

## Blind Separation of Permuted Alias Image Based on K-SVD Dictionary Learning

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### Abstract

*In this work, a new blind separation algorithm for permuted alias image based on dictionary learning is proposed according to a type of permuted alias image with noise diversity. Sparse representation of permuted alias image is obtained by dictionary learning method, since it has high adaptability and its representation result has higher sparsity degrees than that of parameter dictionary. An optimal permuted alias image is achieved by conducting sparse representation with K-SVD dictionary learning algorithm restrained with nonzero element number. The size and the location of permuting region is found by detecting the subtraction image, which is defined as the difference between the reconstructed permuted alias image and the original permuted alias image. The permuting region is optimized by implementing image morphological operation and is separated from the permuted alias image by the threshold. Experimental results show that the permuting sub-images can be efficiently separated from the permuted alias image, which is not affected by the size, location, number of permuting sub-images and noise level of the permuting sub-images.*

**Keywords:** *blind separation, permuted alias image, sparse representation, dictionary learning, K-SVD*

### 1. Introduction

The researches of blind separation of superposed alias image have been concerned for many years. Blind separation of Permuted alias image is a new type of single channel blind source separation (BBS), which causes an increasing concern in signal processing community [1]. The major difference between them is the mixture mode. The traditional single channel blind separation is a technique for estimating individual source components from their single mixtures composed of several components superposed. Blind separation of permuted alias image is a technique to estimate the permuting image from single image made of some images permuted.

Compared with the traditional blind separation for images, it has some new characteristics and challenges. Firstly, the integrality of the permuted image is damaged for some regions are replaced by another image (permuting image). Secondly, the location, size and number of permuted region are unknown or random before being separated. Lastly, the source of the permuted image and the permuting one are completely unknown. All the factors mentioned above cause the essential difference between the traditional blind separation and the blind separation of permuted alias image. Therefore, new methods and theories are needed to solve this new type of blind separation.

Fang, *et al.*, proposed a detection method of the permuting region based on separable characteristic domain, where the activated region was detected with characteristic separation by extracting common factor from various source signals [1]. Therefore, it is important to select appropriate characteristic domain in view of various type of permuted alias images. When the source images are in JPEG format, a blind separation method based on compression factors estimation was proposed in [2] for permuted JPEG image. When the sources contain interpolated images, a novel blind separation method based on interpolation factors estimation was proposed in [3] for permuted interpolated image. For blurring permuted images, a single-channel blind separation scheme using double blur correlation was proposed in [4]. For permuted alias image with morphological diversity, Contourlet and local discrete cosin transform dictionary was respectively used as characteristic field of separation in order to using sparsity diversity existing in sparse representation of the permuting and permuted regions of a permuted alias image [5]. For permuted alias image with defocused and move blurring images, a blind separation algorithm based on four-phase-difference and differential evolution was proposed in [6] for types of permuted alias image with blur difference.

In reality, there are noise level diversity between permuting and permuted regions in a permuted alias image, for the permuting image and permuted image are infected by diverse level noise when they are acquired, transmitted, processed. Noise intensities of different regions in a single image are various. In this paper, for the class permuted alias image with noise difference, a new detection and separation method based dictionary learning is proposed.

## 2. Blind Separation of Permuted Alias Image Mode in Sparse domain

### 2.1. Permuted Alias Image Mode

Mode of permuted alias image is represented based on [1], where  $X$  is composed of 1 permuted image indicated by  $X_p$  and  $n$  permuting images indicated by  $X_{T_1} \cdots X_{T_n}$ .  $A$  indicate permuted alias matrix,  $X_p$  indicate permuted image,  $X_{T_i}$  indicate  $i^{th}$  permuting image. The permuted alias image  $Y$  is shown as :

$$Y = A \bullet X$$

$$= A_p \otimes X_p + A_{T_1} \otimes X_{T_1} + \cdots + X_{T_i} \otimes A_{T_i} + \cdots + X_{T_n} \otimes A_{T_n} \quad (1)$$

$$A = [A_p, A_{T_1}, A_{T_2}, \cdots, A_{T_i}, \cdots, A_{T_n}] \quad (2)$$

$$A_{T_i} = \begin{cases} 1 & (i, j) \in U_{T_i} \\ \vdots & \\ 0 & (i, j) \in U_{T_i}, \cdots, A_{T_i} \\ \vdots & \\ 0 & (i, j) \in U_{T_n} \end{cases}, \cdots, A_{T_i} = \begin{cases} 0 & (i, j) \in U_{T_i} \\ \vdots & \\ 1 & (i, j) \in U_{T_i}, \cdots, A_{T_n} \\ \vdots & \\ 0 & (i, j) \in U_{T_n} \end{cases}, \cdots, A_{T_n} = \begin{cases} 0 & (i, j) \in U_{T_i} \\ \vdots & \\ 0 & (i, j) \in U_{T_i} \\ \vdots & \\ 1 & (i, j) \in U_{T_n} \end{cases} \quad (3)$$

$$A_p = U - [A_{T_1} + A_{T_2}, \cdots, + A_{T_n}] \quad (4)$$

$$X = [X_p, X_{T_1}, X_{T_2}, \cdots, X_{T_i}, \cdots, X_{T_n}]^T \quad (5)$$

where symbol ' $\bullet$ ' is a special multiplication, symbol  $\otimes$  indicate *hadamard* multiplication  $A_p$  is a binary matrix with size  $M \times N$ , where '1' indicate non-permuted region of  $Y$ .  $A_{T_i}$  is a  $M \times N$  binary matrix,  $i^{th}$  permuting matrix, '1' indicate part of permuting image in permuted alias image.

$$U_p \cap U_{T_1} \cap U_{T_2} \cdots \cap U_{T_i} \cdots \cap U_{T_n} = \emptyset \quad (6)$$

$$U_p \cup U_{T_1} \cup U_{T_2} \cdots \cup U_{T_i} \cdots \cup U_{T_n} = U \quad (7)$$

where the intersection of all active region  $U_{T_i}$  is empty set, and the union set of them is matrix of all '1'.

## 2.2. Permuted Blind Separation of Permuted Alias Image Mode in Sparse Domain

In recent years, sparse representation has drawn increasing attention as new trend of signal processing community and has become an efficient tool for describing structure information of signals. Over-complete redundant dictionary is used to represent image, which can be represented as a sparse linear combination of these atoms. The representations which use the least number of atoms is optimum, i.e., the sparsest representation [7-8].

Permuted alias image  $Y$  is composed of two regions, one is part of permuted image  $Y_p$ , the other is part of some permuting images  $Y_T$ .

$$Y = \begin{cases} Y_p = D\alpha_p + \varepsilon_p & Y(i, j) \in U_p \\ Y_T = D\alpha_T + \varepsilon_T + n_T & Y(i, j) \in U_T \end{cases} \quad (8)$$

where  $D$  indicates overcomplete dictionary,  $\alpha_p$  is sparse representation (or approximation) of  $Y_p$ ,  $\alpha_T$  is sparse representation (or approximation) of  $Y_T$ .  $\varepsilon_p$  and  $\varepsilon_T$  are their error of approximation, respectively,  $n_T$  is noise of permuting region and  $U_p = A_p$ ,  $U_T = A_{T_1} + A_{T_2} \cdots + A_{T_m}$ .

From the above mode, it can be seen that the key of separating the permuted images from the permuted alias image is to detect the region denoted  $U_T$  composed of parts of some permuting image with noise. So detecting activating region can be completed by detecting the local noise regions, which can be detected by permuted alias image subtracting the version denoised. However, it is improper to find the noised region by the traditional denoising methods for the location and the size of the noised region both are unknown. The purpose of denoising permuted alias image can be obtained by getting the sparse representation of them with dictionary learning methods for noise is unable to be represented sparsely.

## 3. Sparse Representation base on Dictionary learning

How to select dictionary plays a vital role in whether the image is represented sparsely and how sparsely it is represented, which affects the separating result of permuted alias image. There are two main types of the overcomplete dictionary, one is parameter dictionary, the other is nonparametric dictionary (namely learning dictionary). Parameter dictionary includes DCT, Curvelets, Banedlets, Contourlets transform and so on, which are described as multiscale geometric analysis for images. This type of dictionaries has the advantage of the rapid generation from mathematical mode and easy analysis. Sparse degree variation with various images is insufficient to represent sparsely image [9].

Dictionary learning for sparse image representations has become an extremely active area of research in recent years. One of work of learning overcomplete dictionaries for image representation was the sparse coding method of Olshausen and Field [10], which were described as a model of early visual sensory coding. The probabilistic methods of dictionary learning have been adopted by some researchers. Engan, *et al.*, have proposed a method of optimal directions (MOD) [11], which contained the sparse coding and dictionary updating steps that iteratively optimized the objective ML function. These two improvements make the MOD approach faster compared with the ML method. Kreutz-Delgado proposed Maximum a posteriori (MAP) dictionary learning method, [12] belongs also to the family of two-step iterative algorithms based on probabilistic inference.

K-SVD algorithm was proposed by Aharon and Elad in as a generalization of the K-means for dictionary learning. After the sparse coding step (where any pursuit algorithm can be employed), the dictionary updating is implemented by sequentially updating each column using a singular value decomposition (SVD) to minimize the approximation error. The updating step hence follows a generalized K-means algorithm since each patch can be represented by multiple atoms with different weights, K-SVD algorithm are characteristic in dictionary updating in which the atom and the corresponding sparse coefficient are updated simultaneously[14-18].

K-SVD algorithm process [14]

Task: Find the best dictionary to represent the data samples  $\{y_i\}_{i=1}^N$  as sparse compositions, by solving

$$\min_{D, X} \left\{ \|Y - DX\|_F^2 \right\} \quad \text{subject to } \forall i, \|x_i\|_0 \leq T_0$$

Initialization: Set the dictionary matrix  $D^{(0)} \in R^{n \times K}$  with  $l^2$  normalized columns. Set  $J = 1$ .

Repeat until convergence (stopping rule):

- Sparse Coding Stage: Use one pursuit algorithm to compute the representation vectors  $x_i$  for each example  $y_i$ , by approximating the solution of

$$\min_{x_i} \left\{ \|y_i - Dx_i\|_2^2 \right\}, i = 1, 2, \dots, N, \quad \text{subject to } \|x_i\|_0 \leq T_0.$$

- Codebook Update Stage: For each column  $k = 1, 2, \dots, K$  in  $D^{(J-1)}$ , update it by

- Define the group of examples that use this atom,  $\omega_k = \{i | 1 \leq i \leq N, x_i^k \neq 0\}$

- Compute the overall representation error matrix,  $E_k$ , by

$$E_k = Y - \sum_{j \neq k} d_j x_T^j.$$

- Restrict  $E_k$  by choosing only the columns corresponding to  $\omega_k$ , and obtain  $E_k^R$ .

- Apply the SVD  $E_k^R = U \Lambda V^T$ . Choose the updated dictionary column  $\tilde{d}_k$  to be the first column of  $U$ . Update the coefficient vector  $x_R^k$  to be the first column of  $V$  multiplied by  $\Lambda(1,1)$ .

- Set  $J = J + 1$ .

#### 4. Blind Detection and Separation of Permuted Alias Image

The best sparse dictionary  $D^{opt}$  and the corresponding coefficient  $X^{opt}$  can be obtained by training samples with K-SVD algorithm, and a new approximate permuted alias image  $Y^{opt}$  is reconstructed by  $D^{opt}$  multiplying  $X^{opt}$ .

$$Y^{opt} = D^{opt} X^{opt} \quad (9)$$

Noise intensity within permuting region will be reduced, image within permuted region are perfectly reconstructed. So, the permuting region could be detected and located roughly by the difference image  $Y^{diff}$  derived from original permuted alias image subtracting the reconstructed one.

$$Y^{diff} = Y^{opt} - Y = \begin{cases} Y_p^{opt} - Y_p = \varepsilon_p & Y(i, j) \in U_p \\ Y_T^{opt} - Y_T = \varepsilon_T + n_T & Y(i, j) \in U_T \end{cases} \quad (10)$$

where  $\varepsilon_p$  indicates sparse representation error of image within permuted region  $U_p$ ,  $\varepsilon_T$  indicates sparse representation error of image within permuting region  $U_T$ , the values of  $\varepsilon_T$  and  $\varepsilon_p$  are very little.  $n_T$  indicates the noise intensity within permuting region, which is proportional to the variation of noise. Permuted region can be roughly detected by eliminating disturb of sparse approximation error with the appropriate threshold. However, permuted region is composed of discrete dots for distribution of noise within

$U_T$  submit to Gaussian distribution. Integrate and consecutive binary permuted region can be obtained by filling in binary discrete permuted region by conducting morphological operation for  $U_T$ . The permuted image can be successfully separated from the permuted alias image by original permuted alias image multiplying by binary one.

## 5. Results and Discussion

A series of experiments were carried out aiming at this type of permuted alias images. Three conditions were considered. Separating is conducted with various numbers of non-zero element and the same white-noise standard deviation in permuting region; separating is conducted with the same number of non-zero element and various white-noise standard deviation in permuting region; separating is conducted with various number of non-zero element and the various white-noise standard deviation in several permuting regions respectively. Our experiments were implemented in MATLAB R2008a, on a ThinkPad w530, running Windows 7 operating system on an Intel i7 3630QM processor, with 8GB of RAM. The size of sub-block sampling from images is  $8 \times 8$  pixels.

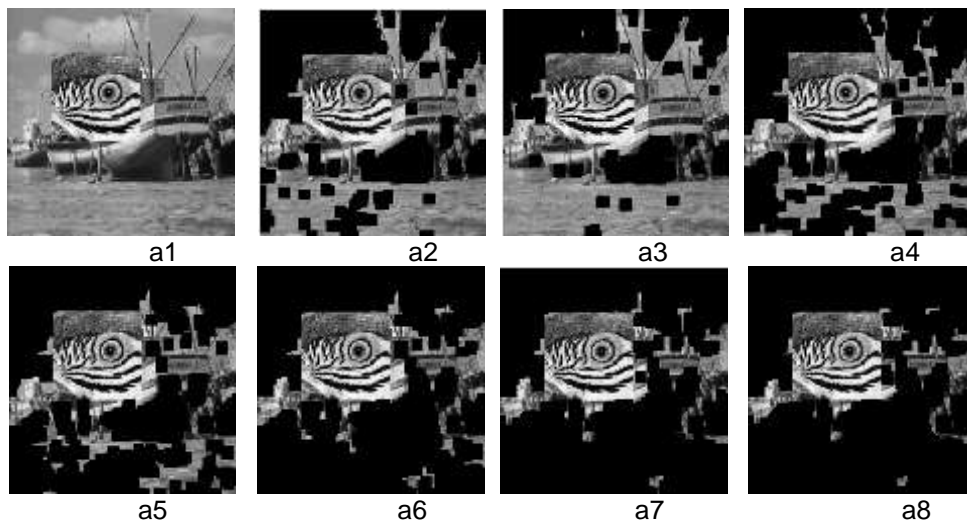
In order to compare the performance of different approaches, an objective evaluation criterion is required. The first evaluation criterion is the correct separation rate ( $CS$ ), which is a ratio of separated permuting and total permuting region, see Eq. (8). The second evaluation criterion is the false separation rate ( $FS$ ) which is a ratio of false detected region and total detected region, see Eq. (9).

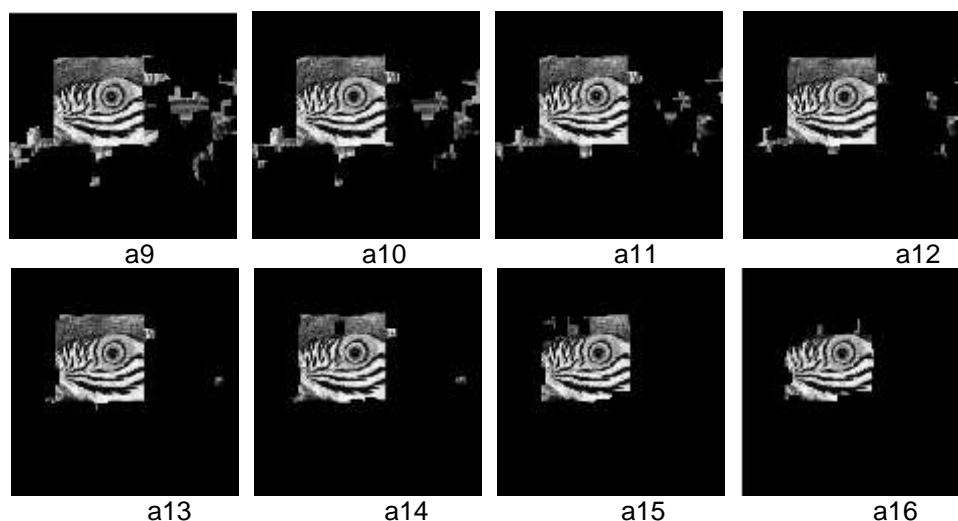
$$CS = \frac{\text{CorrectSeparated Region}}{\text{TotalPermuting Region}} \quad (11)$$

$$FS = \frac{\text{FalseSeparated Region}}{\text{TotalSeparated Region}} \quad (12)$$

By comparing  $CS$  and  $FS$  of different methods together, an objective performance can be achieved.

### (1) Results of Separating with Various Numbers of Non-zero Element and the same White-noise Standard Deviation in Permuting Region





**Figure 1. Results of separating with various numbers of non-zero element and the same white-noise standard deviation in the permuting region**

Figure a1 indicates the original permuted alias image composed of ‘boat’ image and eye part of ‘parrot’ image, in which top left corner coordinate position, size and noise standard deviation of permuting region is (50, 50) pixels, 100×100 pixels and 20. The separating results of the proposed method are shown from sub-image a2 to sub-image a16, which are the separating images successively with nonzero element numbers from 1 to 15 and the same white-noise standard deviation in permuting region. It can be observed from the images (a2-a5) that, the permuting images can be separated completely with nonzero numbers from 1 to 4. The values of correct separation rates CS range from 99.1% to 98.6%. Meanwhile, a lot of images of non-permuting image are falsely separated, false separation rate FS reach up to 61.1% when the separation is conducted with one nonzero element. With the increase of the used nonzero elements, the value of CS decreases gradually and the value of FS descents quickly. Optimal results can be achieved that more than ninety-five percent of the permuting image can be correctly separated and less than five percent of non-permuting image can be falsely separated when the number of the used nonzero elements reaches to about 10. With the increasing of the used nonzero elements continuously, instead, the separating effect becomes worse and the value of CS decreases to 65.2% although the value of FS decreases to less than 1%.

**(2) Results of the Separation with Various White-noise Standard Deviation and the same Numbers of Nonzero Element in Permuting Region**

**Table 1. CS and FS of Figure 1**

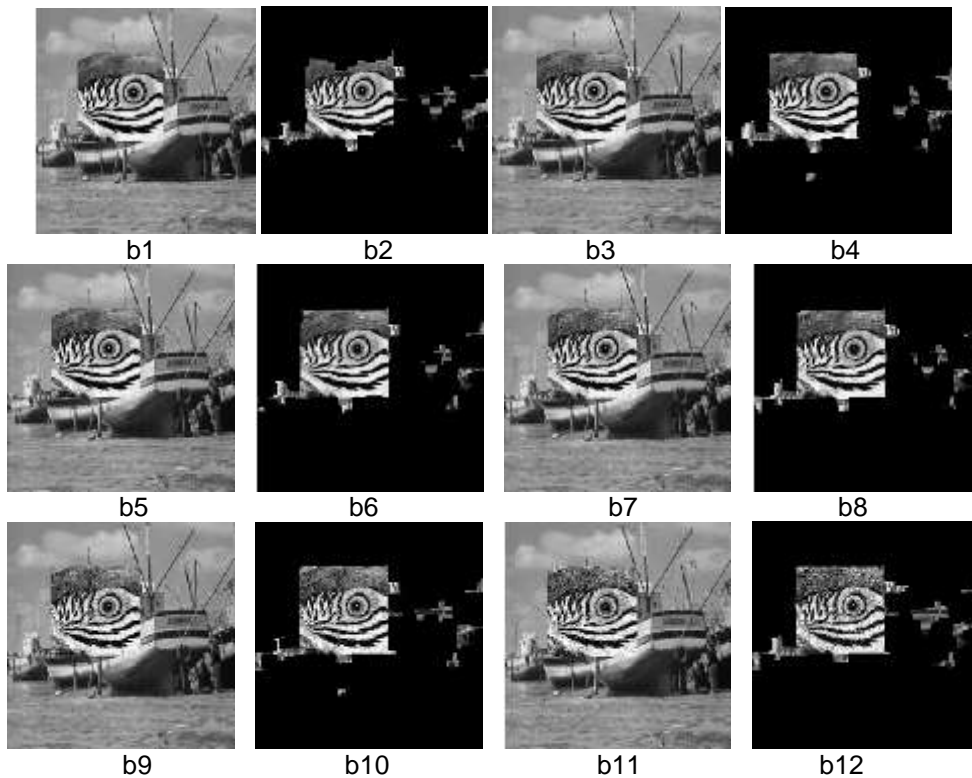
	CS	FS		CS	FS
a2	99.1%	61.1%	a10	97.3%	5.9%
a3	99.1%	49.1%	a11	96.6%	3.4%
a4	98.6%	36.1%	a12	95.6%	2.2%
a5	98.6%	26.2%	a13	94%	0.9%
a6	98.3%	17.2%	a14	89%	0.2%
a7	98%	13.5%	a15	77.6%	0.02%

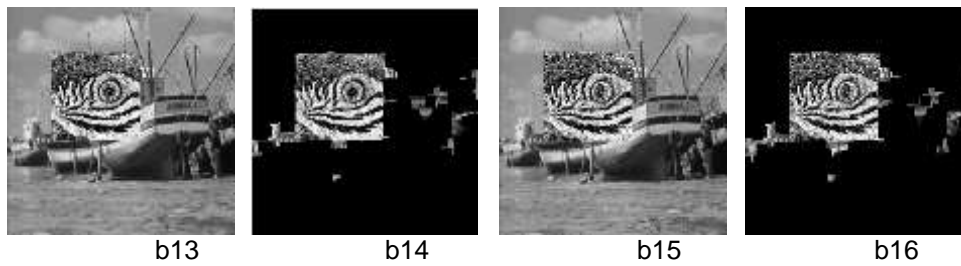
	CS	FS		CS	FS
a2	99.1%	61.1%	a10	97.3%	5.9%
a8	98.1%	11.2%	a16	65.2%	0%
a9	97.6%	8.2%			

The values of noise standard deviation of permuting region in sub-figure b1, b3, b5, b7, b9, b11, b13, b15 respectively are 5, 10, 15, 20, 30, 40, 50, 70, respectively. The corresponding separating results with 10 non-zero elements are shown in sub-figure b2, b4, b6, b8, b10, b12, b14, b16, respectively. It can be seen from figure 3 that almost all of the permuting images can be completely separated except figure b2 and a very small non-permuting image are falsely separated. For various permuting images containing different standard deviation noise, the result shown in Table II reveals that the proposed algorithm achieves the excellent effects that CS is greater than 95% and FS is less than 5%.

**Table 2. CS and FS of Figure 2**

	CS	FS		CS	FS
b2	78.4%	4.4%	b10	97.3%	4%
b4	94.9%	3.8%	b12	97.4%	4.5%
b6	96.4%	3.6%	b14	97.4%	4.2%
b8	96.6%	3.9%	a16	97.7%	4.4%

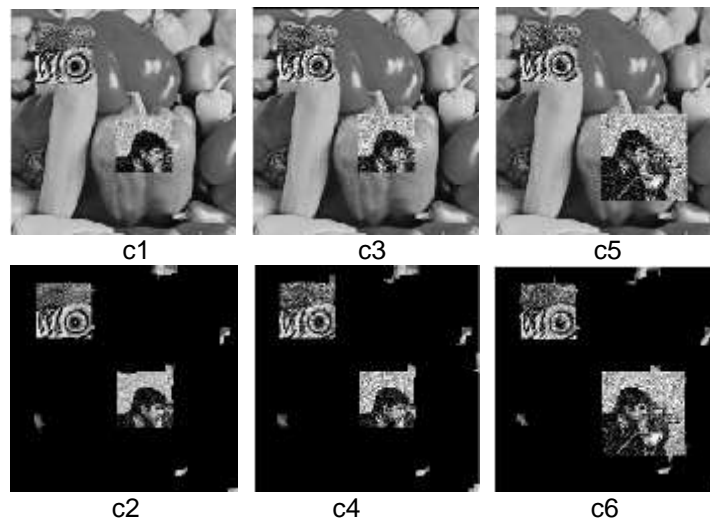




**Figure 2. Results of the Separation with Various White-noise Standard Deviation and the same Numbers of Nonzero Element in Permuting Region**

**(3) Results of Separation with Several Permuting Sub-images in One Permuted Alias Image and the Various White-noise Standard Deviation in Permuting Sub-images**

Figure c1, c3, and c5 indicate three original permuted alias images composed of ‘peppers’ image and part of ‘parrot’ and ‘cameraman’ image. In Figure c1, the two permuting sub-image have the same size and noise standard deviation, which are respectively  $64 \times 64$  and 30. In Figure c3, the two permuting sub-images have the same size, i.e.,  $64 \times 64$ , and the different noise standard deviation, which are 50 and 20, respectively. In Figure c5, the two permuting sub-images have the different sizes, which are  $64 \times 64$  pixels and  $96 \times 96$  pixels, respectively, and the different noise standard deviations which are 50 and 20, respectively. Corresponding separating results using 10 nonzero elements are shown in figure c2, c4 and c6, respectively. It can be seen that the two permuting sub-images can be separated efficiently from the permuted alias image, which doesn’t achieve the 95% of correct separating rate and the 1% of false separating rate, which is affected by size, location, number of permuting image and noise level on the permuting image.



**Figure 3. Results of Separation with Several Permuting Sub-images in one Permuted Alias Image and the Various White-noise Standard Deviation in Permuting Sub-images**

**CONGLUSIONS**

A new blind separation algorithm is proposed for a type of permuted alias image, in which the permuting sub-images are noised. A reconstructed permuted alias image is achieved by getting its sparse representation with the K-SVD dictionary learning



algorithm restrained by the nonzero element number. The location of permuting region is found by detecting the subtraction image, which is defined as the difference between the reconstructed permuted alias image and original permuted alias image. The permuting region is optimized by implementing the image morphological operation and is separated from the permuted alias image by the threshold. The results show that the proposed algorithm has several characteristics as following. Firstly, the proposed algorithm can efficiently separate permuting sub-images with different sizes and locations. Secondly, several permuting sub-images with the same or different standard deviations of the noise can be simultaneously separated from the permuted alias image.

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