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An Approximate Equivalence Based on process Algebra and Numerical Computation and for Differential Semi-algebraic Hybrid Systems

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Abstract

In the paper, approximate ready trace equivalence for differential semi-algebraic hybrid system is proposed. The equivalence can be used to optimize differential semi-algebraic hybrid system. The Concept is proposed on the basis of concrete process algebra and numerical analysis theory. In the approximate ready trace equivalence definition, we consider a cut operator for a polynomial and partial approximation for polynomial. Then we get a strict equivalence between two polynomials. Its advantage is that the new polynomial approximation method overcomes the drawback that traditional approximation method is not transitive, which can be used for automatic reasoning. In order to judge the two differential semi-algebraic hybrid system is equivalent, the axiom system for the approximate ready trace equivalence of differential semi-algebraic hybrid system is presented. This axiom system is a complete axiom system.

Keywords: *axiom system, numerical calculation, approximation ready-trace equivalence, differential semi-algebraic hybrid system, cut operation*

1. Introduction

Hybrid system [7] is a class of complex system with a continuous variable and discrete state transition. Safety verification and liveness verification of hybrid system is paid more and more attention to by researchers. The traditional formal method has been already very mature. On the level of language, there is process algebra which containing CSP, CCS, ACP, LOTOS [20-24] and in the well application of model verification based on character set. In the logic level, mainly the development of temporal logic on the basis of the CTL, CTL*, LTL [10, 12, 15], these logical are well used to reasoning and formal verification based on the of character set. At the structural level, there are Petri Net [18], Kripke Structure [19], Automata [20], Labeled Transition System [3, 4], and Event Structure [7]. All these traditional formal methods have common drawbacks. Firstly, they can't describe the transition of data flow because traditional formal models use abstract actions, which do not meet the demand of description of the system. Secondly, they can’t specify

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conditions because traditional formal models do not contain conditions which need to be described by mathematical specifications. Thirdly, they can't specify continuous behaviors because traditional formal models are made for discrete systems, and do not specify continuous behaviors.

Differential semi-algebraic hybrid system which combines with the differential variable and continuous condition is more complex. The hybrid system model can be used to describe the complex system which has the form of differential equations. In [16], the traditional formal verification method is not able to verify the differential semi-algebraic hybrid system, so if we want to optimize the verification of differential semi-algebraic hybrid system and solve the disadvantages. A new formal verification method needs to be proposed.

Differential semi-algebraic hybrid system is a dynamic system that exhibits both continuous and discrete behaviors. It is seven components $H=(V,S,s_0,V_0,F,Act,Inv)$ where,

(1) *V*: a finite set of real-valued variables;
(2) *S*: a finite set of locations;
(3) s_0: is the initial location or initial state;
(4) X_0 : denotes the initial values of variables over V in q_0;
(5) F: is the set of transitions. Each τ of F is a 3-tuple$\left(q,q^{'},f\right)$, where $q,q^{'}\in Q$ are the pre-location and post-location, f is a polynomial transition relation over $X\cup X^{'}$, where $X,X^{'}$ are the sets of variables in $q,q^{'}$;
(6) *Act*: a labeling function assigns to each location a set of activities;
(7) *Inv*: a labeling function assigns to each location an invariant;

Example 1: the model of thermostat (figure1) is a differential semi-algebraic hybrid system as in figure 3, $H_1=(\{X\},\{l_0,off,on\},l_0,M,\{f_1,f_2\},\varnothing,\{inv(off)=\{x>m\},inv(on)=\{x<M\})$, where three locations: l_0 denotes initial state; Off denotes the heater is off; On denotes the heater is on;

Transition relations are labeled on the edges specified by guarded commands such as x = m from the state Off to On. States invariants are specified by logical formulas $x\geq m, x<M$. $f:X=-kx$, $f:X:=-K(h-x)$

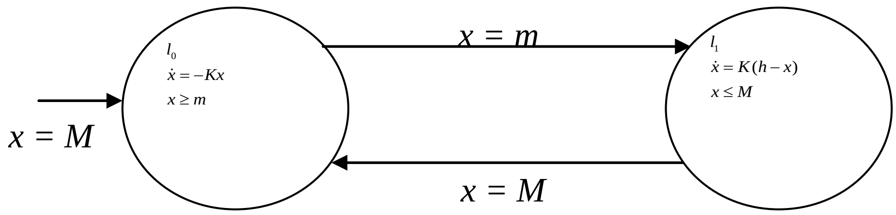


Figure 1. The Model of Thermostat

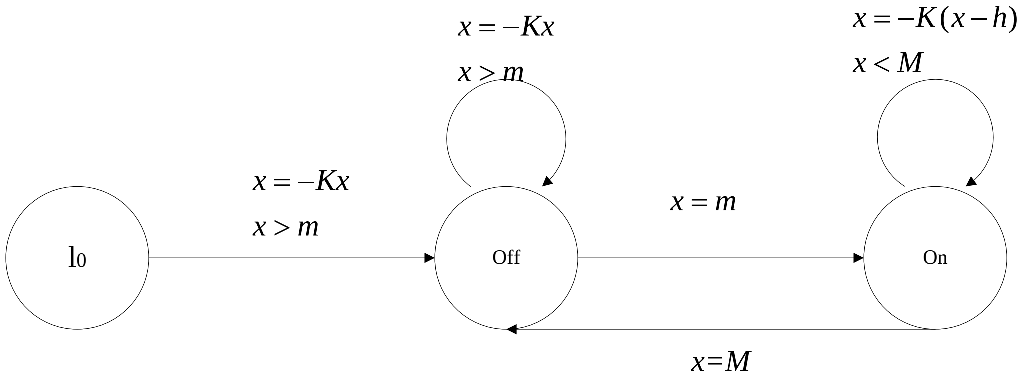


Figure 2. The Transition System Model of Thermostat

There are two characters for the system above: Firstly, each location is labeled with evolution laws. Values of the variables change continuously with time according to associated laws. Each location is also labeled with an invariant condition which must hold when the control resides at the location. Secondly, transitions are labeled with guarded sets of assignments. A transition is enabled when the associated guard is true and its execution modifies the values of the variables according to the assignments.

From the above examples, we know that differential semi-algebraic hybrid system model can well describe the continuous data flow, continuous variables and discrete state transition conditions. The model overcomes withdraw that the traditional formal methods cannot describe the continuous change of flow defects.

Some researchers have related to the research on differential-algebraic hybrid system Zohar Manna mainly studied the invariant generation algorithm [2, 7] of hybrid system based on the theory of polynomial rings. But it can't be used for verifying invariance of differential inequality and differential equations. Mysore has studied uncertainty and certainty [12] of hybrid system to prove the majority of hybrid systems even in low dimensional cases are uncertain. But time automata, multi frequency automata, and rectangular automata, linear systems have been proved to be determined.

André Platzer defines a hybrid dynamic logic [13] and differential algebraic dynamic logic [14] with a series of operator; the definition of differential algebraic dynamic logic is studied for hybrid systems with differential algebraic program. At the same time the early hybrid system modeling Timed Automaton, Multirate Automaton, Rectangular Automaton, and Linear Hybrid Systems all have the drawbacks of traditional formal methods [16].

Analysis of hybrid systems mainly contains safety [9], reachability and activity analysis [4]. Security means that a bad thing never happens; activity mainly means good things will happen. The security research and activity can be attributed to the reachability analysis. So our work mainly focuses on the analysis of reachability. The reachability analysis of important method is to approximately compute the reachable set. General research methods are Taylor approximation [8, 11, 12] and the zero approximation [16].

The reachability problem of hybrid system is starting a U state to a goal state set T. Then found meet safety and activity. Solving the reachability problem includes optimal control, reasoning and model checking. Optimal control makes the target system become simpler through the approximate bisimulation theory. Reasoning technology is often based on automata theory, logic and process algebra [24]. Computation tree logic [12] is mainly used for quantifier elimination the system in the process of real system and can't satisfy the approximation of differential semi algebraic hybrid system in the process. And concrete process algebra is mainly aimed at reasoning techniques from transition system to produce; especially there is a very good application in the control system which has the characteristics of the process. The concrete process algebra [24] is successful in the use of approximation ready trace equivalence and axiom system of linear algebraic transition system [1], so the method based on process algebra is used in the paper. For a differential semi-algebra hybrid system we would give a model based on transition system model, and then the work is focus on the approximation ready trace equivalence of the model.

Approximation works for hybrid system is studied [8, 11, 17]. In Ref [11], the work is on approximation verification of hybrid system. In [11], the work is on Taylor approximation and Wu method approximation for hybrid systems. But all of these works can't be used to reasoning.

Approximate trace equivalence [1], approximate complete trace equivalence [1] and approximate bisimulation equivalence [8] for the differential - algebraic hybrid system have been studied. The equivalences of the above do not hold for transitivity. So the work above can't be used for reasoning. And we have done approximation ready trace for linear algebraic hybrid system, which can be used for automatic reasoning. Its advantage is that the approximation equivalence method overcomes the drawback of above approximation methods, which are not transitive.

But most of the mathematical problems in engineering are nonlinear, even with differential and inequality, called differential semi-algebra hybrid system. We do work on approximation ready trace equivalence of differential semi-algebra hybrid system to optimize the differential semi-algebra hybrid system. It is easier to do equivalence of hybrid system on graph [9], so we do equivalence on the labeled transition system model of differential semi-algebra hybrid system.

The next chapters are according to the following schedule. In the second part, numerical calculation, process algebra and symbolic computation are mainly introduced. In the third part, meet the error conditions of differential semi-algebraic hybrid system and polynomial approximation of differential semi-algebra hybrid system are introduced. The approximation ready-trace equivalence between two differential semi-algebraic hybrid systems is defined and an algorithm to judge whether two systems are approximate is presented. In the fourth part, the axiom system used to reason is proposed and it is proven that the axiom system is complete. In the fifth part, the related theory presented through an experiment is presented to show that the equivalence and the axiom system can verify differential semi-algebra hybrid system. In the sixth part, conclusion is given and further work is introduced.

2. Approximations and Ready Trace Equivalence

It is difficult to define approximate ready-trace equivalence of differential semi-algebraic hybrid system. There three difficult points. Firstly, in order to optimize and validate system model of the differential - algebraic hybrid system and to have the ability of inference, the equivalence must satisfy the reflexive, symmetry and transitive. Secondly the error must be under controllable. In the engineering, only in the controllable error, it is pregnant to approximate compute. Thirdly, it must meet the practical operability, the equivalence defined must can be used to reasoning.

2.1 Cut Operation π for Polynomials and Taylor Expansions

Given a sequence, any n dimension m order polynomials, is arranged according to the arrangement of frequency from high to low. In the paper d（） is a distance function satisfying following：(1) $d(f,g)\geq 0$, (2) $d(f)-d(g)>0$

Definition2: Cut operation π : let $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R$, we define $\pi_m(f)=\sum_{i=n-m+1}^{n}a_i x^i, a_i\in R$ as n terms of π cut.

Definition3：π equivalence: let $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R$, $g(x)=\sum_{i=0}^{n}b_i x^i, b_i\in R$,if x be in domain D, $\pi_m(f)=\pi_m(g)$, f ,g is called π equivalence about π_m on D , noted $f=_{\pi_m} g$

Proposition 1: π equivalence is an equivalent relation.

Proof: reflexivity: from the definition of π cut, for any i, $\pi_i(f)=\pi_i(f)$, reflexivity hold.

Symmetry: if $\pi_i(f)=\pi_i(g)$, then $\pi_i(f)-\pi_i(g)=0$ $-\pi_i(f)+\pi_i(g)=0$,i.e. $\pi_i(g)=\pi_i(f)$

Transitivity: if $\pi_i(g)=\pi_i(f)$, $\pi_i(f)=\pi_i(h)$, then $\pi_i(g)-\pi_i(f)=\pi_i(h)-\pi_i(f)$

So $\pi_i(g)=\pi_i(h)$ hold.

π approximation equivalence: let $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R$, $g(x)=\sum_{i=0}^{n}b_i x^i, b_i\in R$, the error $\varepsilon>0$, $\pi_i(f)=\pi_i(g)$, and the division d , if $d(f-\pi_i(f)+\pi_i(h)-h)<\varepsilon$, $d(g-\pi_i(g)+\pi_i(h)-h)<\varepsilon$ hold, f, g is called π approximation equivalent noted $\pi_i(f)\approx\pi_i(g)$.

Proposition 2: π approximation equivalence is an equivalent relation.

Proof: reflexivity, $\forall j\in N$, $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R, \varepsilon>0$, $\pi_j(f)\approx\pi_j(f)$

Symmetry: $\forall n\in N$, $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R, g(x)=\sum_{i=0}^{n}b_i x^i, b_i\in R$, $\pi_j(f)\approx\pi_j(g)$, from definition above, $\forall n\in N$, $f(x)=\sum_{i=0}^{n}a_i x^i, a_i\in R, g(x)=\sum_{i=0}^{n}b_i x^i, b_i\in R$ and $\varepsilon>0\in R$, $\pi_i(f)=\pi_i(g)$, division d.

$d(f-\pi_i(f)+\pi_i(h)-h)<\varepsilon$, $d(g-\pi_i(g)+\pi_i(h)-h)<\varepsilon$, because $\pi_i(f)=\pi_i(g)$, then $\pi_i(g)=\pi_i(f)$; Because $d(f-\pi_i(f)+\pi_i(h)-h)<\varepsilon$, $d(g-\pi_i(g)+\pi_i(h)-h)<\varepsilon$ then $\pi_j(g)\approx\pi_j(f)$.

Transitivity: $d(f-\pi_2(f)=0.01=0$, $d(h-\pi_2(h)\ )=0$, $h(x)=\sum_{i=0}^{n}c_i x^i, c_i\in R,$

$\pi_j(f)=\pi_j(g)$, $\pi_j(g)=\pi_j(h)$, from the definition 3, $\varepsilon>0$, $\pi_i(f)=\pi_i(g)$, $\pi_i(g)=\pi_i(h)$, $\pi_i(f)=\pi_i(h)$, the division d , because $d(f-\pi_i(f)+\pi_i(v)-v)<\varepsilon$, $d(g-\pi_i(g)+\pi_i(v)-v)<\varepsilon$

$d(h-\pi_i(h)+\pi_i(v)-v)<\varepsilon$, so $d(f-\pi_i(f)+\pi_i(v)-v)<\varepsilon$,
$d(h-\pi_i(h)+\pi_i(v)-v)<\varepsilon$, and $\pi_i(f)=\pi_i(h)$ so $\pi_j(f) \square \pi_j(h)$.Transitivity is hold.

Example 2：let $f=x^2+x+0.01, g=x^2+0.999x, h=x^2+x$, the error $\varepsilon=0.1$.

Remark：let division $d:\begin{cases} f(x)=0.1z, if\ \ 0\le f(x)-0.1z<0.1, z\in Z \\ f(x)=0.1z+0.1\ \ if\ 0\le f(x)-0.1z-0.1<0.1, z\in Z \end{cases}$,

$\pi_1(f)=\pi_1(g)=\pi_1(h)=x^2$,
$0<x<1$, $d(f-\pi_1(f)=x+0.01, d(g-\pi_1(g))=0.99x, d(h-\pi_1(h))=x$,in some region, for example x=9.98, $\pi_1(f) \square \pi_1(h) \square \pi_1(g)$ but x=10 $\pi_1(f) \square \pi_1(h)$ hold, and $\pi_1(f) \square \pi_1(g)$ do not hold.

When $\pi_2(f)=\pi_2(h)=x^2+x, \pi_2(g)=x^2+0.99x$, so $d(f-\pi_2(f)=0.01=0,$
$d(h-\pi_2(h))=0$ and $\pi_2(f) \square \pi_2(h)$.

Lemma 1：Let function f on field of real numbers be continues and nth-order differential, the error e>0, then there exists polynomial team $\{p_j \mid j=1,2,\cdots,m\}$ satisfy the following:

$$|f-\sum_{i=j} p_i|<\varepsilon$$

If *f(x)* at point $x=x_0$ is *n* order differentiable, then

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = f(x_0)+f'(x_0)(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\cdots\sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\cdots$$

is called $f(x)$'s Taylor's series at the point $x=x_0$.

Multivariate version of Taylor expansion: Let $f:R^n\to R$ be a k times differentiable function at point $p\in R^n$, $p=x^a=(x_1^{a_1},\cdots,x_n^{a_n})\in R^n$. Then there exists $h_p:R^n\to R$ such that:

$$\begin{cases} f(x)=\sum_{|a|=0}^{k}\frac{D^a f(p)}{a!}(x-p)^a+\sum_{|a|=k}h_p(x)(x-p)^a \\ \lim_{x\to p}h_p(x)=0 \end{cases}$$

$$|a|=a_1+\cdots+a_n;\ \ D^a f=\frac{\partial^{|a|}f}{\partial x_1^{a_1}\cdots\partial x_n^{a_n}};\ \ a!=a_1!\cdots a_n!$$

By lemma 1 we know, all continuously differential functions can be expanded with Taylor polynomial expansion, so the differential hybrid system, we only need to study the differential polynomial form semi-algebraic hybrid system.

2.2 Ready Trace Equivalence of Labeled Transition System

In [3, 5, 6, 10], it describes ready trace equivalence of labeled transition system is studied. For two labeled transition system *f*, g, if $RT(f)=RT(g)$, *f*, g is called ready trace equivalent. Then axioms system for ready trace semantics is given in Table1, and it is proved that this axiom system is complete. Labeled transition system axiom system is based on the set of operators $\Sigma_1=\oplus$, ., $\|$, $\lfloor\!\lfloor$, $|$, ∂_H, π_n, θ, which were introduced in [3].

Table1: $BPA_\delta+R1,2+PR1-4+RTR$

$x \oplus y = y \oplus x$	A_1
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$	A_2
$x \oplus x = x$	A_3
$(x \oplus y).z = x.z \oplus y.z$	A_4
$(x.y).z = x.(y.z)$	A_5
$\delta \oplus x = x$	A_6
$\delta.x = \delta$	A_7
$f.(g.x \oplus u) \oplus f.(g.y \oplus v) = f.(g.x \oplus g.y \oplus u) \oplus f.(g.x \oplus g.y \oplus v)$	R_1
$f.(g \oplus u) \oplus f.(g.y \oplus v) = f.(g \oplus g.y \oplus u) \oplus f.(g \oplus g.y \oplus v)$	R_2
$f.x \oplus f.(y \oplus z) = f.x \oplus f.(x \oplus y) \oplus f.(y \oplus z)$	S
$x \parallel y = x \lfloor\!\lfloor y \oplus y \lfloor\!\lfloor x \oplus x \mid y$	CM_1
$f \lfloor\!\lfloor x = f.x$	CM_2
$(f.x) \lfloor\!\lfloor y = f.(x \parallel y)$	CM_3
$(x \oplus y) \lfloor\!\lfloor z = x \lfloor\!\lfloor z \oplus y \lfloor\!\lfloor z$	CM_4
$(f.x) \mid g = (f \mid g).x$	CM_5
$f \mid (g.x) = (f \mid g).x$	CM_6
$(f.x) \mid (g.y) = (f \mid g).(x \parallel y)$	CM_7
$(x \oplus y) \mid z = x \mid z \oplus y \mid z$	CM_8
$x \mid (y \oplus z) = x \mid y \oplus x \mid z$	CM_9
$\partial_F(f) = f \quad if \quad f \notin F$	D_1
$\partial_F(f) = \delta \quad if \quad f \in F$	D_2
$\partial_F(x \oplus y) = \partial_F(x) \oplus \partial_F(y)$	D_3
$\partial_F(x.y) = \partial_F(x).\partial_F(y)$	D_4
$\pi_m(f) = f$	PR_1
$\pi_1(f.x) = f$	PR_2
$\pi_{m+1}(f.x) = f\pi_m(x)$	PR_3
$\pi_m(x \oplus y) = \pi_m(x) \oplus \pi_m(y)$	PR_4
$\dfrac{\pi_1(x) = \pi(y)}{z.(x \oplus y) = z.x \oplus z.y}$	RTR

2.3 Approximate Ready Trace Equivalence of Linear Algebra Transition System

In [4], it describes approximate ready trace equivalence of linear algebra transition system is studied. For two linear algebra transition systems f, g, if $RT(f)$ □ $RT(g)$, f, g is called approximate ready trace equivalence. Then axioms system for approximate ready trace semantics is given in Table2, and it is proved that this axiom system is complete. Labeled transition system axiom system is based on the set of operators $\Sigma_1 = \oplus, ., \|, \lfloor\!\lfloor, |, \partial_H, \pi_n, \theta$, which were introduced in [3].

Table 2. $BPA_\delta + R1,2 + PR1-4 + RTR + Ap1-8$

$x \oplus y = y \oplus x$	A_1
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$	A_2
$x \oplus x = x$	A_3
$(x \oplus y).z = x.z \oplus y.z$	A_4
$(x.y).z = x.(y.z)$	A_5
$\delta \oplus x = x$	A_6
$\delta.x = \delta$	A_7
$f.(g.x \oplus u) \oplus f.(g.y \oplus v) = f.(g.x \oplus g.y \oplus u) \oplus$ $f.(g.x \oplus g.y \oplus v)$	R_1
$f.(g \oplus u) \oplus f.(g.y \oplus v) = f.(g \oplus g.y \oplus u) \oplus$ $f.(g \oplus g.y \oplus v)$	R_2
$f.x \oplus f.(y \oplus z) = f.x \oplus f.(x \oplus y) \oplus f.(y \oplus z)$	S
$x \parallel y = x \lfloor\!\lfloor y \oplus y \lfloor\!\lfloor x \oplus x \mid y$	CM_1
$f \lfloor\!\lfloor x = f.x$	CM_2
$(f.x) \lfloor\!\lfloor y = f.(x \parallel y)$	CM_3
$(x \oplus y) \lfloor\!\lfloor z = x \lfloor\!\lfloor z \oplus y \lfloor\!\lfloor z$	CM_4
$(f.x) \mid g = (f \mid g).x$	CM_5
$f \mid (g.x) = (f \mid g).x$	CM_6
$(f.x) \mid (g.y) = (f \mid g).(x \parallel y)$	CM_7
$(x \oplus y) \mid z = x \mid z \oplus y \mid z$	CM_8
$x \mid (y \oplus z) = x \mid y \oplus x \mid z$	CM_9
$\partial_F(f) = f \quad if \quad f \notin F$	D_1
$\partial_F(f) = \delta \quad if \quad f \in F$	D_2
$\partial_F(x \oplus y) = \partial_F(x) \oplus \partial_F(y)$	D_3
$\partial_F(x.y) = \partial_F(x).\partial_F(y)$	D_4
$\pi_m(f) = f$	PR_1
$\pi_1(f.x) = f$	PR_2
$\pi_{m+1}(f.x) = f\pi_m(x)$	PR_3
$\pi_m(x \oplus y) = \pi_m(x) \oplus \pi_m(y)$	PR_4
$\dfrac{\pi_1(x) = \pi(y)}{z.(x \oplus y) = z.x \oplus z.y}$	RTR
$\dfrac{x = y}{x \square y}$	AP_1
$x \square x$	AP_2
$\dfrac{x \square y}{y \square x}$	AP_3
$\dfrac{x \square y, y \square z}{x \square z}$	AP_4
$\dfrac{x \square y}{x \oplus z \square y \oplus z}$	AP_5
$\dfrac{x \square y}{x \oplus y \square x, x \oplus y = y}$	AP_6
$\dfrac{f \square g}{f \mid h \square g \mid h}$	AP_7
$\dfrac{f \square g}{f \mid h \square g \mid h}$	AP_8

Lemma3: the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$ is complete.

Proof: the proof is in [3]

3. Approximate Ready Trace Equivalence of Differential Semi-algebra Hybrid Systems

Firstly, differential semi-algebra hybrid system is modeled using labeled transition system, and then on the transition system, we translate differential equation to polynomial equation using Taylor series expansion under the error. Then we judge two polynomials whether being approximate equivalent. At last, we got two approximately ready-trace equivalent systems, while l each ready trace of the models is approximate equivalent. The basic idea is as following figure 3.

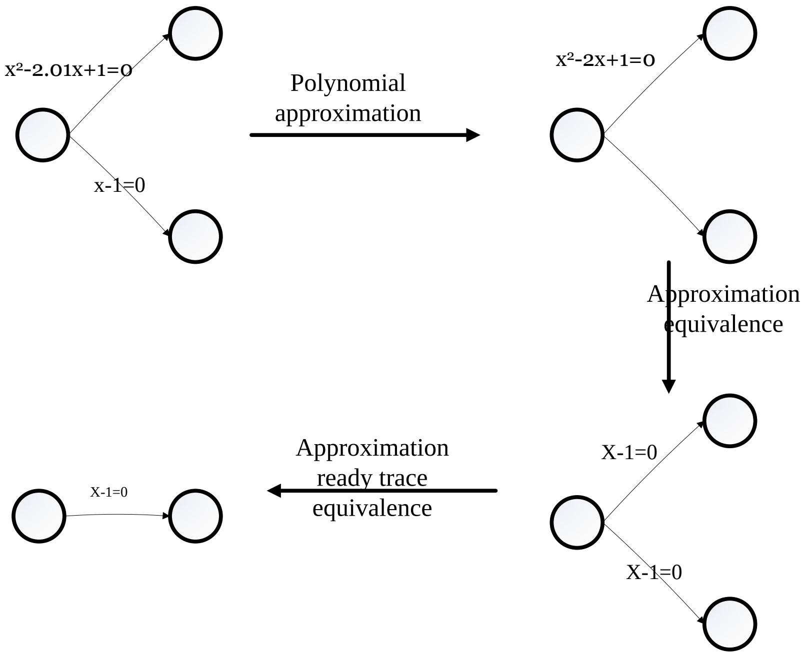


Figure 3. The Basic Idea

3.1 Approximation for Differential Semi-algebra Hybrid Systems

Let H be a differential semi-algebra transition system. I(p) denotes the set of all first transitions. If p is a deadlock, noted δ, and $I(p)=\varnothing$. $I(p)=\varphi$ means that the system terminated successfully.

Definition 4: $\pi = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} \ldots \xrightarrow{f_{n-1}} s_n, n \geq 0$ is a path of $p \in H_{IP}$, a ready trace $rt(\pi)$ of π is the alternating sequence of $I(s_i)$ and f_i : $I(s_0), f_0, I(s_1), f_1, \ldots, f_{n-1}, I(s_n)$

The ready trace of P, notation RT(p) , is $\{rt(\pi) \mid \pi\ a$ path in p, starting from the root.}. If $RT(p)=RT(g)$, we say p,g are ready trace equivalence, notation $p =_{rt} g$

Definition 5: let $\tau_{1:} p \xrightarrow{f} q, \tau_{2:} p \xrightarrow{f'} q'$ be two transitions of differential semi-algebra transition systems, and the error $(d(p),\varepsilon)$, if $d(f,f') \leq \varepsilon$, then $\tau_{1:} p \xrightarrow{f} q, \tau_{2:} p \xrightarrow{f'} q'$ is called approximate on $(d(p),\varepsilon)$ noted $\tau_1 \square_{(d(p),\varepsilon)} \tau_2$

Definition 6：Let S_1, S_2 be two differential semi-algebra transition system, the error $(d(p),\varepsilon)$, if for each transition τ_1 of S_1, there exists a transition τ_2 of S_2, satisfying $\tau_1 \square_{(d(p),\varepsilon)} \tau_2$, S_1, S_2 are called approximate on $(d(p),\varepsilon)$ noted $S_1 \square_{(d(p),\varepsilon)} S_2$.

The advantage of approximation is that when the input value is given and the output results is in only care about , through the process of zero approximation, the output meet the given error, it make the calculation convenient and the model is optimized.

Approximation is different from simulation and emulation, which verify the character set. The approximation is rough calculation data stream. Together with above two points, we can achieve the verification of differential semi-algebraic hybrid system.

Example 3：The following is the airplane flying modeling in Figure 4. A speed v, time t, then the differential algebraic transition model of the plane taking off is as Figure 5.

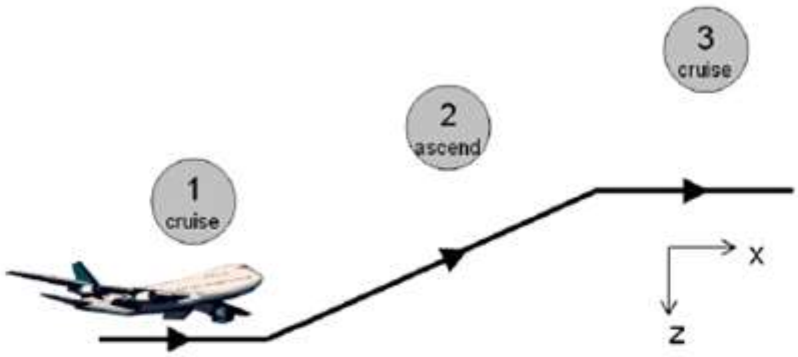


Figure 4. The Airplane Flying Modeling

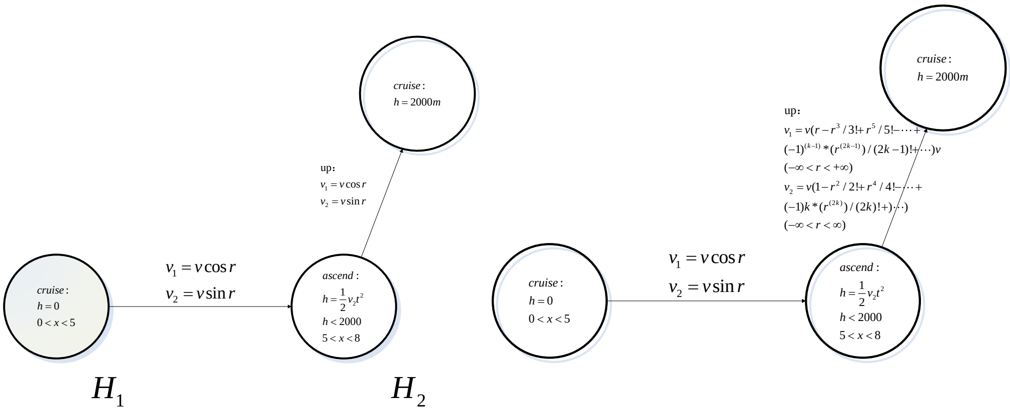


Figure 5. The Differential Algebraic Transition Model of the Plane Taking Off

Let e be the error, from Taylor expansion, for any error 0<e<1, there exists N, when n>N, $d(p_1 - p_2) < e$, i.e. $RT(H_1) \approx RT(H_2)$, H_1 can be approximately computed to H_2.

3.2 Equivalence of Differential Semi-algebra Hybrid Systems

Equivalence of hybrid system is important for optimization of hybrid system. Equivalence can simplify programming objective by choosing a simpler model. The property once is kept between two equivalent models, and then we can take simpler model place of complex model.

Definition 7: Let H_1, H_2 be two differential semi-algebraic hybrid systems, $RT(H_1),\ RT(H_2)$ is ready trace sets of $H_1,\ H_2$, if $RT(H_1) = RT(H_2)$, H_1, H_2 is called ready trace equivalence. In [3] it has been demonstrated that ready trace equivalence for labeled transition system is a strict equivalence relation. Because of the differential - algebraic hybrid system on the structure and the labeled transition system being the same structure, so ready trace equivalence of the differential semi algebraic hybrid system is an equivalence relation. And axioms system for ready trace semantics of labeled transition system is suitable for differential semi-algebraic hybrid system. But because the differential semi algebraic hybrid system has more detailed data flow, data often involves uncertainty calculation and approximation calculation, so approximate ready−trace equivalence for differential semi-algebraic hybrid system is needed to be studied.

3.3 Approximate Ready Trace Equivalence of Differential Semi-algebra Hybrid Systems

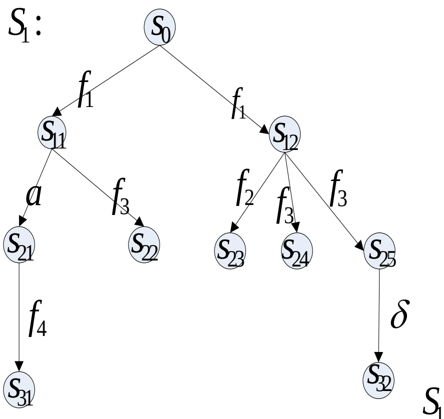
Definition 8: The error $\varepsilon > 0$, $\varepsilon \in R$ and $t_1 : s_0 f_0 \cdots s_m f_m, t_2 : s_0' f_0' \cdots s_m' f_m'$ are two traces of differential semi-algebra transition system. If $i \in 1, 2, \cdots m, d(f_i, f_i') < \varepsilon$, and the states are the same, then , t_1, t_2 is called approximate on $\varepsilon > 0$, $\varepsilon \in R$ if under the division λ, $f_i =_{\lambda} f_i'$, t_1, t_2 is called approximate equivalence under $f_i =_{(\lambda,\varepsilon)} f_i'$.

Definition 9: let two differential semi-algebra hybrid system $H = (V, S, s_0, V_0, F_1, Act, Inv)$, $H = (V, S, s_0, V_0, F, Act, Inv)$, and division (d, ε). If for $\forall rt \in RT(S)$, $\exists rt \in RT(S)$, satisfied $rt \square_{(d,\varepsilon)} rt$, then H is included by H under (d, ε), denoted $RT(S) \square_{(d,\varepsilon)} RT(S)$,if $RT(S) \square_{(d,\varepsilon)} RT(S)$, $RT(S) \square_{(\|A\|,\varepsilon,\|\square\|)} RT(S)$, i.e. $RT(S) =_{(d,\varepsilon)} RT(S)$. Then $H = (V, S, s_0, V_0, F_1, Act, Inv)$ and $H = (V, S, s_0, V_0, F, Act, Inv)$ are approximate ready trace equivalence under division (d, ε), denoted $S \square_{(d,\varepsilon)} S$

Example 4: let S_1, S_2, S_3, S_4 be four differential semi-algebra transition systems as in figure6, figure7, and figure8. S_1, S_2 is ready trace equivalence. Where $S_1 = (\overrightarrow{X} = (x_1, x_2), Q = \{s_0, s_{11}, \ldots, s_{32}\}, s_0, \overrightarrow{X_0} = (2,1), \Gamma), S_2 = (\overrightarrow{X} = (x_1, x_2), Q = \{s_0, s_{11}, s_{12}, \ldots, s_{32}\}, s_0, \overrightarrow{X_0} = (2,1), \Gamma)$.

$$f_1 : \begin{cases} x_1 = x_1^2 + 2.01 x_1 x_2 + x_2^2 \\ x_2 = x_1 - x_2 \end{cases},\quad f_2 : \begin{cases} x_1 = x_1 + x_2 \\ x_2 = x_2 \end{cases},\quad f_3 : \begin{cases} x_1 = x_1 + x_2 \\ x_2 = x_2 \end{cases},\quad f_4 : \begin{cases} f_4(X) = d_{x_1}(f_1) - 0.01 x_2 \\ x_2 = 0 \end{cases}$$

,



$S_1 =_{RT} S_2$

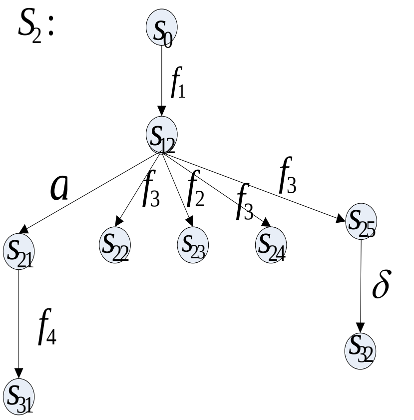


Figure 6 **and** **Figure 7**

$RT(S_1)=\left\{\{f_1\}f_1\{a,f_2,f_3\}a\{f_4\}f_4,\{f_1\}f_1\{a,f_2,f_3\}f_2,\{f_1\}f_1\{a,f_2,f_3\}f_3\{\emptyset,\delta\}\delta,\{f_1\}f_1\{a,f_2,f_3\}f_3\right\}$,

$RT(S_2)=\left\{\{f_1\}f_1\{a,f_2,f_3\}a\{f_4\}f_4,\{f_1\}f_1\{a,f_2,f_3\}f_2,\{f_1\}f_1\{a,f_2,f_3\}f_3\{\emptyset,\delta\}\delta,\{f_1\}f_1\{a,f_2,f_3\}f_3\right\}$

$RT(S_3)=\{\{f_1,f_1^{'}\}f_1\{a,f_3\}a\{f_4\}f_4,\{f_1,\ f_1^{'}\}f_1\{a,f_3\}f_3,\{f_1,\ f_1^{'}\}f_1^{'}\{f_2,f_3\}f_3\{\emptyset,\delta\}\delta,$
$\{f_1,f_1^{'}\}f_1^{'}\{f_2,f_3\}f_3,\{f_1,f_1^{'}\}f_1^{'}\{f_2,f_3\}f_2\}$

Absolutely, the states of S_2 is less than states of S_1 and S_2 is easier computed than S_1.

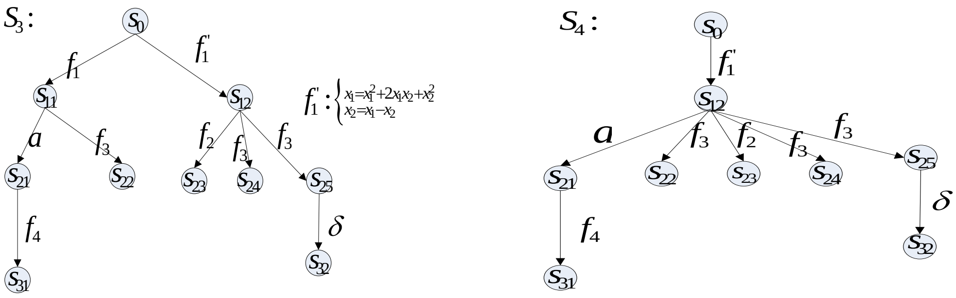


Figure 8

Let $\varepsilon=0.3$, and degree d=n+0.1m, then we got by computing $\pi_1(f)\approx\pi_1(f_1^1)$, $f-\pi_1(f_1)\approx f^{'}-\pi_1(f_1^{'})$, so S_1,S_2,S_3,S_4 is approximate ready-trace equivalence, and we can take S_4 place of S_1,S_2,S_3 when computing the four systems.

From above example absolutely S_4 is simpler than S_1,S_2,S_3, and under the error the four systems are approximate ready-trace equivalence. To judge the two hybrid systems being approximate ready trace equivalence, axiom system for approximate ready trace equivalence is given in the next part.

Proposition 3: An approximate ready trace equivalence of differential semi-algebra hybrid system is an equivalent relation.

Proof: (1) Any hybrid system *S*, absolutely $S\approx_{(d,\varepsilon)}S$ is hold.

(2) For two differential semi-algebra hybrid systems, S_1,S_2, if $S_1\approx_{(d,\varepsilon)}S_2$, then for the error *e* any $s_1\in RT(S_1)$, there exists $s_2\in RT(S_2)$, $s_1\approx_{(d,\varepsilon)}s_2$, let

$s_1 \square_{(d,\varepsilon)} s$,then $s \square_{(d,\varepsilon)} s_2$, so $s_2 \square_{(d,\varepsilon)} s_1$, so for any $s_2 \in RT(S_2)$, there exists $s_1 \in RT(S_1)$, $s_2 \square_{(d,\varepsilon)} s_1$, i.e. $S_2 \square_{(d,\varepsilon)} S_1$.

(3) For two differential semi-algebra hybrid systems S_1, S_2, S_3 , $S_1 \square_{(d,\varepsilon)} S_2$, $S_2 \square_{(d,\varepsilon)} S_3$, from $S_1 \square_{(d,\varepsilon)} S_2$,let $S_1 \square_{(d,\varepsilon)} H$, $S_2 \square_{(d,\varepsilon)} H$,from $S_2 \square_{(d,\varepsilon)} S_3$,so $S_3 \square_{(d,\varepsilon)} H$, then $S_1 \square_{(d,\varepsilon)} S_3$.

4. Axiom Systems for Approximate Ready Trace Equivalence of Differential Semi-algebraic Hybrid Systems

The axiom system of ready trace equivalence for labeled transition system and the axiom system of approximate ready trace equivalence for linear transition system are proposed in Ref [1, 8]. Here the axiom system of ready trace equivalence for differential hybrid system is proposed. It is different that the axiom is with high order polynomial and nonlinear. And here the cut operation for polynomial is added in the axiom system. The axiom is as in Table 3.

Table 3. $BPA_\delta + R1,2 + PR1-4 + RTR + Ap1-8 + PAP1-6$

$x \oplus y = y \oplus x$	A_1
$(x \oplus y) \oplus z = x \oplus (y \oplus z)$	A_2
$x \oplus x = x$	A_3
$(x \oplus y).z = x.z \oplus y.z$	A_4
$(x.y).z = x.(y.z)$	A_5
$\delta \oplus x = x$	A_6
$\delta.x = \delta$	A_7
$f.(g.x \oplus u) \oplus f.(g.y \oplus v) = f.(g.x \oplus g.y \oplus u) \oplus f.(g.x \oplus g.y \oplus v)$	R_1
$f.(g \oplus u) \oplus f.(g.y \oplus v) = f.(g \oplus g.y \oplus u) \oplus f.(g \oplus g.y \oplus v)$	R_2
$f.x \oplus f.(y \oplus z) = f.x \oplus f.(x \oplus y) \oplus f.(y \oplus z)$	S
$x \parallel y = x \lfloor\lfloor y \oplus y \lfloor\lfloor x \oplus x \mid y$	CM_1
$f \lfloor\lfloor x = f.x$	CM_2
$(f.x) \lfloor\lfloor y = f.(x \parallel y)$	CM_3
$(x \oplus y) \lfloor\lfloor z = x \lfloor\lfloor z \oplus y \lfloor\lfloor z$	CM_4
$(f.x) \mid g = (f \mid g).x$	CM_5
$f \mid (g.x) = (f \mid g).x$	CM_6
$(f.x) \mid (g.y) = (f \mid g).(x \parallel y)$	CM_7
$(x \oplus y) \mid z = x \mid z \oplus y \mid z$	CM_8
$x \mid (y \oplus z) = x \mid y \oplus x \mid z$	CM_9
$\partial_F(f) = f \quad if \quad f \notin F$	D_1
$\partial_F(f) = \delta \quad if \quad f \in F$	D_2
$\partial_F(x \oplus y) = \partial_F(x) \oplus \partial_F(y)$	D_3
$\partial_F(x.y) = \partial_F(x).\partial_F(y)$	D_4
$\pi_m(f) = f$	PR_1
$\pi_1(f.x) = f$	PR_2
$\pi_{m+1}(f.x) = f\pi_m(x)$	PR_3
$\pi_m(x \oplus y) = \pi_m(x) \oplus \pi_m(y)$	PR_4
$\dfrac{\pi_1(x) = \pi(y)}{z.(x \oplus y) = z.x \oplus z.y}$	RTR
$\dfrac{x = y}{x \square y}$	AP_1
$x \square x$	AP_2
$\dfrac{x \square y}{y \square x}$	AP_3
$\dfrac{x \square y, y \square z}{x \square z}$	AP_4
$\dfrac{x \square y}{x \oplus z \square y \oplus z}$	AP_5
$\dfrac{x \square y}{x \oplus y \square x, x \oplus y = y}$	AP_6
$\dfrac{f \square g}{f \mid h \square g \mid h}$	AP_7
$\dfrac{f \square g}{f \mid h \square g \mid h}$	AP_8

$u+v \square v+u$	PAP_1
$\dfrac{u \square v}{v \square u}$	PAP_2
$\dfrac{u \square v, v \square w}{u \square w}$	PAP_3
$\dfrac{\pi_i(u)=\pi_i(v), i>j}{\pi_j(u)=\pi_j(v)}$	PAP_4
$\pi_i(au)=a\pi_i(u)$	PAP_5
$\pi_i((a+b)u)=a\pi_i(u)+b\pi_i(u)$	PAP_6

Theory: the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR + Ap1-8 + PAP1-6$ is a complete axiom system.

Prof.: From Lemma 3, we know that the axiom system the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$ is complete system. From Lemma 1, the proposition that the relation $\square$ is equivalence relation has been proven. So the axiom system $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$ in table 2 is complete system. Then if we proof that the axiom PAP1-6 is not conflict with the axioms system $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$, then the theory is proven.

For PAP1, the expression denotes that when two processes are equivalence then they are approximate equivalence, absolutely, it is not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$.

For PAP 2-4, the expression denotes that three characters of equivalence, there no doubt that they are not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$.

For PAP 5-6, because of the change of approximation is only on one step, recording to the definition of approximate ready trace equivalence, they are not they are not conflict with $BPA_\delta + R1,2 + PR1-4 + RTR + AP1,8$.

Then the theory 1 is proven.

5. Experiments

In Figure 9, the system from Ref. [11] describes the following situation. The two aircraft 1and the aircraft 2 flight in 1 mutually vertical direction toward the X axis. When the aircraft 2 reach the origin, if the distance between two aircrafts is less than the safe distance of two aircraft, it will be dangerous, so the aircraft 2 flew towards up. If the distance between two aircrafts is more than the safe distance, the two aircraft will eventually fly follow the X axis. Aircraft 1 and aircraft 2 are initially at coordinates $(-d_1, 0)$ and at coordinates $(-d_2 - r, -r)$. Aircraft 1 is moving along the positive x axis with velocity v_1 and aircraft 2 is moving along the positive y axis with velocity v_2. Aircraft 1 travels a total distance of $(d_1 + d_r)$. At some point within distance v_2 from its initial position aircraft 2 may choose to accelerate or decelerate at a constant rate a. Then its velocity changes till it reaches the point X which is at distance d_2 from the initial point. Let its velocity at point X by v. After this, the aircraft follows a circular trajectory with velocity v along the boundary of a circle with center $c_0 = (0, r)$ and radius r till it reaches the origin. Then it continues to travel along the positive x-axis with velocity v.

The two aircraft must merge safely. The distance between two aircrafts must be at lest d. The problem is verified: given a value of acceleration a, does there exists a time t to start the acceleration (or deceleration) so that the two aircraft merge safely?

We ask when the plane 1 has not reached the destination at any time; the distance between the two aircraft is at least d to safety assurance. We will solve the problem as follows: given that the acceleration a value, and the existence of the time t started to accelerate (decelerate), given the initial value of $\mathrm{v_1 = v_2 = 100,\ z_1 = z_2 = r = d_r = 1000,\ a = 40,\ error\ e = 10,\ d = 100}$, safety distance from the two aircraft can fly safely during the flight? In the next, we will formalize the system and verify the problem using approximate ready-trace equivalence theory above.

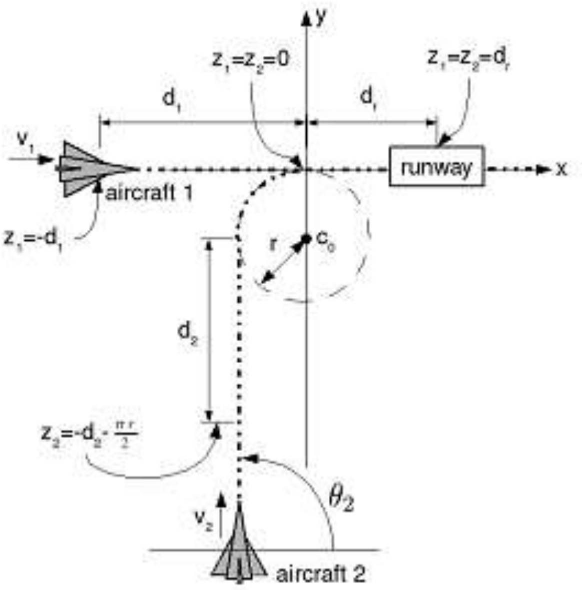


Figure 9

The formal model of the system is a differential algebraic formalized model of mixed system, and contains a polynomial function. Our first step is to establish differential algebraic transition system model under the error. Then approximate ready-trace equivalence model is given. Finally it is proven through the experiment that the approximate ready trace equivalence can verify differential semi-algebra hybrid system model under the error.

The hybrid automata model of the system $H_0 = (Loc, Var, Ede, Act, Inv)$ is as following, where

Loc={init,accel,turn,final,},Var={z_1,z_2,v},Edg={(init;accel);(accel;turn);(turn;fnal)}Act:={init,accel,turn,final},

$Inv = \{inv(init) = z_1 \le d_r \wedge z_2 \le -d_2, inv(accel) = z_1 \le d_r \wedge z_2 \le -d_2, inv(turn) = z_1 \le d_r \wedge z_2 \le 0, inv(final) = z_1 \le d_r\}$

$flow(init,(z_1,z_2,v_2))(t) = (z_1 + v_1 t, z_2 + v_2 t, v_2)$,

$flow(accel,(z_1,z_2,v_2))(t) = (z_1 + v_1 t, z_2 + v_2 t + \frac{1}{2}at^2, v_2 + at)$

$flow(turn,(z_1,z_2,v_2))(t) = (z_1 + v_1 t, z_2 + v_2 t, v_2)$, $flow(final,(z_1,z_2,v_2))(t) = (z_1 + v_1 t, z_2 + v_2 t, v_2)$

Verifying problem $dist(z_1, z_2) < d$ whether being hold, where

$$dist(z_1,z_2)=\begin{cases}\sqrt{(z_1+r)^2+(z_2+\frac{\pi}{2}r-r)^2} & \text{when } z_2 \le -\frac{\pi}{2}r \\ \sqrt{(z_1+r\sin\theta)^2+r^2(1-\cos\theta)^2} & \text{when } z_2 \le -\frac{\pi}{2}r \le z_2 \le 0 \\ \sqrt{(z_1-z_2)^2} & \text{when } z_2 > 0\end{cases}$$

We translate the above model to differential semi-algebra transition system model $H_1 = (V, S, s_0, V_0, F, Act, Inv)$ where: $V = \{z_1, z_2, v_2, dist\}$, $S = \{s_i \mid 0 \le i \le 20, i \in N\}$, $F = \{f_i, f_i' \mid 1 \le i \le 4, i \in N\}$, $V_0 = \{1000, 1000, 100, dist\}$, $Act = \{accel, turn, down, cruise\}$, $Inv = \{Inv(f_i), Inv(f_i') \mid i = 1, 2, 3, 4\}$.

$$f_1:\begin{cases}z_2 \le -\frac{\pi}{2}r, \\ z_1 \le d_r \wedge z_2 \le -d_2 \\ z_{1:} = z_1 + v_1 t \\ z_{2:} = z_2 + v_2 t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+r)^2+(z_2+\frac{\pi}{2}r-r)^2} > d\end{cases}, \quad f_1':\begin{cases}z_2 \le -\frac{\pi}{2}r, \\ z_1 \le d_r \wedge z_2 \le -d_2 \\ z_{1:} = z_1 + v_1 t - 0.5t \\ z_{2:} = z_2 + v_2 t - 0.5t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+r)^2+(z_2+\frac{\pi}{2}r-r)^2} > d\end{cases}$$

$$f_2=\begin{cases}z_2 \le -\frac{\pi}{2}r, \\ z_1 \le d_r \wedge z_2 \le -d_2 \\ z_{1:} = z_1 + v_1 t \\ z_{2:} = z_2 + v_2 t + 0.5at^2 \\ v_2 := v_2 + at \\ dist = \sqrt{(z_1+r)^2+(z_2+\frac{\pi}{2}r-r)^2} > d\end{cases}, \quad f_2'=\begin{cases}z_2 \le -\frac{\pi}{2}r, \\ z_1 \le d_r \wedge z_2 \le -d_2 \\ z_{1:} = z_1 + v_1 t - 0.5t \\ z_{2:} = z_2 + v_2 t + 0.5at^2 - 0.5t \\ v_2 := v_2 + at \\ dist = \sqrt{(z_1+r)^2+(z_2+\frac{\pi}{2}r-r)^2} > d\end{cases}$$

$$f_3:\begin{cases}-\frac{\pi}{2}r \le z_2 \le 0, \\ z_1 \le d_r \wedge z_2 \le 0 \\ z_{1:} = z_1 + v_1 t \\ z_{2:} = z_2 + v_2 t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+r\cos\theta_2)^2+r^2(1-\cos\theta_2)^2} > d\end{cases}, \quad f_3':\begin{cases}-\frac{\pi}{2}r \le z_2 \le 0, \\ z_1 \le d_r \wedge z_2 \le 0 \\ z_{1:} = z_1 + v_1 t - 0.5t \\ z_{2:} = z_2 + v_2 t - 0.5t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+r\cos\theta_2)^2+r^2(1-\cos\theta_2)^2} > d\end{cases}$$

$$f_4:\begin{cases}z_2 > 0, \\ z_1 \le d_r \\ z_{1:} = z_1 + v_1 t \\ z_{2:} = z_2 + v_2 t \\ v_2 := v_2 \\ dist := \sqrt{(z_1-z_2)^2} > d\end{cases}, \quad f_4':\begin{cases}z_2 > 0, \\ z_1 \le d_r \\ z_{1:} = z_1 + v_1 t - 0.5t \\ z_{2:} = z_2 + v_2 t - 0.5t \\ v_2 := v_2 \\ dist := \sqrt{(z_1-z_2)^2} > d\end{cases}$$

$$f_5:\begin{cases} z_2 > 0, \\ z_1 \le d_r \\ z_{1:} = z_1 + v_1 t \\ z_{2:} = z_2 + v_2 t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+z_2)^2} > d \end{cases}, \quad f_5^{'}:\begin{cases} z_2 > 0, \\ z_1 \le d_r \\ z_{1:} = z_1 + v_1 t - 0.5t \\ z_{2:} = z_2 + v_2 t - 0.5t \\ v_2 := v_2 \\ dist := \sqrt{(z_1+z_2)^2} > d \end{cases}$$

The differential semi-algebra transition system model is as following Figure 11.

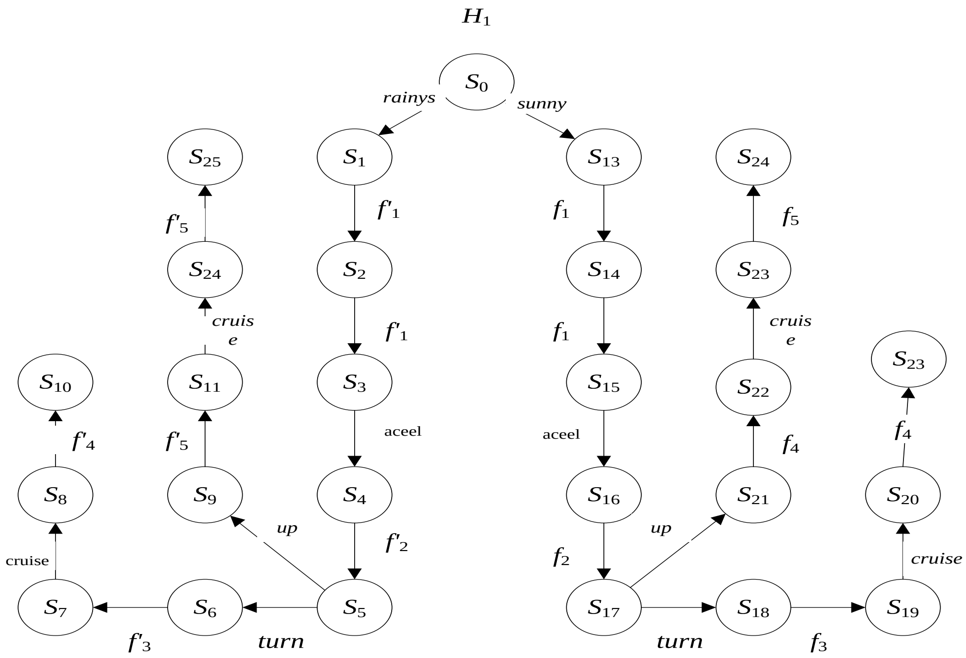


Figure 10

$RT(H_1) = \{\pi_1, \pi_2, \pi_1^{'}, \pi_2^{'} \mid \pi_1 = \{sunny, rain\}sunny\{f_1\}f_1\{f_1\}f_1\{accel\}accel\{f_2\}f_2\{accel, turn\}$
$turn\{f_3\}f_3\{cruise\}cruise\{f_4\}f_4,$
$\pi_2 = \{sunny, rain\}sunny\{f_1\}f_1\{f_1\}f_1\{accel\}accel\{f_2\}f_2\{accel, turn\}accel\{f_4\}f_4\{up\}up$
$\{f_4\}f_4\{cruise\}cruise\{f_5\}f_5,$
$\pi_1^{'} = \{sunny, rain\}rainy\{f_1^{'}\}f_1^{'}\{f_{11}^{'}\}f_{11}^{'}\{accel\}accel\{f_2^{'}\}f_2^{'}\{accel, turn\}turn\{f_3^{'}\}f_3^{'}\{cruise\}cruise,$
$\pi_2^{'} = \{sunny, rain\}sunny\{f_1^{'}\}f_1^{'}\{f_1^{'}\}f_1^{'}\{accel\}accel\{f_2^{'}\}f_2^{'}\{up, turn\}up\{f_4^{'}\}f_4^{'}\{cruise\}cruise\{f_5^{'}\}f_5^{'}\}$

We approximate the function $dist := \sqrt{(z_1 + r\cos\theta_2)^2 + r^2(1-\cos\theta_2)^2}$ using Taylor expansions. For a 5-th order Taylor approximation around zero, we obtain an upper bound of 0.25<10 as the approximation error in this range.

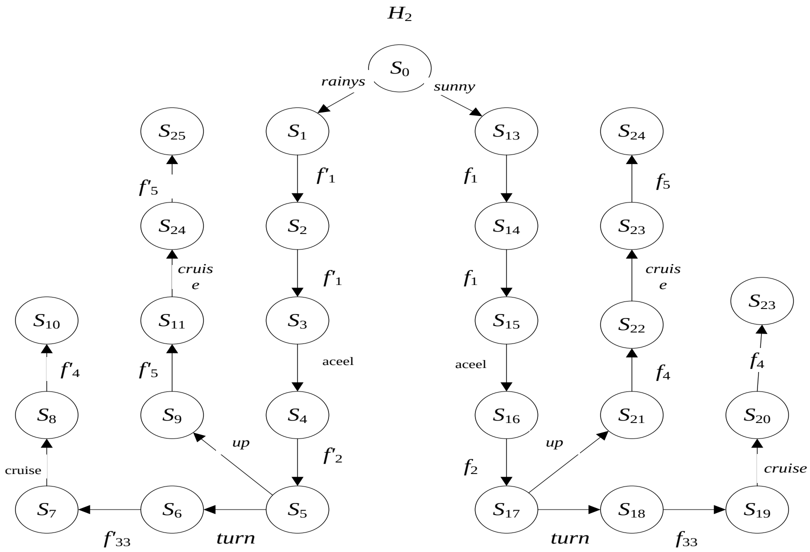


Figure 11

From definition of approximate ready trace equivalence, we got $\pi_1 \approx \pi_1', \pi_2 \approx \pi_2'$ through computing.

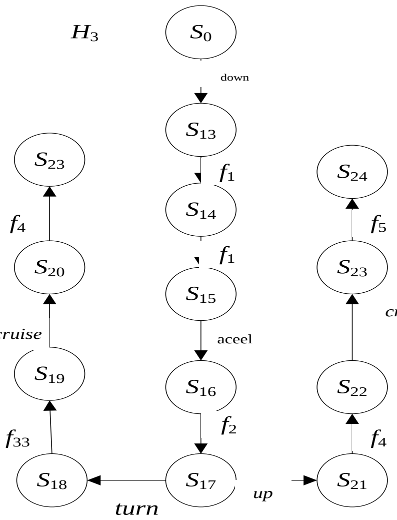


Figure 12

The following process can be got using axiom system $BPA_\delta + R1,2 + PR1-4 + RTR + Ap1-8 + PAP1-6$ to reason.

$$H2: suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_3 cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5) \oplus$$
$$rainy\ \ f_1^{'} f_1^{'} acced\ \ f_2^{'}(turn\ \ f_3^{'} cruise f_4^{'} \oplus up\ \ f_4^{'} cruise\ \ f_5^{'})$$
$$\square\ \ suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5) \oplus$$
$$suny\ \ f_1^{'} f_1^{'} acced\ \ f_2^{'}(turn\ \ f_{33}^{'} cruise f_4^{'} \oplus up\ \ f_4^{'} cruise\ \ f_5^{'}) \qquad (PAP_1)$$
$$\square\ \ suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5) \oplus$$
$$suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33}^{'} cruise f_4^{'} \oplus up\ \ f_4^{'} cruise\ \ f_5^{'}) \qquad (PAP_3)$$
$$= suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5 \oplus$$
$$turn\ \ f_{33}^{'} cruise f_4^{'} \oplus up\ \ f_4^{'} cruise\ \ f_5^{'}) \qquad (R1)$$
$$= suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5 \oplus$$
$$turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5) \qquad (AP5)$$
$$= suny\ \ f_1 f_1 acced\ \ f_2(turn\ \ f_{33} cruise f_4 \oplus up\ \ f_4 cruise\ \ f_5) \quad (A3)$$

It is the system in Figure12.

In the example above, for the distance error e=100 and ε =10, i=5, we get $\pi_1 \square_{RT} \pi_1^{'}, \pi_2 \square_{RT} \pi_2^{'}$. After reasoning using axiom system, we got approximate system H_3 . The number of states of H_3 is less than H_1 . As a result, a non-linear differential hybrid system is optimized to a differential semi-algebra hybrid system. And so we can study differential semi-algebra hybrid system instead of studying complex differential non-linear hybrid system.

Conclusions

In this paper, approximate ready trace equivalence for differential semi algebraic hybrid system is studied in the paper. Firstly, we use transition system modeling differential semi-algebra hybrid system. Then, based on signal computation and numerical calculation, we do the work of approximate ready trace equivalence of differential semi-algebraic hybrid system. The approximate ready trace equivalence is a strict equivalence relation. It has transitivity, reflexivity and symmetry of characters. It overcomes the drawback that traditional approximation method before is not transitive, and it can be used for reasoning. What’s more, the error is under control. Then axiom system for the approximate ready trace equivalence is given and its completeness is proven.

In the following work, we will focuses on three directions. One is other equivalence need to be studied, for example, ready equivalence and failure equivalence. And the other a tool for reasoning of differential semi-algebra hybrid system is need developed. At last, more numerical computation method based on WU method [16] would be proposed.

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