International Journal of Hybrid Information Technology
Vol.8, No.11 (2015), pp.13-26
http://dx.doi.org/10.14257/ijhit.2015.8.11.02

Optimal Policy for Plug-in Hybrid Electric Vehicles Charging Station Scheduling Problem

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Abstract

Advances in the development of electric vehicles along with policy incentives will see a wider uptake of this technology in the transport sector in the coming years. However, the widespread adoption of electric vehicles will add a substantial energy load to power grids. As a result, many technical problems related to the impact of this technology on the power grid need to be addressed, especially the management and allocation of the energy to the plug-in hybrid electric vehicles (PHEVs), electric vehicles (PEVs). In this paper, we formulated the optimal power allocation to PHEVs/PEVs for the PHEVs/PEVs charging stations scheduling problem as a nonlinear resource allocation continuous problem. We used pegging algorithm to solve the optimal power allocation to the PHEVs/PEVs. A mathematical framework for the objective function (i.e., minimizing the average depth of discharge (DoD) at the next time step) was also given. The authors characterized the performance of optimal power allocation to PHEVs/PEVs problem and pegging algorithm using MATLAB simulation, and compared it with other charging methods.

Keywords: *Plug-in hybrid Electric vehicles (PHEVs), Plug-in Electric vehicles (PEVs), Electric vehicles (EVs) and Power allocation*

1. Introduction

In many countries, electric vehicle technology has been considered as a key component in the effort to reduce harmful greenhouse gas emissions and the dependence of the vehicles on imported petroleum within the transport sector. As a result, many automotive manufacturers have begun to place an increasing emphasis on the development of various types of electric vehicles (EVs). These include battery electric vehicles, which operate purely from battery power, and plug-in hybrid electric vehicles which operate based on power from a combination of an on-board battery and a combustion engine. The batteries for both technologies can be recharged from external energy sources, *e.g.*, an electricity network. Ambitious targets and incentives for introducing EVs into the transport sector have been proposed in many countries [1-3]. Such targets along with the likely increase in the cost of fossil fuels over the coming years will see the wide spreading of EV technology.

US government has put a lot of effort to accelerate the introduction and penetration of advanced electric drive vehicles into the market. The US Department of Energy projects, which is approximately 1 million PHEVs/PEVs, will be on the road by 2015 and 425,000 PHEVs/PEVs will be sold in 2015. At this rate, PHEVs would account for 2.5% of all new vehicle sales in 2015 [4]. The Electric Power

ISSN: 1738-9968 IJHIT

Research Institute (EPRI) projects, which accounts for 62% of the entire U.S. vehicle fleet, will consist of PHEVs by 2050 using a moderate penetration scenario [5].

Accordingly, there is a growing need to address the implications of this emerging technology on the power grid. A large market penetration of PHEVs/PEVs imposes an additional stress on the power systems. And a large number of PHEVs/PEVs has the potential to threaten the stability of existing power grids. Moreover, due to the variations in the needs of the PHEVs/PEVs connected to the power grids at any given time, the demand pattern will also have a significant impact on the electricity market.

In order to maximize the customer satisfaction and minimize the disturbances to the grid, a sophisticated controller is needed to be designed to allocate power appropriately. This controller must take into consideration the real-world constraints (*i.e.,* communication and infrastructure variations among individual vehicles), and accommodate for the difference in arrival/departure times and the number of PHEVs in the parking deck. The algorithm should be robust to uncertainty and capable of making decisions in real-time with a limited communication bandwidth. And it should also work seamlessly with the existing utilities.

Recently, the issues of optimal power allocation to PHEVs/PEVs are particularly attractive in research. P. Richardson, *et al.,* [6] proposed a method for controlling the rate at which electric vehicles charge can lead to a better utilization of existing networks. W. Su, *et al.,* [7] conducted a comprehensive survey on the electrification of transportation in a Smart Grid environment. In [8-11], the authors proposed computational intelligence-based centralized charging algorithms (*e.g.,* Estimation Distribution Algorithm EDA, Particle Swarm Optimization and Gravitational Search Algorithm GSA) to achieve the optimal power allocation at a large-scale public PHEVs/PEVs charging facility. W. Su, *et al.,* [12-14] evaluated the impact of the integration of PHEVs/PEVs on the power grids under a variety of charging scenarios. In other research W. Su, *et al.,* [15] analyzed the optimal performance of the proposed charging algorithms under a certain operating conditions and various types of battery models. In addition, a reliable communication network and control of public charging needed to enable the successful integration of a large number of EVs is proposed in ref. [16].

In this paper, we solved the power allocation to PHVEs/VEs based on model in Ref. [8-10]. However our study differs from their model in two aspects. First, instead of maximizing the average state-of-charge (SoC) for all vehicles at the next time step, the objective function considered in this paper is the minimizing of the depth of discharge (DoD) for all vehicles at the next time step. This modification makes objective function more convenient for further processes. Second, we demonstrated that the objective function is a convex nonlinear function. We used the KKT (Karush–Kuhn–Tucker) conditions and pegging algorithm to solve the optimal power allocation to the PHEVs/EVs.

The rest of this paper is organized as follows. First, the formulation and characterization of an optimal charging policy of the model is given in section two. Second, a numerical analysis and discussions is provided in section three. Finally, the conclusion and future work is given in section four.

2. Problem Formulation

2.1. Battery Charging Process

In the optimal power allocation to PHEVs, the SoC and DoD are two important parameters of battery. They measure the percentage of battery energy power that has been used and indicate how far a vehicle can drive on it. The SoC and DoD are defined as the remaining capacity of a batter in ref. [19].

$$SoC=Remaining\ Capacity/Rated\ Capacity \tag{1}$$

And DoD indicates the percentage of the total battery capacity that has been discharged.

$$DoD = 1 - SoC \tag{2}$$

If the Ah capacity is used, the change of DoD can be expressed as:

$$\Delta DoD = DoD_i(t) - DoD_i(t+1) = \frac{1}{C_i}\int_t^{t+1} \mathrm{i}_i(t)\,dt \tag{3}$$

Where, C_i is rated capacity of battery.

The charging current is assumed to be constant over charging time interval Δt.

$$\left[DoD_i(t) - DoD_i(t+1)\right]C_i = I_i(t)\Delta t \tag{4}$$

$$DoD_i(t+1) = DoD_i(t) - \frac{I_i(t)\Delta t}{C_i} \tag{5}$$

Where, $I_i\ (t)$ is the charging current over Δt. Without loss of generality, the battery is modeled to be a capacitor circuit. C_b is capacitance in Farads (the Farad being the capacitance unit of measure).

$$C_b\frac{dU_i(t)}{dt} = I_i(t) \tag{6}$$

Since the variable is the power allocated $P_i(t)$ to PHEVs, the relation between charging current $I_i\ (t)$ and power $P_i\ (t)$ can be expressed as follow.

$$I_i(t) = \frac{P_i(t)}{\bar{U}_{\mathrm{i}}(t)} = \frac{P_i(t)}{0.5\left[U_i(t+1) + U_i(t)\right]} \tag{8}$$

Substituting (7) into (8), we obtain:

$$U_i^2(t+1) = U_i^2(t) + \frac{2P_i(t)\Delta t}{C_b} \tag{9}$$

$$U_i(t+1) = \sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)} \tag{10}$$

$$DoD_i(t+1) = DoD_i(t) - \frac{2P_i(t)\Delta t}{C_i\left[U_i(t+1) + U_i(t)\right]} \tag{11}$$

2.2. Objective Function

The objective function in this paper is the minimizing of the depth of discharge (DoD) for all vehicles in the next time step. In order to maximize the customer satisfaction, the authors consider the energy price, charging time and current DoD in the model. Therefore, the proposed function aims at ensuring some fairness in the energy allocation at each time step. This helps to ensure that a reasonable level of battery power is attained even in the event of less remaining charging time. The objective function is defined as:

Minimize (P):

$$J(t) = \sum_{i=1}^{n(t)} w_i(t)\,DoD_i(t+1) \tag{12}$$

Substituting (11) into (12), the objective function becomes:

$$J(t)=\sum_{i=1}^{n(t)} w_i(t)\left\{DoD_i(t)-\frac{2P_i(t)\Delta t}{C_i\left[\sqrt{\frac{2P_i(t)\Delta t}{C_b}+U_i^2(t)}+U_i(t)\right]}\right\} \tag{13}$$

$$w_i(t)=f\left(C_{r,i}(t),T_{r,i}(t),D_i(t)\right) \tag{14}$$

$$C_{r,i}(t)=DoD_i(t)*C_i \tag{15}$$

Where, n(t) is the sum of PHEVs at the time step t; $C_{r,i}(t)$ is the remaining battery capacity to be filled for *i*-th PHEV at time step t; C_i is the rated battery capacity (Ah) of the *i*-th PHEV; $T_{r,i}(t)$ is the remaining time for charging the *i*-th PHEV at time step t; $D_i(t)$ is the price difference between the real-time energy price and the price that a specific customer at the *i*-th PHEV charger is willing to pay at time step t; $w_i(t)$ is the charge weighting term of the *i*-th PHEV at time step t (this is a function of the energy price, the remaining charging time, and the present DoD); $DoD_i(t+1)$ is the depth of discharge of the *i*-th PHEV at time step t+1.

The weighting term gives a response proportional to the attributes of a specific PHEV. For example, if a particular vehicle has a higher initial DoD and less remaining charging time but the driver is willing to pay a higher price, the controller could consider to give the priority to charge and allocate more power to this PHEV charger:

$$w_i(t)\propto\left[C_{r,i}(t)+D_i(t)+1/T_{r,i}(t)\right] \tag{16}$$

Since the three terms $C_{r,i}(t)$, $D_i(t)$, $1/T_{r,i}(t)$ are not on the same scale, it is necessary to normalize all the terms in order to assign similar importance to each of them:

$$c_{r,i}(t)=\frac{C_{r,i}(t)-Min\left[C_{r,i}(t)\right]}{Max\left[C_{r,i}(t)\right]-Min\left[C_{r,i}(t)\right]} \tag{17}$$

$$d_i(t)=\frac{D_i(t)-Min\left[D_i(t)\right]}{Max\left[D_i(t)\right]-Min\left[D_i(t)\right]} \tag{18}$$

$$t_{r,i}(t)=\frac{1/T_{r,i}(t)-Min\left[1/T_{r,i}(t)\right]}{Max\left[1/T_{r,i}(t)\right]-Min\left[1/T_{r,i}(t)\right]} \tag{19}$$

The parking deck operators may also have different interests and assign different importance factors to $c_{r,i}(t)$, $t_i(t)$, and $d_i(t)$, depending on their own preferences. Thus:

$$w_i(t)=\alpha_1 c_{r,i}(t)+\alpha_2 t_{r,i}(t)+\alpha_3 d_i(t) \tag{20}$$

2.3. System Constrains

The primary energy constraints considered in this paper include the power available from the maximum power of charging station (P_{S_max}) and the maximum power (P_i^{max}) that can be absorbed by a specific vehicle.

Constraint (21) prevents overload for the charging station, which means that the total power allocated to PHEVs at the each time step must be equal to or less than station maximum power multiply with safety coefficient η. η is assumed to be constant at any time step.

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Subject to:

$$\sum_{i=1}^{n(t)} P_i(t) \le \eta . P_{S\_max} \tag{21}$$

$$0 \le P_i(t) \le P_i^{max}(t) \tag{22}$$

2.4. Structure of Optimal Policy

To characterize the optimality of the model, we first process the following two properties of objective function.

Property 1: The objective (13) is a continuous differentiable function of decision variables $P_i(t)$.

Proof: It is obvious that objective function (13) is continuous and differentiable in $P_i(t)$ except for the interval of $P_i(t)$ specified by:

$$\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t) \ge 0 \tag{23}$$

Since $P_i(t)$ is amount of power allocated to i-th vehicle so it is always nonnegative, and expression (23) holds true for arbitrary value of $P_i(t) \in [0, P_i^{max}(t)]$.

Property 2: The objective (13) of (P) is a joint convex function with respect to decision variables $P_i(t)$.

Proof: Detail of proof (see appendix A).

Proposition: For a typical capacitated charging station problems (P), there always exists an optimal charging policy, represented by amount of electric power allocated to i-th vehicle in time step t, denoted as $P_i^*(t)$. $P^*(t)=[P_1^*(t), P_2^*(t),\ldots P_n^*(t)]$ satisfies KKT (Karush–Kuhn–Tucker) conditions.

$$\frac{\partial J(t)}{\partial P_i(t)} + \lambda \frac{\partial P_i(t)}{\partial P_i(t)} + \mu_i - \theta_i = 0 \quad i = 1, \tag{24}$$

$$\lambda \left[\sum_{i=1}^{n(t)} P_i(t) - \eta P_{S-max} \right] = 0 \tag{25}$$

$$\mu_i \left[P_i(t) - P_i^{max} \right] = 0 \tag{26}$$

$$\theta_i \left[0 - P_i(t) \right] = 0 \tag{27}$$

$$\sum_{i=1}^{n(t)} P_i(t) \le \eta P_{S-max} \tag{28}$$

$$0 \le P_i(t) \le P_i^{max} \tag{29}$$

$$\lambda \ge 0; \ \mu_i \ge 0; \ \theta_i \ge 0 \tag{30}$$

Where, λ, μ_i, and θ_i are the Lagrange multipliers.

Lets $\Omega=\{P_i(t), \lambda, \mu_i, \theta_i\}$, where $P_i(t) \in [0, P_i^{max}(t)]$, λ, μ_i, θ_i:$\in \Re^+$ be the set of feasible solutions to (24) – (30). It is obvious that Ω is nonempty set (*i.e.*, there exists at least one feasible solution). Indeed, considering the case $0 < P_i(t) < P_i^{max}(t)$, we could select $P_i(t)$ in such a way that they are satisfied (28). Combining above $P_i(t)$ with (26) and (27) yields $\mu_i = \theta_i = 0$ for i = 1, ...,n; Substitute into (24) we have:

$$\frac{\partial J(t)}{\partial P_i(t)} + \lambda \frac{\partial P_i(t)}{\partial P_i(t)} = 0 \quad i = 1, \dots, n \tag{31}$$

Substitute (A22) in appendix Proof of property 2 into (31) we get:

$$\lambda = \frac{w_i(t)\Delta t}{C_i \sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)}} \tag{32}$$

For any positive value of parameters $w_i(t)$, Δt, C_i and $U_i(t)$ and nonnegative $P_i(t)$, (31) imposes $\lambda \geq 0$. This also satisfies remaining equations (25) and (30) for λ, μ_i , θ_i. Therefore, there always exists at least one feasible solution for the problem (P). Optimal solution will be the one among those feasible solutions and can be derived by following algorithm.

2.5. Finding Optimal Policy Algorithm

In this section, pegging algorithm in [18] is employed. Because, J(t) is monotonicity function of $P_i(t) \in [0, P_i^{max}(t)]$, therefore, the condition for the algorithm is satisfied. Pegging algorithm first solves ***P'***- the relaxation of ***P***. In this case, constraint (21) is binding, and bounds constraints (22) are omitted. It can be verified in [17] that if the optimal solution to ***P'*** satisfies the bounds constraints (19), then it is also an optimal solution to ***P***. Otherwise, we can fix the $P_i(t)$ that violate the bounds at the lower bound or the upper bound and solve the modified bound relaxation problem iteratively and eventually find the optimal solution to (P).

Lets $\underline{O}^k = \{\underline{P_i(t)}: 0 < \underline{P_i(t)} < P_i^{max}(t)\}$ be the index set of free variables; $\underline{V}^k = \{\underline{P_i(t)}: P_i(t) = P_i^{max}(t)\}$ and $L^k = \{P_i(t): P_i(t) = 0\}$ be the index sets of variables already pegged to upper bounds and lower bounds (i.e., zero) at the k-th iteration. The bound relaxation problem ***(RP)*** is:

$$(\mathbf{RP_k})\ \text{Min} \sum_{i \in O^k} J_i(P_i(t)) + \sum_{i \in U^k} J_i(P_i^{max}(t)) + \sum_{i \in L^k} J_i(0) \tag{33}$$

Subject to

$$\sum_{i \in O^k} P_i(t) = \eta P_{S-max}^k \tag{34}$$

Where: $$\eta P_{S-max}^k = \eta P_{S-max} - \sum_{i \in U^k} P_i^{max}(t) \tag{35}$$

In this step, it is observed that $0 < P_i(t) < P_i^{max}(t)$, $i \in O^k$, then (26) and (27) imply that $\mu_i = \theta_i = 0$, and we can solve (31) for $P_i^k(\lambda)$ in term of , thus:

$$P_i^k(\lambda) = \left[\frac{w_i^2(t)\Delta t^2}{(\lambda C_i)^2} - U_i^2(t) \right] \frac{C_b}{2\Delta t} \tag{36}$$

Substitute (36) into (34) we get:

$$\lambda^k = \sqrt{\frac{\sum_{i \in O^k} \frac{w_i^2(t).\Delta t.C_b}{c_i^2}}{2\eta P_{S-max}^k + \sum_{i \in O^k} \frac{U_i^2(t)C_b}{\Delta t}}} \tag{37}$$

Substitute λ^k from (37) into (36) we get $P_i^k(\lambda)$. Iteration is terminated when all $P_i^k(\lambda)$ are satisfying boundary constraints. Finally, $P_i^*(t)$ can be summarized as:

$$P_i^*(t) = \begin{cases} 0 & \text{If } i \in L^k \\ P_i^k(\lambda) & \text{If } i \in O^k \\ P_i^{max}(t) & \text{If } i \in V^k \end{cases} \tag{38}$$

It can be verified in [18] that $P_i^*(t)$ is an optimal solution to the problem (P).

2.6. Computational Procedure

Step 1: Set k = 1, $O^k = \{1, 2, \dots, n\}$, $V^k = \varnothing$, $L^k = \varnothing$. Using (34) and (37) calculate λ1 and corresponds $P_i^1(t)$. If $P_i^1(t)$ satisfies $0 \le P_i^1(t) \le P_i^{max}(t)$, stop and go to step 5. Otherwise, go to step 2.

Step 2: Calculate

$$O_A^k = \left\{ i \in O^k \,\middle|\, P_i^k(t) < 0 \right\} \tag{39}$$

$$O_B^k = \left\{ i \in O^k \,\middle|\, P_i^k(t) > P_i^{max}(t) \right\} \tag{40}$$

$$S_A^k = \sum_{i \in O_A^k} -P_i^k(t) \tag{41}$$

$$S_B^k = \sum_{i \in O_B^k} \left[P_i^k(t) - P_i^{max}(t) \right] \tag{42}$$

Step 3: If $S_A^k \ge S_B^k$, set $O^{k+1} = O^k \setminus O_A^k$, $L^{k+1} = L^k \cup O_A^k$; $V^{k+1} = V^k$. Otherwise, if $S_A^k < S_B^k$, set $O^{k+1} = O^k \setminus O_B^k$, $V^{k+1} = V^k \cup O_B^k$; $L^{k+1} = L^k$ and go to Step 4.

Step 4: Solve ***(RP$_k$)*** to obtain a solution $P_i^k(t)$ ($i \in O^k$). If $P_i^k(t)$ satisfies $0 \le P_i^k(t) \le P_i^{max}(t)$, stop go to Step 5. Otherwise, go to Step 2.

Step 5: Optimal solution summarize

$$P_i^*(t) = \begin{cases} 0 & \text{If } i \in L^k \\ P_i^k(t) & \text{If } i \in O^k \\ P_i^{max}(t) & \text{If } i \in V^k \end{cases} \tag{43}$$

3. Numerical Analysis

In this section, the authors analyze the optimal power allocation strategy of results and compare the results with other methods such as equally allocation method (EAM), first come first serve strategy (FCFS).

3.1. Equally Allocation Method

Equally allocation method, the total capacity of the charging station at each time step will be equally allocated among the vehicles waiting at the station, with the constraint $P_i(t) \le P_i^{max}(t)$. The equal allocation method can be expressed as follows:

$$P_i(t) = \frac{\eta P_{S\text{-}max}}{n(t)} \tag{44}$$

$$st. \qquad P_i(t) \le P_i^{max}(t)$$

Where $P_i(t)$ is power allocated to i-th PHEV at the time step t, n(t) is sum number of PHEVs at the time step t, $P_i^{max}(t)$ is the maximum power that PHEVs can get at each time step and P_{S_max} is the maximum power of charging station, η is the overall charging efficiency of the charging station.

3.2. First Come, First Serve Strategy

In the first come, first serve strategy (FCFS), the total vehicles charged in each time step includes the vehicles left from the previous time steps and the ones that have just arrived. The vehicle comes first will be served first and will only leave when it is fully charged or the customer requiring time is out (remaining charging time equal zero). The number of total vehicles charged in each time step is counted by the division of total power of station and the maximum power (P_i^{max}) that can be absorbed by a specific vehicle plus the binary variable n_0 (n_0 is 1 as the remainder of the division is nonzero and n_0 is 0 as the as the remainder of the division is zero). The first come, first serve strategy is demonstrated in the following formula.

$$n_{ch}(t) = \frac{\eta P_{S-max}}{P_i^{max}(t)} + n_0 \tag{45}$$

$$n_0 = \begin{cases} 0 & \sum P_i^{max}(t) = \eta P_{S-max} \\ 1 & \sum P_i^{max}(t) < \eta P_{S-max} \end{cases} \tag{46}$$

Where, $n_{ch}(t)$ is the total number of vehicles to be charged at time step t, n_0 is a binary variable.

3.3. Data and Results Analysis

Due to lack of real market data, some of parameters are estimated or simulated according to published work and public data [7-9]. Battery chargers fall into three categories by voltage and power level. Level 2 is typically described as the primary or standard method for both private and public charging, and specifies a single-phase branch circuit with typical voltage 240 VAC. In this paper, all the battery chargers are assumed to be Level 2 and maximum PHEV charger limit $P_i^{max}(t)$ is 6.7kW. The station capacity (P_{S-max}) in the test $P_{S\_max} = 0.6 * n(t) * P_i^{max}(t)$.

For this test, the number of vehicles *m(t)* arriving to station at each time step to request charging service is stochastic. In such a case, arrival times are assumed to follow Poisson distribution with mean parameter lambda λ, the probability mass function of *m(t)* is given by:

$$f = (n(t) \mid \lambda) = \frac{\lambda^{m(t)} e^{-\lambda}}{m(t)!} \tag{47}$$

Where, m(t) is the number of arrived PHEVs to the charging station each time step; e is Euler's number (e = 2.71828...).

Moreover, the initial DoD of arrived vehicles are assumed to follow log-normal distribution with mean μ and standard deviation σ. Therefore, the probability that initial DoD can be computed as:

$$f(DoD_i|\mu,\ \sigma) = \frac{1}{DoD_i\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln DoD_i - \mu)^2}{2\sigma^2}} \tag{48}$$

In this test, we considered for 96 time step corresponding to 24 hours, the sample time was set for each time step is 15 minutes. The remaining charge time of the vehicle arrived to station at each time step was defined as a continuous random number between 1 and 8 hour. The price that the customers were willing to pay for electricity was defined as a continuous random number between \$1 and 1.5\$. The battery capacity was assumed to be identical for all vehicles. Weighting terms $w_i(t)$ were the summation of three terms normalized attributes, and followed the relation:

$$w_i(t) = \alpha_1 c_i(t) + \alpha_2 t_{r,i}(t) + \alpha_3 d_i(t),$$

Where α_1, α_2, α_3 are the coefficient factor and used to evaluate the importance of each attribute of the weighting term. Their values depend on station characteristics, demand type, *etc*. In this paper, we assumes that α_1=0.25; α_2=0.1; α_3=0.65.

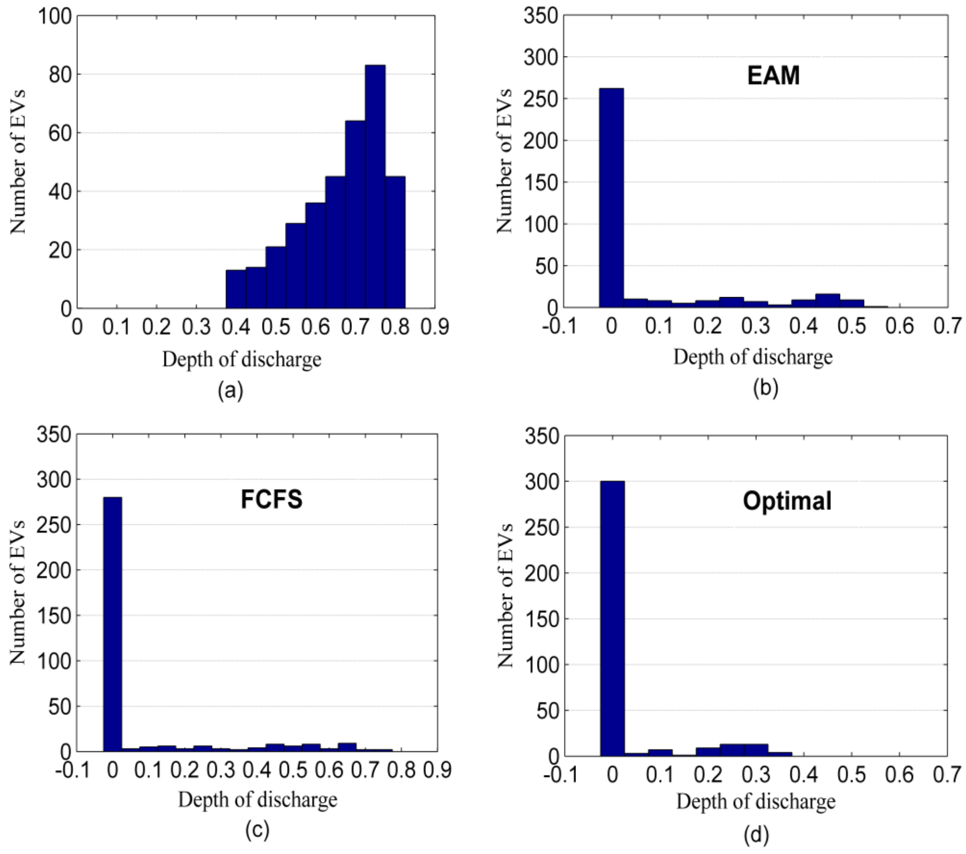

Figure 1. Initial DoD and Departure DoD of 350 PHEVs/EVs for 500 PHEVs/EVs Case

Figure 1 demonstrates initial value of DoD and departure value of DoD as a vehicle leaves the station. Figure shows the initial values of DoD which is generated randomly and follows log-normal distribution. Figure 1 (b), (c) and (d) illustrate the values of DoD as the vehicles leave the station when EAM, FCFS and Optimal are applied. It is obviously that the number of vehicle with DoD values equal to zero is smallest in EAM method. However, in EAM method, the DoD values are always smaller than 0.6. For the method of FCFS, there are more vehicles with DoD equal to zero, but there are more vehicle with DoD greater than 0.6 or even equal to 0.8 which means that the vehicles are not charged after leaving the station. Meanwhile, as the optimal method is applied, the number of vehicle leaving station with DoD equal to zero is maximum (about 300 out of 350 vehicle leaving the station). In addition, all the vehicles leave the station with DoD smaller than 0.4, which means the vehicles are charged with a certain amount of power regardless of the time in the station.

The difference in DoD obtained with three method could be explained as follow. In EAM method, in each time period, the total power of the station is distributed equally to the vehicles in the station as long as Pi(t) ≤ Pimax(t). The more the number of vehicle in the station, the less the power is distributed for each vehicle in a period of time. Thus, those vehicles staying for a long time in the station could be charged fully (DoD = 0) and those vehicles stay for a shot time are distributed a small amount of power and could only be partially charged. At the mean time, in FCFS method, a vehicle that comes first is charged first with a highest rate ($P_i(t)= P_i^{max}(t)$) until it is fully charged. This results in to a severe charging problem for the vehicle that comes later and gets a short charging time since it would only be charged when the vehicle that came before is fully charged. In other word, a vehicle with a short charging time is distributed a very small amount of power, even when it leaves, it is not charged as shown in Figure 1 (c). In optimal method, each vehicle is assigned with a charge weighting term $w_i(t)$ which depends on the parameter such as: remaining charging time $T_{r,i}(t)$, remaining battery capacity to be filled $C_{r,i}(t)$ (current value of DoD) and especially cost willing to pay $D_i(t)$ that the customer is willing to pay to be charged first. In the optimal method, according to the value of $w_i(t)$, the optimal power charging for each vehicle in each period is obtained by pegging algorithm. In each period, the changes in value of t and d lead to the change in the charging priority for each vehicle. Hence, most of the vehicles are charged with a reasonable amount of power after leaving the station.

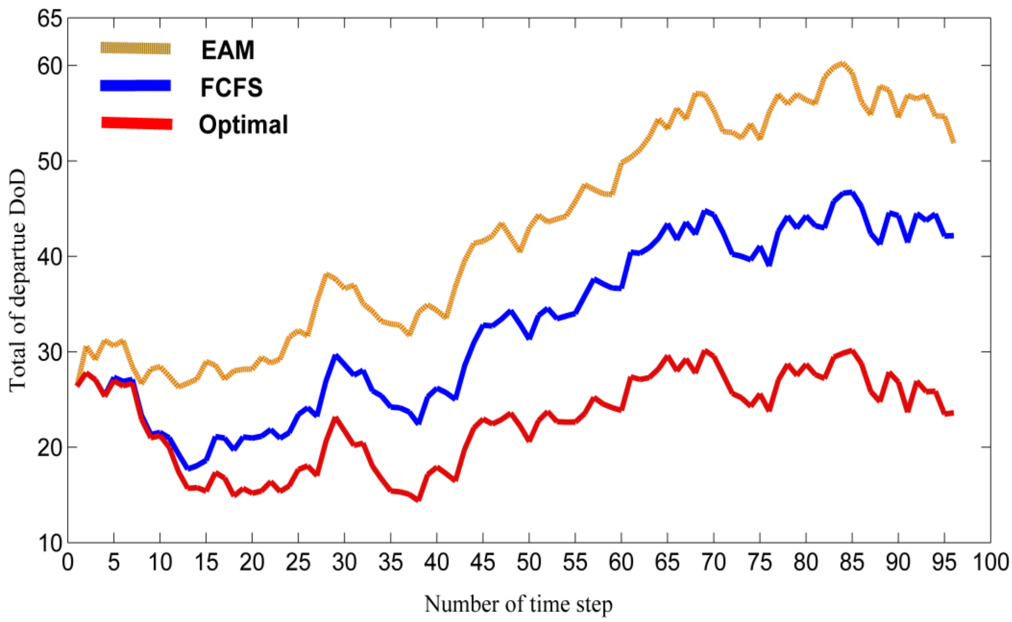

Figure 2. Total of DoD at the Each Time Step

The objective function built in this paper is minimum of DoD in next time step, which means that the power allocated to PHEVs is maximum. As can be seen in Figure 2, the total value of DoD of optimal power allocation to the PHEVs is always smaller than that of other methods. This implies that the power allocation to PHEVs is significant and the requirement of objective function is satisfied. In addition, the total value of DoD is small which means that the power has been optimally utilized and energy loss in the allocated power for PHEVs process is small.

There are some different properties of the power allocation system in the charging station as compared to the practical serving system. For example, there are number of customers who want to leave the station for long time (8 hours at work) and there are some who want to charge quickly and willing to pay more for the priority. Hence, a suitable and flexible charging strategy is necessary to satisfy all the costumers.

Figure 3 demonstrates some parameters of 31 EVs, such as: the time that the customer wants to leave the vehicle in the station (remaining charging time), the initial DoD and the departure DoD. In order to analyze the ability to serve the customer, 31 vehicles from the 70th to 100th out of 500 vehicles were picked. As can be seen in Figure 3, the initial remaining charging time Figure 3 (a) and the initial DoD Figure 3 (b) of vehicles arrived to charging station is different. For the vehicles with long charging time and small value of DOD, in all 3 methods FCFS, EAM and Optimal, when the vehicles leave the station they are almost fully charged (DoD = 0).

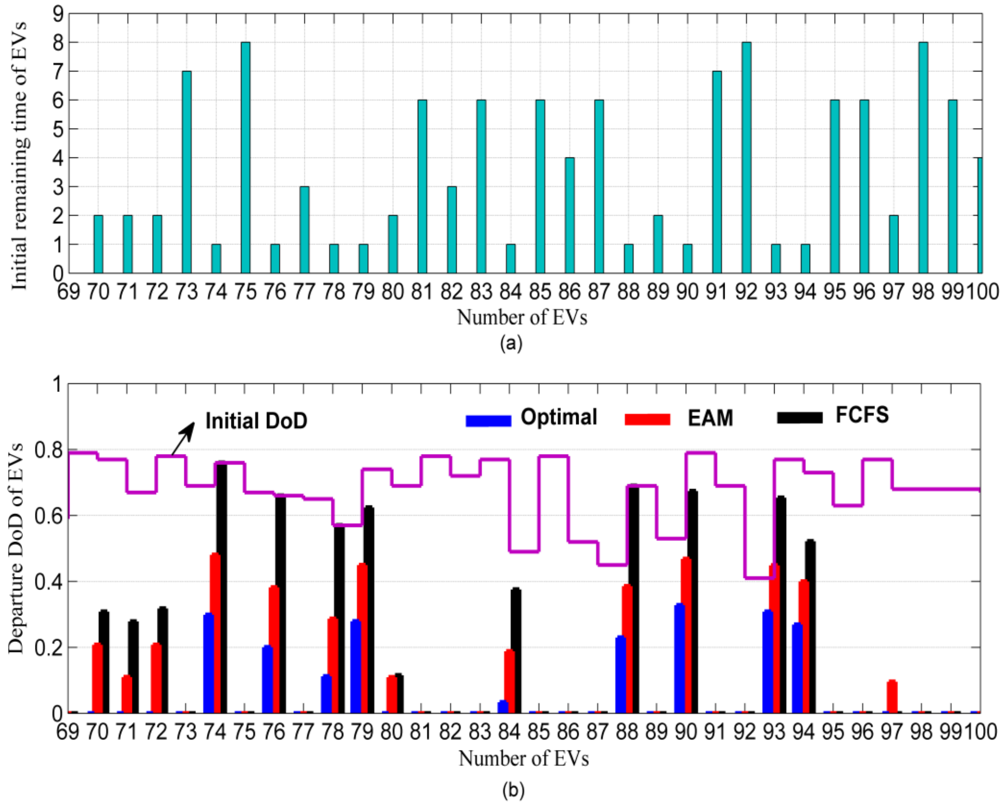

Figure 3. Comparison of the Satisfaction of Customer Demand Charging Methods

However, for the vehicles with a short charging time and large value of DOD, in the FCFS and EAM methods, when the vehicles leave station, they are allocated only a small amount of capacity, even not being charged. Meanwhile, in the Optimal method, the vehicles left the station are charged appropriately. To clarify the problem analyzed above, we considered some of vehicles such as: the vehicles 70th, 71th, 72th that have charging time of 2 hours, and initial DoD are large. Figure 3 (b) shows that only in the Optimal method the vehicles are fully charged (DoD = 0) when the charging time has expired. Meanwhile, in the other methods, the vehicles are not fully charged. In addition, we also considered the 74th vehicle with charging time of 1 hour, when the charging time has expired, it is easily seen that in the Optimal method, DoD value of the 74th vehicle is nearly equal 0.3. In EAM method is 0.5, while DoD in FCFS method is initial DoD (it is not charged). As analyzed above, in the three power allocation methods, we can be affirmed that the ability to respond the demand of the customer of the optimal power allocation to PHEVs/EVs is much better than that of the other methods. Besides, this method can increase the profits for charging station by allowing the customers to pay high prices to be charged in priority.

We compare the pegging algorithm with two other methods: equally allocation method (EAM) and first come first service method (FCFS) in Section 3. The results indicate that the pegging algorithm for the power allocation to PHEVs/EVs problem is superior to the other methods. In addition, our computational testing also shows that the pegging algorithm gives exact solution instead of approximation solution produced by other methods as PSO and GA. Moreover, this algorithm is very easy to apply, especially suitable for solving the nonlinear resource allocation problem.

4. Conclusion

In this paper, we described the performance evaluation of a PHEV charging station from a mathematical perspective. In our model, we considered the constraints such as energy price, remaining battery capacity, and remaining charging time. In order to manage the power allocated to the PHEVs in real-time, we fomulated the PHEVs/EVs charging station scheduling problem as a nonlinear resource allocation continuous problem. Then we applied pegging algorithm to solve the optimal power allocation to the PHEVs. We used a PC to perform the calculations for pegging algorithm testing. The simulation results demonstrate that, for a specific charging station with given parameters, operator is able to find exact optimal charging policy for any instant of time by applying solving procedure presented in this paper. Furthermore, the simulation results also show that the model fits the real-world charging stations by the efficiency in allocating capacity for electric vehicles and the ability to respond customer demand. However, in the paper we considered optimal power allocation to PHEVs for each time step. Besides, the characteristics of electricity supplier, such as real-time pricing policy and supply limitation on peak hours have not been included in the model. The optimal power allocation to PHEVs for full time step and reflecting supplier characteristics in charging policy are our future works. Besides, we will try to evaluate the influence of priority factors and establish their reasonable scaling for specific charging station.

Appendix

Proof of property 2: Because objective function are separable into $J_i(t)$ so that we can partially examine its convexity.

$$J_i(t) = w_i(t)\left\{ DoD_i(t) - \frac{2P_i(t)\Delta t}{C_i\left[\sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)} + U_i(t)\right]} \right\} \qquad (A11)$$

Taking first derivative of $J_i(t)$ with respect to $P_i(t)$ we have:

$$\frac{dJ_i(t)}{dP_i(t)} = -w_i(t)\left\{ \frac{2\Delta t\left[C_i\sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)} + U_i(t)\right] - 2P_i(t)\Delta t\frac{\Delta t}{\sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)}}}{\left(C_i\sqrt{\frac{2P_i(t)\Delta t}{C_b} + U_i^2(t)} + U_i(t)\right)^2} \right\} \qquad (A2.1)$$

Implementing some modifications (A2.1) we obtain.

$$\frac{dJ_i(t)}{dP_i} = -w_i(t)\left\{\frac{\Delta t}{C_i\sqrt{\frac{2P_i\Delta t}{C_b}+U_i^2(t)}}\right\} = -\frac{w_i(t)\Delta t}{C_i\sqrt{\frac{2P_i\Delta t}{C_b}+U_i^2(t)}} \qquad (A2.2)$$

Taking second derivative and some simple modifications we have:

$$\frac{d^2J_i(t)}{d^2P_i(t)} = \frac{w_i(t)\Delta t^2}{C_i^2\left(\frac{2P_i\Delta t}{C_b}+U_i^2(t)\right)\sqrt{\frac{2P_i\Delta t}{C_b}+U_i^2(t)}} \qquad (A2.3)$$

It is clear that right hand sides of (A2.3) is always positive in the range $0 \le P_i(t) \le P_i^{max}(t)$, and we can conclude that $J_i(t)$ is convex function of $P_i(t)$. Along with property 1 we see that objective function $J(t)=\sum J_i(t)$ is a jointly convex function of decision variables $P_i(t)$. Proof is finished.

Acknowledgments

The research presented in this paper is supported in part by the National Natural Science Foundation of China (61221063, U1301254, 61473218, 61304212).

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