# The Delay-dependent Absolute Stability Condition for T-S Fuzzy Lurie Control Systems with Time-varying Delay 

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#### Abstract

In this paper, the problem of delay-dependent absolute stability condition for a new class of Takagi-Sugeno (T-S) fuzzy Lurie control systems with time-varying delay is investigated. New Lyapunov functions are defined by utilizing the Lyapunov-Krasovskii functional (LKF). Meanwhile, free-weighting matrix method and some techniques are used to obtain a new absolute stability condition, which is different from previous research. Finally, a numerical example will be provided to demonstrate feasibility and less conservativeness of the proposed result.


Keywords: Takagi-Sugeno(T-S) fuzzy Lurie systems, Absolute stability, Time-varying delay, free-weighting matrix method, Lyapunov functional, Linear matrix inequality (LMI)

## 1. Introduction

As we all known that, the theory of absolute stability originated from the stability analysis of space robot. It occupies an important place among exact mathematical methods being used in the design and analysis of control systems. So the absolute stability of Lurie systems has been extensively studied [1,2] in recent years. As the existence of time-delay is often the main source of instability and poor performance, and time-delay phenomenon is frequently encountered in various of engineering systems such as chemical process, mechanical systems, long transmission lines and so on, some scholars have presented some stability condition of Lurie systems with time-delay over the past years [3, 4]. In general, the delay-dependent stability condition is considered to be less conservative than the delay-independent case. Based on this, a considerable number of delay-dependent absolute stability conditions have been proposed in [5-7].

On the other hand, it is well known that the Takagi-Sugeno (T-S) fuzzy models are powerful tools, which is described in [8] for the first time. One can use such models to describe a nonlinear system in the form of a weighted sum of some simple linear subsystems, and some analysis methods in the linear systems can be extended to the T-S fuzzy systems effectively. The control systems with time-varying delay have strong application background, so many scholars have also paid much attention to the study of these systems. Based on these two points, the T-S fuzzy systems with time-varying delay have also attracted great interest by researchers [9-11].

However, to the authors' knowledge, the study of the absolute stability for Lurie control systems using T-S fuzzy model has rarely discussed up to now, which motivates the present study. In this paper, a new class of T-S fuzzy Lurie control systems with timevarying delay is studied. Appropriate Lyapunov functions are defined and new absolute stability conditions are derived with the free-weighting matrix method and novel
techniques. Some useful information was ignored, when $h(t)(\cdot)$ is enlarged to $h_{M}(\cdot)$ in [9], which would lead to increased conservativeness. In this paper, we need not enlarge $h(t)(\cdot)$ to $h_{M}(\cdot)$, so that the error can be reduced. The advantage of this method is that we can reduce the conservatism. Finally, a numerical example will be provided to demonstrate feasibility and less conservativeness of the proposed result.

## 2. Problem Formulation

In this section, we consider a class of T-S fuzzy Lurie control systems with timevarying delay, which is described by a T-S fuzzy model composed of a set of fuzzy implication. The $i$ th rule of the T-S fuzzy model for each $i=1,2, \cdots, r$ is represented as follows:

Plant Rule $i$ :If $s_{1}(t)$ is $\mu_{i 1}, s_{2}(t)$ is $\mu_{i 2}, \cdots, s_{g}(t)$ is $\mu_{\text {ig }}$ THEN

$$
\left\{\begin{array}{l}
\dot{x}(t)=A_{i} x(t)+A_{h i} x(t-h(t))+b_{i} f_{i}(\sigma(t)), t \geq 0  \tag{1}\\
\dot{\sigma}(t)=c^{T} x(t), \\
x(\theta)=\varphi(\theta), \quad \theta \in\left[-h_{M}, 0\right]
\end{array}\right.
$$

where $s_{1}(t), s_{2}(t), \cdots, s_{g}(t)$ are the premise variables, and each $\mu_{i j}(j=1,2, \cdots, g)$ is a fuzzy set. $x(t) \in R^{n}$ denotes the state vector; $u(t) \in R^{p}$ is the system control input vector; $A_{i}, A_{h i}$ and $B_{i}(i=1,2, \cdots, r)$ are the coefficient matrices with appropriate dimensions; $b_{i} \in R^{n} ; c \in R^{n} ; h(t) \in\left[h_{m}, h_{M}\right]$ and $h^{\prime}(t) \leq h$ is the time-varying delay; and $\varphi(\cdot) \in C\left(\left[-h_{M}, 0\right], R^{n}\right)$ is a continuous vector valued initial function; the nonlinearity functions $f_{i}(\cdot)$ satisfy the following sector condition:

$$
\begin{equation*}
f_{i}(\cdot) \in K[0, \infty]=\left\{f_{i}(\cdot) \mid f_{i}(\cdot)=0,0<\sigma f_{i}(\sigma(t))<\infty, \sigma \neq 0\right\} . \tag{2}
\end{equation*}
$$

By using a center-average defuzzifier, product fuzzy inference, fuzzy model in (1) can be represented in the following form:

$$
\Sigma:\left\{\begin{array}{l}
\dot{x}(t)=\sum_{i=1}^{r} h_{i}(s(t))\left[A_{i} x(t)+A_{h i} x(t-h(t))+b_{i} f_{i}(\sigma(t))\right], t \geq 0  \tag{3}\\
\dot{\sigma}(t)=c^{T} x(t), \\
x(\theta)=\varphi(\theta), \quad \theta \in\left[-h_{M}, 0\right]
\end{array}\right.
$$

The fuzzy basis functions are described by:

$$
\begin{equation*}
h_{i}(s(t))=\frac{\omega_{i}(s(t))}{\sum_{j=1}^{r} \omega_{i}(s(t))}, \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
& \omega_{i}(s(t))=\prod_{j=1}^{g} \mu_{i j}\left(s_{j}(t)\right), \\
& s(t)=\left[s_{1}(t), s_{2}(t), \cdots, s_{g}(t)\right]^{T}, i=1,2, \cdots, r,
\end{aligned}
$$

in which $\mu_{i j}\left(s_{j}(t)\right)$ is the grade of membership of $s_{j}(t)$ in $\mu_{i j}$. It is assumed in this paper that

$$
\omega_{i}(s(t)) \geq 0, \sum_{j=1}^{r} \omega_{i}(s(t))>0, i=1,2, \cdots, r, \forall t \geq 0 .
$$

Therefore, the fuzzy basis functions satisfy

$$
\sum_{i=1}^{r} h_{i}(s(t))=1 \text { with } h_{i}(s(t)) \geq 0, i=1,2, \cdots, r, \forall t \geq 0 .
$$

Before stating the main results of this paper, the following lemma is introduced which will be used:
Lemma1. (see[10]) For any constant matrices
$Q_{11}, Q_{22}, Q_{12} \in R^{n \times n}, Q_{11} \geq 0, Q_{22}>0,\left[\begin{array}{ll}Q_{11} & Q_{12} \\ * & Q_{22}\end{array}\right]$
$\geq 0$, scalar $\tau_{1} \leq \tau(t) \leq \tau_{2}$, and vector function $\dot{x}:\left[-\tau_{2}, 0\right] \rightarrow R^{n}$ such that the following integration is well defined, then (1)

$$
\begin{align*}
& -\left(\tau_{2}-\tau_{1}\right) \int_{t-\tau_{2}}^{t-\tau_{1}}\left[\begin{array}{l}
x(t) \\
\dot{x}(t)
\end{array}\right]^{T}\left[\begin{array}{ll}
Q_{11} & Q_{12} \\
* & Q_{22}
\end{array}\right]\left[\begin{array}{l}
x(t) \\
\dot{x}(t)
\end{array}\right] d t \\
& \leq\left[\begin{array}{l}
x\left(t-\tau_{1}\right) \\
x(t-\tau(t)) \\
\int_{t-\tau(t)}^{t-\tau_{1}} x(t) d t
\end{array}\right]^{T}\left[\begin{array}{ccc}
-Q_{22} & Q_{22} & -Q^{T}{ }_{12} \\
* & -Q_{22} & Q^{T} \\
* & * & -Q_{11}
\end{array}\right]\left[\begin{array}{l}
x\left(t-\tau_{1}\right) \\
x(t-\tau(t)) \\
\int_{t-\tau(t)}^{t-\tau_{1}} x(t) d t
\end{array}\right]  \tag{2}\\
& \quad\left(\tau_{1}-\tau_{2}\right) \int_{t-\tau(t)}^{t-\tau_{1}} \dot{x}^{T}(t) Q_{22} \dot{x}(t) d t \\
& \quad \leq\left[\begin{array}{l}
x\left(t-\tau_{1}\right) \\
x(t-\tau(t))
\end{array}\right]^{T}\left[\begin{array}{cc}
-Q_{22} & Q_{22} \\
* & -Q_{22}
\end{array}\right]\left[\begin{array}{l}
x\left(t-\tau_{1}\right) \\
x(t-\tau(t))
\end{array}\right]
\end{align*}
$$

Lemma 2. (see [11]) $M_{1}, M_{2}$ and $\Omega$ are constant matrices of appropriate dimensions and $0 \leq h_{m} \leq h(t) \leq h_{M}$, then

$$
\left(h(t)-h_{m}\right) M_{1}+\left(h_{M}-h(t)\right) M_{2}+\Omega<0
$$

if and only if

$$
\left(h_{M}-h_{m}\right) M_{1}+\Omega<0
$$

and

$$
\left(h_{M}-h_{m}\right) M_{2}+\Omega<0
$$

hold.

## 3. Main Results

Theorem1. For prescribed scalars $h_{m}, h_{M}$, the system $\Sigma$ is absolutely stable, if there exist positive definite matrices $\tilde{P}>0, \tilde{Q}_{s}>0(s=1,2,3), \tilde{R}_{k}>0$, and matrices $\tilde{M}_{k i j}, \tilde{N}_{k i j}(k=1,2 ; i=$
$1,2, \cdots, r ; j=1,2, \cdots, r)$ of appropriate dimensions and scalar $\beta>0$ such that the following LMIs hold:

$$
\begin{gather*}
\Omega^{i i}(k)=\left[\begin{array}{ll}
\Omega_{11}^{i i} & * \\
\Omega_{21}^{i i}(k) & \Omega_{22}
\end{array}\right]<0, \mathrm{i}=\mathrm{j}  \tag{5}\\
\Omega^{i j}(k)=\left[\begin{array}{ll}
\Omega_{11}^{i j}+\Omega_{11}^{j i} & * \\
\Omega_{21}^{i j}(k)+\Omega_{21}^{j i}(k) & 2 \Omega_{22}
\end{array}\right]<0, \quad \mathrm{i} \neq \mathrm{j} \tag{6}
\end{gather*}
$$

$\Omega_{11}^{i j}=\left[\begin{array}{ccccc}\Xi_{1}^{i j} & * & * & * & * \\ \tilde{R}_{1} & \Xi_{2}^{i j} & * & * & * \\ A_{h i}^{T} \tilde{P}+A_{h i}^{T} \Lambda A_{i} & \tilde{M}_{2 i j}-\tilde{M}_{1 i j}^{T} & \Xi_{3}^{i j} & * & * \\ 0 & 0 & \tilde{N}_{2 i j}-\tilde{N}_{1 i j}^{T} & \Xi_{4}^{i j} & * \\ b_{i}^{T} \tilde{P}+\beta c^{T} A_{i}+b_{i}^{T} \Lambda A_{i} & 0 & \beta c^{T} A_{h i}+b_{i}^{T} \Lambda A_{h i} & 0 & 2 \beta b_{i}^{T} c+b_{i}^{T} \Lambda b_{i}\end{array}\right]$,

Let * denotes the elements below the main diagonal of a symmetric block matrix.

$$
\begin{aligned}
& \Omega_{21}^{i j}(1)=\sqrt{h_{M}-h_{m}} \tilde{M}_{1 j}^{T}, \Omega_{21}^{i j}(2)=\sqrt{h_{M}-h_{m}} \tilde{N}_{i j}^{T}, \\
& \Omega_{22}=-\tilde{R}_{2}, \\
& \Xi_{1}^{i j}=\tilde{P} A_{i}+A_{i}^{T} \tilde{P}-\tilde{R}_{1}+\tilde{Q}_{1}+\tilde{Q}_{2}+\tilde{Q}_{3}+A_{i}^{T} \Lambda A_{i}, \\
& \Xi_{2}^{i j}=-\tilde{Q}_{1}-\tilde{R}_{1}+\tilde{M}_{1 i}+\tilde{M}_{1 i j}^{T}, \\
& \Xi_{3}^{i j}=-(1-h) \tilde{Q}_{2}-\tilde{M}_{2 i j}-\tilde{M}_{2 i j}^{T}+\tilde{N}_{1 i j}+\tilde{N}_{1 i j}^{T}+A_{h i}^{T} \Lambda A_{h i}, \\
& \Xi_{4}^{i j}=-\tilde{Q}_{3}-\tilde{N}_{2 i j}-\tilde{N}_{2 i j}^{T}, \\
& \Lambda=h_{m}^{2} \tilde{R}_{1}+\left(h_{M}-h_{m}\right),
\end{aligned}
$$

and

$$
\tilde{M}_{i j}^{T}=\left[\begin{array}{lllll}
0 & \tilde{M}_{1 j}^{T} & \tilde{M}_{2 i j}^{T} & 0 & 0
\end{array}\right], \tilde{N}_{i j}^{T}=\left[\begin{array}{lllll}
0 & 0 & \tilde{N}_{1 i j}^{T} & \tilde{N}_{2 i j}^{T} & 0
\end{array}\right] .
$$

Proof. Choose the Lypunov- krasovskii functional candidate as follows:

$$
V(t)=V_{1}(t)+V_{2}(t)+V_{3}(t)+V_{4}(t)
$$

with

$$
\begin{aligned}
& V_{1}(t)=x^{T}(t) \tilde{P} x(t) \\
& V_{2}(t)=\int_{t-h_{m}}^{t} x^{T}(s) \tilde{Q}_{1} x(s) d s+\int_{t-h(t)}^{t} x^{T}(s) \tilde{Q}_{2} x(s) d s+\int_{t-h_{M}}^{t} x^{T}(s) \tilde{Q}_{3} x(s) d s \\
& V_{3}(t)=h_{m} \int_{t-h_{m}}^{t} \int_{s}^{t} \dot{x}^{T}(\theta) \tilde{R}_{1} \dot{x}(\theta) d \theta d s+\int_{t-h_{m}}^{t-h_{m}} \int_{s}^{t} \dot{x}^{T}(\theta) \tilde{R}_{2} \dot{x}(\theta) d \theta d s \\
& V_{4}(t)=2 \beta \int_{0}^{\sigma(t)} f_{i}(\sigma) d \sigma
\end{aligned}
$$

Taking the derivation of $V(t)$ along the trajectory of system $\Sigma$, we have

$$
\begin{align*}
& \dot{V}_{1}(t)=2 \dot{x}^{T}(t) \tilde{P}^{x}(t)  \tag{7}\\
& \dot{V}_{2}(t) \leq x^{T}(t)\left(\tilde{Q}_{1}+\tilde{Q}_{2}+\tilde{Q}_{3}\right) x(t) \\
& -x^{T}\left(t-h_{m}\right) \tilde{Q}_{1} x\left(t-h_{m}\right) \\
& \quad-(1-h) x^{T}(t-h(t)) \tilde{Q}_{2} x(t-h(t)) \\
& \quad-x^{T}\left(t-h_{M}\right) \tilde{Q}_{3} x\left(t-h_{M}\right) \tag{8}
\end{align*}
$$

We suppose $f(t, s)=\int_{s}^{t} \dot{x}^{T}(\theta) \tilde{R}_{1} \dot{x}(\theta) d \theta$, then

$$
\begin{aligned}
& \frac{d}{d t}\left[h_{m} \int_{t-h_{m}}^{t} \int_{s}^{t} \dot{x}^{T}(\theta) \tilde{R}_{1} \dot{x}(\theta) d \theta d s\right]=\frac{d}{d t}\left[h_{m} \int_{t-h_{m}}^{t} f(t, s) d s\right] \\
& =h_{m}\left[\int_{t-h_{m}}^{t} \frac{\partial f(t, s)}{\partial t} d s+f(t, t)-f\left(t, t-h_{m}\right)\right] \\
& =h_{m}\left[\int_{t-h_{m}}^{t} \dot{x}^{T}(t) \tilde{R}_{1} \dot{x}(t) d s\right]-h_{m} \int_{t-h_{m}}^{t} \dot{x}^{T}(s) \tilde{R}_{1} \dot{x}(s) d s \\
& =h_{m}^{2} \dot{x}^{T}(t) \tilde{R}_{1} \dot{x}(t)-h_{m} \int_{t-h_{m}}^{t} \dot{x}^{T}(s) \tilde{R}_{1} \dot{x}(s) d s
\end{aligned}
$$

So

$$
\begin{gather*}
\dot{V}_{3}(t)=\dot{x}^{T}(t)\left(h_{m}^{2} \tilde{R}_{1}+\left(h_{M}-h_{m}\right) \tilde{R}_{2}\right) \dot{x}(t) \\
-h_{m} \int_{t-h_{m}}^{t} \dot{x}^{T}(s) \tilde{R}_{1} \dot{x}(s) d s-\int_{t-h_{M}}^{t-h_{m}} \dot{x}^{T}(s) \tilde{R}_{2} \dot{x}(s) d s  \tag{9}\\
\dot{V}_{4}(t)=2 \beta \dot{x}^{T}(t) c f_{i}(\sigma(t)) \tag{10}
\end{gather*}
$$

Using the free-weighting matrix method and (7)~(10), we have

$$
\begin{align*}
\dot{V}(t) & =\dot{V}_{1}(t)+\dot{V}_{2}(t)+\dot{V}_{3}(t)+\dot{V}_{4}(t) \\
& \leq 2 \dot{x}^{T}(t) \tilde{P}^{x} x(t)+x^{T}(t)\left(\tilde{Q}_{1}+\tilde{Q}_{2}+\tilde{Q}_{3}\right) x(t) \\
& -x^{T}\left(t-h_{m}\right) \tilde{Q}_{1} x\left(t-h_{m}\right)-(1-h) x^{T}(t-h(t)) \tilde{Q}_{2} x(t-h(t)) \\
& -x^{T}\left(t-h_{M}\right) \tilde{Q}_{3} x\left(t-h_{M}\right)+\dot{x}^{T}(t)\left[h_{m}^{2} \tilde{R}_{1}+\left(h_{M}-h_{m}\right) \tilde{R}_{2}\right] \dot{x}(t) \\
& -h_{m} \int_{t-h_{m}}^{t} \dot{x}^{T}(s) \tilde{R}_{1} \dot{x}(s) d s-\int_{t-h_{M}}^{t-h_{m}} \dot{x}^{T}(s) \tilde{R}_{2} \dot{x}(s) d s \\
& +2 \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{M}_{i j}\left[x\left(t-h_{m}\right)-x(t-h(t))\right. \\
& \left.-\int_{t-h(t)}^{t-h_{m}} \dot{x}(s) d s\right]+2 \sum_{i=1}^{r} \sum_{i=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{N}_{i j}[x(t-h(t)) \\
& \left.-x\left(t-h_{M}\right)-\int_{t-h_{M}}^{t-h(t)} \dot{x}(s) d s\right]+2 \beta \dot{x}^{T}(t) c f_{i}(\sigma(t)) \tag{11}
\end{align*}
$$

Where

$$
\eta^{T}(t)=\left[\begin{array}{lll}
x^{T}(t) & x^{T}\left(t-h_{m}\right) & x^{T}(t-h(t))
\end{array} x^{T}\left(t-h_{M}\right) f_{i}(\sigma(t))\right]
$$

By using Lemma1, there holds

$$
\begin{align*}
& -h_{m} \int_{t-h_{m}}^{t} \dot{x}^{T}(s) \tilde{R}_{1} \dot{x}(s) d s \\
& \leq\left[\begin{array}{l}
x(t) \\
x\left(t-h_{m}\right)
\end{array}\right]^{T}\left[\begin{array}{ll}
-\tilde{\mathrm{R}}_{1} & \tilde{\mathrm{R}}_{1} \\
\tilde{\mathrm{R}}_{1} & -\tilde{\mathrm{R}}_{1}
\end{array}\right]\left[\begin{array}{l}
x(t) \\
x\left(t-h_{m}\right)
\end{array}\right] \\
& =-\left[x^{T}(t)-x^{T}\left(t-h_{m}\right)\right] \tilde{\mathrm{R}}_{1}\left[x(t)-x\left(t-h_{m}\right)\right] \tag{13}
\end{align*}
$$

Using the method in [12], we can obtain the following inequalities

$$
\begin{align*}
& -2 \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{M}_{i j} \int_{t-h(t)}^{t-h_{m}} \dot{x}(s) d s \\
& \leq\left(h(t)-h_{m}\right) \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{M}_{i j}^{T} \tilde{R}_{2}^{-1} \tilde{M}_{i j} \eta(t)  \tag{14}\\
& +\int_{t-h(t)}^{t-h_{m}} \dot{x}^{T}(s) \tilde{R}_{2} \dot{x}(s) d s
\end{align*}
$$

$$
\begin{align*}
& -2 \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{N}_{i j} \int_{t-h_{M M}}^{t-\tau(t)} \dot{x}(s) d s \\
& \leq\left(h_{M}-h(t)\right) \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \tilde{N}_{i j}^{T} \tilde{R}_{2}^{-1} \tilde{N}_{i j} \eta(t)  \tag{15}\\
& +\int_{t-h h_{M}}^{t-h(t)} \dot{x}^{T}(s) \tilde{R}_{2} \dot{x}(s) d s
\end{align*}
$$

Adding up (13)~(15) into (11), we have

$$
\begin{align*}
\dot{V}(t) \leq & \sum_{i=1}^{r} \sum_{j=1}^{r} h_{i}(s(t)) h_{j}(s(t))\left\{\eta ^ { T } ( t ) \left[\Gamma_{11}^{i j}\right.\right. \\
& +\left(h(t)-h_{m}\right) \tilde{M}_{i j}^{T} \tilde{R}_{2}^{-1} \tilde{M}_{i j} \\
& \left.\left.+\left(h_{M}-h(t)\right) \tilde{N}_{i j}^{T} \tilde{R}_{2}^{-1} \tilde{N}_{i j}\right] \eta(t)\right\} \\
= & \sum_{j=1}^{r} h_{i}^{2}(s(t)) \eta^{T}(t) \mathrm{H}_{1 i i} \eta(t)+\sum_{i, j=1}^{r} \sum_{i<j} h_{i}(s(t)) h_{j}(s(t)) \eta^{T}(t) \mathrm{H}_{2 j} \eta(t) \tag{16}
\end{align*}
$$

Where

$$
\begin{aligned}
& \quad \mathrm{H}_{1 i i}=\Gamma_{1 i}^{i i}+\left(h(t)-h_{m}\right) \tilde{M}_{i i}^{T} \tilde{R}_{2}^{-1} \tilde{M}_{i i}+\left(h_{M}-h(t)\right) \tilde{N}_{i i}^{T} \tilde{R}_{2}^{-1} \tilde{N}_{i i} \\
& \mathrm{H}_{2 i j}=\Gamma_{11}^{i j}+\Gamma_{11}^{j i}+\frac{1}{2}\left(h(t)-h_{m}\right)\left(\tilde{M}_{i j}^{T}+\tilde{M}_{j i}^{T}\right) \tilde{R}_{2}^{-1}\left(\tilde{M}_{i j}+\tilde{M}_{j i}\right) \\
& +\frac{1}{2}\left(h_{M}-h(t)\right)\left(\tilde{N}_{i j}^{T}+\tilde{N}_{j i}^{T}\right) \tilde{R}_{2}^{-1}\left(\tilde{N}_{i j}+\tilde{N}_{j i}\right)
\end{aligned}
$$

By Lemma2 and Schur complements, the LMIs (5) and (6) hold. This completes the proof.

## 4. Numerical Example

The T-S fuzzy Lurie system $\Sigma$ considered in this example is with two rules for $i=j=2$.
And we suppose

$$
\begin{aligned}
& A_{1}=\left[\begin{array}{cc}
-1.19684 & 1.3 \\
1 & -0.9901
\end{array}\right], A_{2}=\left[\begin{array}{cc}
-0.9 & 1.3 \\
1 & -1
\end{array}\right], A_{h 1}=\left[\begin{array}{cc}
-0.9 & 0.5 \\
0.54 & -1
\end{array}\right], A_{h 2}=\left[\begin{array}{ll}
1.3 & -1.8 \\
0.6 & -1.04
\end{array}\right] \\
& h=0.5101, h_{m}=0.01773, h_{M}=1.7464, \quad c=\left[\begin{array}{c}
-0.5 \\
-0.747
\end{array}\right], b_{1}=\left[\begin{array}{c}
1 \\
2.0007
\end{array}\right], b_{2}=\left[\begin{array}{l}
1 \\
2
\end{array}\right],
\end{aligned}
$$

Using the MATLAB LMI Toolbox to solve the LMI in (8), we obtained a set of feasible solutions as follows:

$$
\begin{aligned}
& \tilde{P}=\left[\begin{array}{cc}
0.0834 & -0.0814 \\
-0.0814 & 0.0865
\end{array}\right], \tilde{Q}_{1}=\left[\begin{array}{cc}
0.0037 & -0.0027 \\
-0.0027 & 0.0048
\end{array}\right], \tilde{Q}_{2}=\left[\begin{array}{cc}
0.1305 & -0.1552 \\
-0.1552 & 0.1911
\end{array}\right] \\
& \tilde{Q}_{3}=\left[\begin{array}{cc}
0.0099 & -0.0069 \\
-0.0069 & 0.0104
\end{array}\right], \tilde{R}_{1}=\left[\begin{array}{cc}
1.1616 & -0.3834 \\
-0.3834 & 1.1999
\end{array}\right], \tilde{R}_{2}=\left[\begin{array}{cc}
0.0201 & -0.0123 \\
-0.0123 & 0.0193
\end{array}\right], \beta=0.0412 .
\end{aligned}
$$

In this example, there are feasible solutions when $h_{M}$ is in an interval, and the maximum value of $h_{M}$ is 1.7464 . In the numerical experiment, we found that the maximum value of $h_{M}$ obviously increases with the increase of $h$. It reflects that the method used in this paper can make less conservatism of the conclusion.

## 5. Conclusions

In this paper, the delay-dependent absolute stability condition for a new class of T-S fuzzy Lurie control systems with time-varying delay is investigated. By using novel
techniques, the error caused by enlarging $h(t)(\cdot)$ to $h_{M}(\cdot)$ is avoided. The advantage of proposed method is that the conservatism can be reduced. Mean-while, we use the method of adding zero and introduce free weighting matrices effectively, rather than adding zero with a fixed weight matrix based on the Newton Leibniz formula, which is different from some existing research. In this way, we can get a less conservative result. With Lyapunov-Krasovskii functional and linear matrix inequality approach, a new delay-dependent absolute stability condition is obtained. Finally, a numerical experiment has shown that the maximum value of $h_{M}$ obviously increases with the increase of $h$ and the proposed result is feasible and less conservative.

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