combination forecast model, Support vector machine, Real-coded quantum evolutionary algorithm*

1. Introduction

The development of subgrade settlement has complex characteristics, such as non-linear, non-stabilization, and including numerous uncertain information, so it is very difficult to forecast the subgrade settlement accurately. At present, the methods of subgrade settlement prediction mainly include layer-wise summation method (LWSM), numerical analysis method (NAM) and modeling method based on the observed data (MMBD) [1, 2].

LWSM and NAM demand the precise geotechnical parameters and model of elasto plastic constitutive relation, but the geotechnical parameters and the model of elasto plastic constitutive relation is difficult to be acquired, so their application is limited. The observed data of subgrade settlement is an integrated reaction of all kinds of factors, and contains abundant information, then MMBD is paid more attention. MMBD includes experience formula method (EFM) (such as hyperbolic method, exponential curve method, growth-curve approach and parabolic method), system analysis and control theory method (grey system method and nerve network method). However, each forecasting model has their own advantages and their scopes of application, but there are some shortcomings too [3, 4].

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Combined forecast makes full use of the advantages of single-phase prediction model and overcomes its shortcomings, improving the forecasting precision and reducing the forecasting risk. Combined forecasting method combines different single-phase prediction models through considering the characteristics of each single-phase prediction model, so the predicted results could make full use of the obtained information from each single-phase prediction model, it has strong adaptability and good stability. We use machine learning and optimized calculation to establish combined forecasting model of subgrade settlement. Firstly, we lead Usher, Logistics, Gompertz of S-type single-phase prediction model into combined forecast according to the development law and sedimentation curve characteristic of subgrade settlement, and calculating the corresponding prediction results; then making each single-phase prediction results into input vector, using SVM to build combined forecasting model of subgrade settlement, and adopting real-coded quantum evolutionary algorithm to optimize SVM parameters, to improve the prediction accuracy. Engineering case analysis shows that the combined forecasting model has better prediction accuracy and stability which uses S-type single-phase prediction model and based on SVM and real-coded quantum evolutionary algorithm [5, 6].

2. Support Vector Machine (SVM)

Set up a sample set as $\{x_i, y_i\}$, $x_i \in X = R^n$ as input, $y_i \in Y = R$ as output, $i = 1, \cdots, l$, l as sample numbers, and the sample satisfies a certain unknown but determined joint probability distribution $P(x, y)$, the problem of regression is to find a function $f(x) = (w \cdot x) + b$, $w \in R^n$, $b \in R$ as generalized parameter, $(w \cdot x)$ as inner product operation, minimizing expected risk.

$$R[f] = \int L(x, y, f(x)) dP(x, y) \tag{1}$$

Where $L(x, y, f(x))$ is the given loss function, common loss function is $\varepsilon -$ insensitive loss function.

$$L(x, y, f(x)) = |y - f(x)|_\varepsilon = \max\{0, |y - f(x)| - \varepsilon\} \tag{2}$$

where ε as a positive number of preset.

The basic idea of SVM is to take the input sample x from input space X to map to a high-dimensional feature space H through a nonlinear mapping $\phi(\cdot)$, and to realize linear function regression in feature space H. After selecting nonlinear function $\phi(\cdot)$, according to the statistical theory and minimum structural risk criteria, adopting $\varepsilon -$ insensitive loss function, and introducing upper and lower relaxation variables ξ_i and ξ_i^* to describe fitting loss except ε, determining linear regression function.

$$f(x) = (w \cdot \phi(x)) + b \tag{3}$$

It is to solve optimization problem of variable w, b, ξ, ξ^*.

$$\begin{aligned} \min_{w,b,\xi_i,\xi_i^*} \quad & \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \\ s.t. \quad & y_i - ((w \cdot \phi(x)) + b) \le \varepsilon + \xi_i, i = 1, \cdots, l \\ & ((w \cdot \phi(x)) + b) - y_i \le \varepsilon + \xi_i^*, i = 1, \cdots, l \\ & \xi_i, \xi_i^* \ge 0, i = 1, \cdots, l \end{aligned} \tag{4}$$

where C is penalty coefficient, controlling penalty degree except ε, to realize the balance between complexity and fitting accuracy of function $f(x)$.

Optimizing question (4) as the quadratic programming problem under linear restriction condition, we could obtain the dual problem according to optimization theory.

$$\begin{aligned} &\max_{\alpha_i,\alpha_i^*} -\frac{1}{2}\sum_{i=1}^{l}\sum_{j=1}^{l}\left(\alpha_i-\alpha_i^*\right)\left(\alpha_j-\alpha_j^*\right)K\left(x_i\cdot x_j\right) \\ &\qquad +\sum_{i=1}^{l} y_i\left(\alpha_i-\alpha_i^*\right)-\varepsilon\sum_{i=1}^{l}\left(\alpha_i+\alpha_i^*\right) \\ &\text{s.t.} \quad \sum_{i=1}^{l}\left(\alpha_i-\alpha_i^*\right)=0 \\ &\qquad 0\le\alpha_i\le C, 0\le\alpha_i^*\le C, i=1,\cdots,l \end{aligned} \tag{5}$$

where $\alpha_i \ge 0$, $\alpha_i^* \ge 0$, $\alpha_j \ge 0$ and $\alpha_j^* \ge 0$ are Lagrange dual variables, and $i, j = 1,\cdots,l$, $K(x_i, x_j) = (\phi(x_i)\cdot\phi(x_j))$ are kernel functions that constituted through the map function meets Mercer condition, common kernel function is RBF function, its expression is:

$$K(x_i, x) = \exp\left(-\|x-x_i\|^2 / (2\sigma^2)\right) \tag{6}$$

where σ is the width coefficient of RBF kernel function.

If $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_1^*, \cdots, \overline{\alpha}_l, \overline{\alpha}_l^*)^T$ is the optimum solution of dual problem (5), we could calculate b according to KKT condition.

$$\begin{cases} b = y_i - \sum_{j=1}^{l}\left(\overline{\alpha}_j - \overline{\alpha}_j^*\right)K\left(x_j\cdot x_i\right)+\varepsilon, if\ \overline{\alpha}_i \in (0, C) \\ b = y_i - \sum_{j=1}^{l}\left(\overline{\alpha}_j - \overline{\alpha}_j^*\right)K\left(x_j\cdot x_i\right)-\varepsilon, if\ \overline{\alpha}_i^* \in (0, C) \end{cases} \tag{7}$$

Regression function is:

$$f(x) = \sum_{i=1}^{nsv}\left(\overline{\alpha}_i - \overline{\alpha}_i^*\right)K\left(x_i\cdot x\right)+b \tag{8}$$

where x_i is support vector, nsv is the number of support vectors.

The ε -insensitive coefficient, RBF kernel function width coefficient σ , penalty coefficient C affect the performance of SVM directly. The performance of SVM depends on ε , σ and C , and there is the best matching of the parameters.

3. Real-coded Quantum Evolutionary Algorithm (RQEA)

In Ref. [7, 8], a real-coded chromosome, whose allele is composed of one component x_i of variable vector *X* and probability amplitudes $(\alpha_i, \beta_i)^T$ of one qubit, $i = 1,2,\cdots,n$, in RQEA is represented as:

$$q = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} \tag{9}$$

where n is the length of chromosome, and lies on the dimensions of variable vector X .

The single-gene mutation is adopted to update population at each iteration in RQEA. Assume that RQEA maintains a population $p(t) = \{p_j^t\}$ at the *t*-th iteration, where $j = 1,2,\cdots,N$, N is the population size. Select the *i*-th gene $(x_{ji}^t, \alpha_{ji}^t, \beta_{ji}^t)^T$ of p_j^t, and update the value of $x_{j,i}^t$ using Gaussian mutation, which is expressed as:

$$x_{j,i}^{t+1,k} = x_{ji}^t + (x_{i,\max} - x_{i,\min})N(0, (\sigma_{j,i}^k)^2) \tag{10}$$

where $k \in \{\alpha, \beta\}$, $(\sigma_{j,i}^k)^2$ denotes Gaussian distribution variance, and its value is designed as:

$$(\sigma_{j,i}^{k})^{2} = \begin{cases} \left|\alpha_{j,i}^{t}\right|^{2}, & k = \alpha \\ \left|\alpha_{j,i}^{t}\right|^{2} \big/ 5, & k = \beta \end{cases} \tag{11}$$

To avid generation the infeasible solution, the value of $x_{j,i}^{t+1,k}$ is clipped according to Eq. (12). Until the value of $x_{j,i}^{t+1,k}$ lies in the feasible solution space, Eq. (5) has to be performed repeatedly.

$$\begin{cases} x_{j,i}^{t+1,k} = 2x_{i,\max} - x_{j,i}^{t+1,k}, & x_{j,i}^{t+1,k} > x_{i,\max} \\ x_{j,i}^{t+1,k} = 2x_{i,\min} - x_{j,i}^{t+1,k}, & x_{j,i}^{t+1,k} < x_{i,\min} \end{cases} \tag{12}$$

If the feasible solution derived from Eq.(10)~(12) $(x_{j,1}^{t},\cdots,x_{j,i}^{t+1,k},\cdots x_{j,n}^{t})^{T}$ is superior to the feasible solution $(x_{j,1}^{t},\cdots,x_{j,i}^{t},\cdots x_{j,n}^{t})^{T}$, then the valid evolution is carried out, and $x_{j,i}^{t} = x_{j,i}^{t+1,k}$, $\alpha_{j,i}^{t+1} = \alpha_{j,i}^{t}$, $\beta_{j,i}^{t+1} = \beta_{j,i}^{t}$. Otherwise, the invalid evolution is done ,the feasible solution $(x_{j,1}^{t},\cdots,x_{j,i}^{t},\cdots x_{j,n}^{t})^{T}$ is retained ,and $(\alpha_{ji}^{t},\beta_{ji}^{t})^{T}$ is updated by quantum rotation gates as:

$$\begin{bmatrix} \alpha_{j,i}^{t+1} \\ \beta_{j,i}^{t+1} \end{bmatrix} = \begin{bmatrix} \cos\left(\Delta\theta_{j,i}^{t}\right) & -\sin\left(\Delta\theta_{j,i}^{t}\right) \\ \sin\left(\Delta\theta_{j,i}^{t}\right) & \cos\left(\Delta\theta_{j,i}^{t}\right) \end{bmatrix} \begin{bmatrix} \alpha_{j,i}^{t+1} \\ \beta_{j,i}^{t+1} \end{bmatrix} \tag{13}$$

where $\Delta\theta_{j,i}^{t}$ is the rotation angle, and the value of $\Delta\theta_{j,i}^{t}$ is design as:

$$\Delta\theta_{j,i}^{t} = \operatorname{sgn}(\alpha_{j,i}^{t}\beta_{j,i}^{t})\theta_{0}\exp\left(-\frac{\left|\beta_{j,i}^{t}\right|}{\left|\alpha_{j,i}^{t}\right| + \gamma}\right) \tag{14}$$

where $\operatorname{sgn}(\cdot)$ is the sign function and determines the direction of $\Delta\theta_{j,i}^{t}$, θ_0 is the initial rotation angle; γ is evolutionary scale. θ_0, γ and $(\alpha_{j,i}^{t}\beta_{j,i}^{t})^{T}$ decide the size of $\Delta\theta_{j,i}^{t}$ together , control further the convergence rate.

In RQEA, discrete crossover is performed at period τ_c. Select individual p_u^t and p_v^t, $u \neq v$, $u = 1,2,\cdots,N$, $v = 1,2,\cdots,N$, at random in population, let p_u^t and p_v^t as parents, exchange ever corresponding gene of them by 0.5 probability, and generate new individual c^t.

4. Combined Forecasting Model based on SVM and RQEA

4.1. The Selection of Single Forecasting Model

Subgrade settlement developing process presents "S" type curve in linear loading process, the model which has "S" type curve characteristics could reflect the relationship between subgrade settlement and time, single-phase forecasting model contains Logistics model, Usher model, Gompertz model, the expressions of each model are:

Usher model:

$$S(t) = \frac{k}{\left(1 + ae^{-bt}\right)^{\frac{1}{c}}} \tag{15}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b, c are the parameters to be estimated.

Gompertz model:

$$S(t) = ke^{-ae^{-bt}} \quad (16)$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

Logistic model:

$$S(t) = \frac{k}{1 + ae^{-bt}} \quad (17)$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

4.2. Optimizing SVM Parameters based on RQEA

SVM parameter selection problem can be seen as combinatorial optimization problem of parameter ε, σ, C, or optimal matching problem, mean square error can be obtained through fitting value $\hat{y}_i$ and sample value y_i as:

$$MSE = \left(\frac{1}{l} \sum_{i=1}^{l} (\hat{y}_i - y_i)^2 \right)^{\frac{1}{2}} \quad (18)$$

It can regard as SVM performance evaluation index, so SVM parameter selection problem can transform into the continuous function optimization problem which generated through ε, σ and C on R^3 space.

$$\begin{aligned} \min \quad & f(\varepsilon, C, \sigma) = \min \quad MSE \\ \text{s.t.} \quad & \varepsilon_{\min} \le \varepsilon \le \varepsilon_{\max} \\ & C_{\min} \le C \le C_{\max} \\ & \sigma_{\min} \le \sigma \le \sigma_{\max} \end{aligned} \quad (19)$$

For a set of given parameter (ε, σ, C), we could use Sequential Minimal Optimization to train sample data to establish SVR model, then calculating output estimate that corresponding to the sample input according to established SVR model, and to calculate MSE. If choosing MSE as the fitness function, we could use RQEA to solve optimization problem (18), which has the characteristics of fast convergence, strong global search ability and good stability, to optimize SVR parameters, and forming SVR based on RQEA.

4.3. Structure of Combined Forecasting Model

Modeling flow chart of combined forecasting model for subgrade settlement based on SVM and RQEA is shown in Figure 1. Construction steps of combined forecasting model are as follows:

Step1 To estimate single forecasting model parameters using RQEA based on Minimum of
mean square error, and to carry on settlement prediction through the model of determined parameters respectively.

Step2 To normalize the forecasting results of each single forecasting model, taking normalized forecasting results which obtained from each single forecasting model at each observation moment as component, constituting SVM model training data set and testing data set.

Step3 To establish combined forecasting model for subgrade settlement based on SVM using training data set, at the same time, optimize the allocation to the parameters of SVM using RQEA.

Step4 To predict the testing set data using trained robust SVM, in order to evaluate and compare the predicted results.

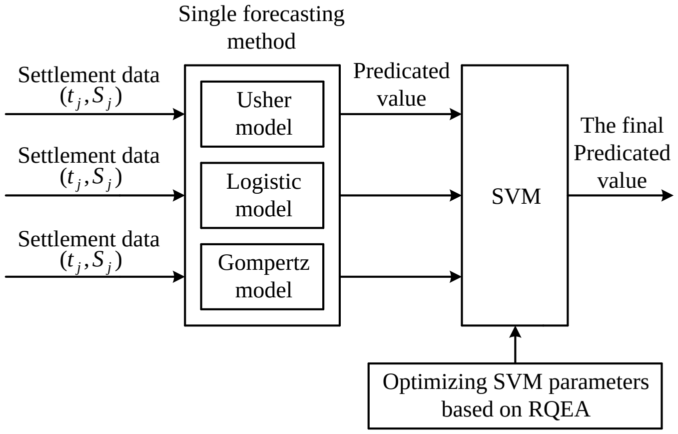

Figure 1 Combined forecasting model based on RQEA and SVN

5. The Application of Combined Forecasting Model

In this paper, choose Ning-Hang highway subgrade NH standard K095+520 section as observation point, select 15 group settlement observation data from 2001-08-09 to 2002-12-14 as research object, using Usher model, Logistic model, Gompertz model and combined forecasting model to model and forecast, the first 11 data are used to model and after 4 data are used to forecast.

RQEA proposed is used to Estimating each single forecasting model parameter values, and the value of estimated parameters are shown in Table 1. Forecast results and relative errors of each single forecasting model and combined forecasting model are shown in Table 2. Forecast results contrast of each single forecasting model and combined forecasting model is shown in Figure 2, relative error contrast is shown in Figure 3.

Table 1. Four Kinds of Forecast Methods Comparison of Forecast Results

Model	Parameter estimation			
	a	*b*	*c*	*k*
Usher	5.5590	-3.9067	5.6292	-5.2187
Logistic	6.1583	-4.7323	6.2051	-5.5293
Gompertz	6.3039	3.9047	6.3465	3.2549

Table 2. Four kinds of Forecast Methods Comparison of Forecast Results

No.	Measured values (cm)	Usher Model		Logistics model		Gompertz model		RQEA-SVM model	
		Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）
1	5.35	5.5590	-3.9067	5.6292	-5.2187	5.6878	-6.3139	0.5430	-2.3004
2	5.88	6.1583	-4.7323	6.2051	-5.5293	6.2218	-5.8124	5.9466	-1.1331
3	6.56	6.3039	3.9047	6.3465	3.2549	6.3518	3.1736	6.5311	0.4402
4	7.48	6.9992	6.4282	7.0282	6.0012	6.9780	6.7119	7.5714	-1.2228
5	7.82	7.6306	2.4221	7.6540	2.1229	7.5595	3.3312	7.7292	1.1602
6	8.05	7.9754	0.9271	7.9968	0.6614	7.8854	2.0452	8.0258	0.3003
7	8.38	8.1905	2.2608	8.2105	2.0227	8.0927	3.4282	8.5615	-2.1669
8	8.59	8.6772	-1.0156	8.6915	-1.1821	8.5763	0.1599	8.6687	-0.9166

9	8.84	9.0932	-2.8643	9.0973	-2.9103	9.0107	-1.9309	8.7129	1.4372
10	9.01	9.2409	-2.5629	9.2394	-2.5464	9.1711	-1.7880	9.1628	-1.6961
11	9.18	9.4438	-2.8735	9.4324	-2.7499	9.3979	-2.3735	9.2391	-0.6439
12	**9.81**	**9.7359**	**0.7554**	**9.7046**	**1.0744**	**9.7407**	**0.7069**	**9.7503**	**0.6091**
13	**10.04**	**9.9770**	**0.6274**	**9.9225**	**1.1699**	**10.0423**	**-0.0234**	**9.9929**	**0.4691**
14	**10.06**	**10.0198**	**0.3999**	**9.9604**	**0.9899**	**10.0981**	**-0.3784**	**10.1039**	**-0.4363**
15	**10.11**	**10.1745**	**-0.6382**	**10.0951**	**0.1472**	**10.3066**	**-1.9443**	**10.1556**	**-0.4510**

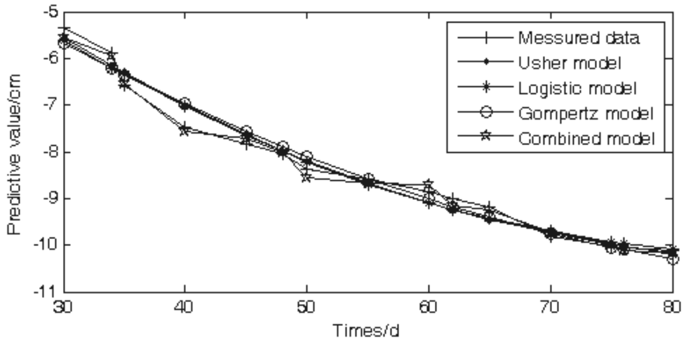

Figure 2. The Predictive Effect of Four Kinds of Forecast Methods

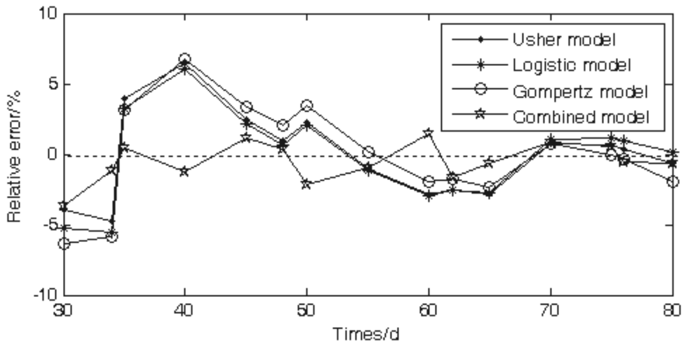

Figure 3. The Relative Error of Four Kinds of Forecast Methods

From Table 2, we can see that the relative error maximum of fitting value of combined forecasting model is 2.3004%, it is better than one of Usher model which is 6.4282%, Logistics model which is 6.0012% and Gompertz model which is 6.7119%, the elative error maximum of predicting value of combined forecasting model is 0.6091%, it is lower than the one of Usher model is 0.7554%, Logistic model is 1.0744% and Gompertz model is 1.9443%. So the fitting and forecasting precision of combined forecasting model is more superior to each single forecasting model.

In Figure 2, the settlement development curves from each forecasting model are describes respectively. We could know that combined forecasting model can reflects the characteristics and law of settlement development curve in both fitting period and forecasting period, but each single forecasting model has good performance at some time.

In Figure 3, the error curves of each forecasting model are illustrates respectively. We could see that the relative error of single forecasting model appears larger fluctuations at the beginning of the model period, and the relative error is also larger during the forecast period, but the relative error of predicted results of combined forecasting model is relatively stable, it reduces the risk of forecasting.

6. Conclusion

This paper has proposed the combined forecasting model for subgrade settlement based on SVM and RQEA. At the first, the growth curve which has S type characteristic is selected as single forecasting model according to the basic settlement law of subgrade and characteristics of settlement curve. Then, the combined forecasting model for subgrade settlement is constructed on the basis of SVM taking the forecasting results of single forecasting model as input vector of SVM, and the parameters of SVM is optimized by RQEA to improve the performance of SVM. Engineering examples show that The proposed combined forecasting model could improve the predicting accuracy, low the predicting risk, at the same time, provide an effective tool to subgrade settlement forecasting, has some practical value.

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