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Design Robust Artificial Intelligence Model-base Variable Structure Controller with Application to Dynamic Uncertainties OCTAM VI Continuum Robot

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Abstract

Design a robust artificial intelligent nonlinear controller for second order nonlinear uncertain dynamical systems is one of the most important challenging works. This paper focuses on the design of a robust chattering free mathematical model-base artificial intelligence (fuzzy inference system) variable structure controller (MFVSC) for highly nonlinear dynamic continuum robot manipulator, in presence of uncertainties. In order to provide high performance nonlinear methodology, variable structure controller is selected. Pure variable structure controller can be used to control of partly known nonlinear dynamic parameters of continuum robot manipulator. In order to reduce/eliminate the chattering, this research is used the artificial intelligence (fuzzy logic) theory. The results demonstrate that the model base fuzzy variable structure controller with switching function is a model-based controllers which works well in certain and partly uncertain system. Lyapunov stability is proved in mathematical model-based fuzzy variable structure controller with switching (sign) function. This controller has acceptable performance in presence of uncertainty (e.g., overshoot=1%, rise time=0.9 second, steady state error = 1.6e-8 and RMS error=4.8e-8).

Keywords: *Variable structure control, fuzzy logic methodology, robust controller, hyper-redundant, continuum robot manipulator*

1. Introduction

Continuum robots represent a class of robots that have a biologically inspired form characterized by flexible backbones and high degrees-of-freedom structures [1]. The idea of creating "trunk and tentacle" robots, (in recent years termed continuum robots [1]), is not new [2]. Inspired by the bodies of animals such as snakes [3], the arms of octopi [4], and the trunks of elephants [5-6], researchers have been building prototypes for many years. A key motivation in this research has been to reproduce in robots some of the special qualities of the biological counterparts. This includes the ability to "slither" into tight and congested spaces, and (of particular interest in this work) the ability to grasp and manipulate a wide range of objects, via the use of "whole arm manipulation" i.e. wrapping their bodies around objects, conforming to their shape profiles. Hence, these robots have potential applications in whole arm grasping and manipulation in unstructured environments such as rescue operations. Theoretically, the compliant nature of a continuum robot provides infinite degrees of freedom to these devices. However, there is a limitation set by the practical inability to incorporate infinite actuators in the device. Most of these robots are consequently underactuated (in terms of numbers of independent actuators) with respect to their anticipated tasks. In other words they must achieve a wide range of configurations with relatively few control inputs. This is partly due to the desire to keep the body structures (which, unlike in conventional rigid-link manipulators or

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fingers, are required to directly contact the environment) "clean and soft", but also to exploit the extra control authority available due to the continuum contact conditions with a minimum number of actuators. For example, the Octarm VI continuum manipulator, discussed frequently in this paper, has nine independent actuated degrees-of-freedom with only three sections. Continuum manipulators differ fundamentally from rigid-link and hyper-redundant robots by having an unconventional structure that lacks links and joints. Hence, standard techniques like the Denavit-Hartenberg (D-H) algorithm cannot be directly applied for developing continuum arm kinematics. Moreover, the design of each continuum arm varies with respect to the flexible backbone present in the system, the positioning, type and number of actuators. The constraints imposed by these factors make the set of reachable configurations and nature of movements unique to every continuum robot. This makes it difficult to formulate generalized kinematic or dynamic models for continuum robot hardware. Chirikjian and Burdick were the first to introduce a method for modeling the kinematics of a continuum structure by representing the curve-shaping function using modal functions [6]. Mochiyama used the Serret- Frenet formulae to develop kinematics of hyper-degrees of freedom continuum manipulators [5]. For details on the previously developed and more manipulator-specific kinematics of the Rice/Clemson "Elephant trunk" manipulator, see [1-2, 5]. For the Air Octor and Octarm continuum robots, more general forward and inverse kinematics have been developed by incorporating the transformations of each section of the manipulator (using D-H parameters of an equivalent virtual rigid link robot) and expressing those in terms of the continuum manipulator section parameters [4]. The net result of the work in [6, 3-5] is the establishment of a general set of kinematic algorithms for continuum robots. Thus, the kinematics (i.e. geometry based modeling) of a quite general set of prototypes of continuum manipulators has been developed and basic control strategies now exist based on these. The development of analytical models to analyze continuum arm dynamics (*i.e.* physics based models involving forces in addition to geometry) is an active, ongoing research topic in this field. From a practical perspective, the modeling approaches currently available in the literature prove to be very complicated and a dynamic model which could be conveniently implemented in an actual device's real-time controller has not been developed yet. The absence of a computationally tractable dynamic model for these robots also prevents the study of interaction of external forces and the impact of collisions on these continuum structures. This impedes the study and ultimate usage of continuum robots in various practical applications like grasping and manipulation, where impulsive dynamics [1, 4] are important factors. Although continuum robotics is an interesting subclass of robotics with promising applications for the future, from the current state of the literature, this field is still in its stages of inception.

A controller is a device which can sense information from linear or nonlinear system (*e.g.,* continuum robot) to improve the systems performance [7-15]. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error[15]. Variable structure controller is an influential nonlinear controller to certain and uncertain systems which it is based on system's dynamic model 14]. Variable structure controller is a powerful nonlinear robust controller under condition of partly uncertain dynamic parameters of system [7]. This controller is used to control of highly nonlinear systems especially for continuum robot. Chattering phenomenon and nonlinear equivalent dynamic formulation in uncertain dynamic parameter are two main drawbacks in pure variable structure controller [14-15]. The chattering phenomenon problem in pure variable structure controller is reduced by artificial intelligence in this research.

In recent years, artificial intelligence theory has been used in variable structure control systems. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant and uncertain plant (*e.g.,* robot manipulator). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model techniques as in model-based controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [10] but also this method can help engineers to design a model-free controller. Control robot arm manipulators using model-based controllers are based on manipulator dynamic model. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic model of robot manipulator, but most of time these models are MIMO, nonlinear and partly uncertain therefore calculate accurate dynamic model is complicated [11]. The main reasons to use fuzzy logic methodology are able to give approximate recommended solution for uncertain and also certain complicated systems to easy understanding and flexible. Fuzzy logic provides a method to design a model-free controller for nonlinear plant with a set of IF-THEN rules [12].

The main goal of this research is to use a robust model base fuzzy VSC to control of Octarm VI continuum manipulator. The research concentrated on the high performance control. An additional goal was to design a nonlinear stable and robust fuzzy based methodology that would control the uncertain manipulator and provide functionality to an Octarm VI. To control the continuum robotic system a model base fuzzy VSC control was created.

This paper is organized as follows; Section 2, is served as an introduction to the variable structure controller formulation algorithm and its application to control of continuum robot, dynamic of continuum robot and proof of stability. Part 3, introduces and describes the methodology (gradient descent optimal variable structure controller) algorithm. Section 4 presents the simulation results and discussion of this algorithm applied to a continuum robot and the final section is describing the conclusion.

2. Theory

A. Dynamic Formulation of Continuum Robot

The Continuum section analytical model developed here consists of three modules stacked together in series. In general, the model will be a more precise replication of the behavior of a continuum arm with a greater of modules included in series. However, we will show that three modules effectively represent the dynamic behavior of the hardware, so more complex models are not motivated. Thus, the constant curvature bend exhibited by the section is incorporated inherently within the model. The mass of the arm is modeled as being concentrated at three points whose co-ordinates referenced with respect to (see Figure 1);

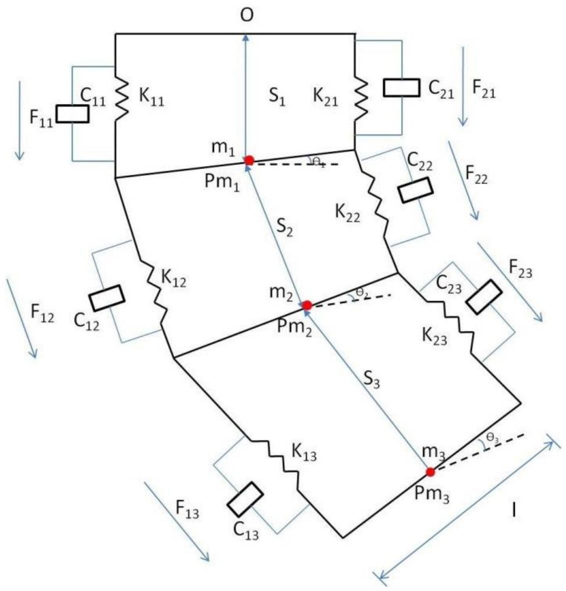

Figure 1. Assumed Structure for Analytical Model of a Section of a Continuum Arm

Where;

l - Length of the rigid rod connecting the two struts, constant throughout the structure,

$k_{1,i}$, $i = 1,2,3$ - Spring constant of actuator 1 at module i,

$k_{2,i}$, $i = 1,2,3$ - Spring constant of actuator 2 at module i,

$C_{1,i}$, $i = 1,2,3$ - Damping coefficient of actuator 1 at module i,

$C_{2,i}$, $i = 1,2,3$ - Damping coefficient of actuator 2 at module i,

m_i , $i = 1,2,3$ - Mass in each module,

I_i , $i = 1,2,3$ - Moment of inertia of the rigid rod in each module.

A global inertial frame (N) located at the base of the arm are given below

$$\boldsymbol{{}_{m1}^{N}P = S_1 . \widehat{n_3}} \quad (1)$$

$$\boldsymbol{{}_{m2}^{N}P = S_2 . \; sin\theta_1 \widehat{n_1} + (S_1 + S_2 \cos\theta_1) . \widehat{n_3}} \quad (2)$$

$$\boldsymbol{{}_{m3}^{N}P = (S_2 . \; sin\theta_1 + S_3 . \sin(\theta_1 + \theta_2)) \, \widehat{n_1} + (S_1 + S_2 \cos\theta_1 + S_3 . \cos(\theta_1 + \theta_2))) . \widehat{n_3}} \quad (3)$$

The position vector of each mass is initially defined in a frame local to the module in which it is present. These local frames are located at the base of each module and oriented along the direction of variation of coordinate *'s'* of that module. The positioning of each of these masses is at the centre of mass of the rigid rods connecting the two actuators. Differentiating the position vectors we obtain the linear velocities of the masses. The kinetic energy (T) of the system comprises the sum of linear kinetic energy terms (constructed using the above velocities) and rotational kinetic energy terms due to rotation of the rigid rod connecting the two actuators, and is given below as

$$\boldsymbol{T = (0.5)m_1\dot{s}_1{}^2 + (0.5)m_2\left(\left(\dot{s}_2 sin\theta_1 + s_2 cos\theta_1\dot{\theta}_1\right)^2 + \left(\dot{s}_1 + \dot{s}_2 cos\theta_1 - s_2 sin\theta_1\dot{\theta}_1\right)^2\right) + (0.5)m_3\left(\left(\dot{s}_2 sin\theta_1 + s_2 cos\theta_1\dot{\theta}_1 + \dot{s}_3 sin(\theta_1+\theta_2) + s_3 cos(\theta_1+\theta_2)\dot{\theta}_1 + s_3 cos(\theta_1+\theta_2)\dot{\theta}_2\right)^2 + \left(\dot{s}_1 + \dot{s}_2 cos\theta_1 - s_2 sin\theta_1\dot{\theta}_1 + \dot{s}_3 cos(\theta_1+\theta_2) - s_3 sin(\theta_1+\theta_2)\dot{\theta}_1 - s_3 sin(\theta_1+\theta_2)\dot{\theta}_2\right)^2\right) + (0.5)I_1\dot{\theta}_1{}^2 + (0.5)I_2\left(\dot{\theta}_1{}^2 + \dot{\theta}_2{}^2\right) + (0.5)I_3\left(\dot{\theta}_1{}^2 + \dot{\theta}_2{}^2 + \dot{\theta}_3{}^2\right).} \tag{4}$$

The potential energy (P) of the system comprises the sum of the gravitational potential energy and the spring potential energy. A small angle assumption is made throughout the derivation. This allows us to directly express the displacement of springs and the velocities associated with dampers in terms of system generalized coordinates.

$$\boldsymbol{P = -m_1 g s_1 - m_2 g(s_1 + s_2 cos\theta_1) - m_3 g\big(s_1 + s_2 cos\theta_1 + s_3 cos(\theta_1 + \theta_1)\big) + (0.5)k_{11}(s_1 + (1/2)\theta_1 - s_{01})^2 + (0.5)k_{21}(s_1 + (1/2)\theta_1 - s_{01})^2 + (0.5)k_{12}(s_2 + (1/2)\theta_2 - s_{02})^2 + (0.5)k_{22}(s_2 + (1/2)\theta_2 - s_{02})^2 + (0.5)k_{13}(s_3 + (1/2)\theta_3 - s_{03})^2 + (0.5)k_{23}(s_3 + (1/2)\theta_3 - s_{03})^2} \tag{5}$$

where,S_{01}, S_{02}, S_{03} are the initial values ofS_1, S_2, S_3 respectively.

Due to viscous damping in the system, Rayliegh's dissipation function [6] is used to give damping energy

$$\boldsymbol{D = (0.5)c_{11}\big(\dot{s}_1 + (1/2)\dot{\theta}_1\big)^2 + (0.5)c_{21}\big(\dot{s}_1 + (1/2)\dot{\theta}_1\big)^2 + (0.5)c_{12}\big(\dot{s}_2 + (1/2)\dot{\theta}_2\big)^2 + (0.5)c_{22}\big(\dot{s}_2 + (1/2)\dot{\theta}_2\big)^2 + (0.5)c_{13}\big(\dot{s}_3 + (1/2)\dot{\theta}_3\big)^2 + (0.5)c_{23}\big(\dot{s}_3 + (1/2)\dot{\theta}_3\big)^2.} \tag{6}$$

The generalized forces in the system corresponding to the generalized co-ordinates are expressed as appropriately weighted combinations of the input forces.

$$\boldsymbol{Q_{s_1} = F_{11} + F_{21} + (F_{12} + F_{22})cos\theta_1 + (F_{13} + F_{23})cos(\theta_1 + \theta_2)} \tag{7}$$

$$\boldsymbol{Q_{s_2} = F_{12} + F_{22} + (F_{13} + F_{23})cos(\theta_2)} \tag{8}$$

$$\boldsymbol{Q_{s_3} = F_{13} + F_{23}} \tag{9}$$

$$\boldsymbol{Q_{\theta_1} = (1/2)(F_{11} - F_{21}) + (1/2)(F_{12} - F_{22}) + (1/2)(F_{13} - F_{23}) + s_2 sin\theta_2(F_{13} + F_{23})} \tag{10}$$

$$\boldsymbol{Q_{\theta_1} = (1/2)(F_{12} - F_{22}) + (1/2)(F_{13} - F_{23})} \tag{11}$$

$$\boldsymbol{Q_{\theta_1} = (1/2)(F_{13} - F_{23})} \tag{12}$$

It can be evinced from the force expressions that the total input forces acting on each module can be resolved into an additive component along the direction of extension and a subtractive component that results in a torque. For the first module, there is an additional torque produced by forces in the third module.

The model resulting from the application of Lagrange's equations of motion obtained for this system can be represented in the form

$$\boldsymbol{F_{coeff}}\,\underline{\boldsymbol{\tau}} = \boldsymbol{D}\left(\underline{\boldsymbol{q}}\right)\underline{\ddot{\boldsymbol{q}}} + \boldsymbol{C}\left(\underline{\boldsymbol{q}}\right)\underline{\dot{\boldsymbol{q}}} + \boldsymbol{G}\left(\underline{\boldsymbol{q}}\right) \quad (13)$$

where τ is a vector of input forces and q is a vector of generalized co-ordinates. The force coefficient matrix F_{coeff} transforms the input forces to the generalized forces and torques in the system. The inertia matrix, D is composed of four block matrices. The block matrices that correspond to pure linear accelerations and pure angular accelerations in the system (on the top left and on the bottom right) are symmetric. The matrix C contains coefficients of the first order derivatives of the generalized co-ordinates. Since the system is nonlinear, many elements of C contain first order derivatives of the generalized co-ordinates. The remaining terms in the dynamic equations resulting from gravitational potential energies and spring energies are collected in the matrix G. The coefficient matrices of the dynamic equations are given below,

$$\boldsymbol{Fcoeff} = \begin{bmatrix} 1 & 1 & cos(\theta_1) & cos(\theta_1) & cos(\theta_1+\theta_2) & cos(\theta_1+\theta_2) \\ 0 & 0 & 1 & 1 & cos(\theta_2) & cos(\theta_2) \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1/2 & -1/2 & 1/2 & -1/2 & 1/2+s_2sin(\theta_2) & -1/2+s_2sin(\theta_2) \\ 0 & 0 & 1/2 & -1/2 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 & 1/2 & -1/2 \end{bmatrix} \quad (14)$$

$$\boldsymbol{D}\left(\underline{\boldsymbol{q}}\right) = \begin{bmatrix} m_1+m_2+m_3 & m_2cos(\theta_1)+m_3cos(\theta_1) & m_3cos(\theta_1+\theta_2) & -m_2s_2sin(\theta_1)-m_3s_2sin(\theta_1)-m_3s_3sin(\theta_1+\theta_2) & -m_3s_3sin(\theta_1+\theta_2) & 0 \\ m_2cos(\theta_1)+m_3cos(\theta_1) & m_2+m_3 & m_3cos(\theta_2) & -m_3s_3sin(\theta_2) & -m_3s_3sin(\theta_2) & 0 \\ m_3cos(\theta_1+\theta_2) & m_3cos(\theta_2) & m_3 & m_3s_3sin(\theta_2) & 0 & 0 \\ -m_2s_2sin(\theta_1)-m_3s_2sin(\theta_1)-m_3s_3sin(\theta_1+\theta_2) & -m_3s_3sin(\theta_2) & m_3s_2sin(\theta_2) & m_2s_2^2+I_1+I_2+I_3+m_3s_2^2+m_3s_3^2+2m_3s_3cos(\theta_2)s_2 & I_2+m_3s_3^2+I_3+m_3s_3cos(\theta_2)s_2 & I_3 \\ -m_3s_3sin(\theta_1+\theta_2) & -m_3s_3sin(\theta_2) & 0 & I_2+m_3s_3^2+I_3+m_3s_3cos(\theta_2)s_2I & I_2+m_3s_3^2+I_3 & I_3 \\ 0 & 0 & 0 & I_3 & I_3 & I_3 \end{bmatrix} \quad (15)$$

$$\boldsymbol{C}\left(\underline{\boldsymbol{q}}\right) = \tag{16}$$

$$\begin{bmatrix} c_{11}+c_{21} & \begin{matrix} -2m_2 sin(\theta_1)\dot{\theta}_1 \\ -2m_3 sin(\theta_1)\dot{\theta}_1 \end{matrix} & \begin{matrix} -2m_3 sin(\theta_1+\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} -m_2 s_2 \\ cos(\theta_1)(\dot{\theta}_1) \\ +(1/2)(c_{11}+c_{21}) \\ -m_3 s_2 \\ cos(\theta_1)(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_1+\theta_2)(\dot{\theta}_1) \end{matrix} & -m_3 s_3 sin(\theta_1+\theta_2) & 0 \\ 0 & c_{12}+c_{22} & \begin{matrix} -2m_3 sin(\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} -m_3 s_3(\dot{\theta}_1) \\ +(1/2) \\ (c_{12}+c_{22}) \\ -m_3 s_2(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} -2m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_2) \end{matrix} & 0 \\ 0 & 2m_3 sin(\theta_2)(\dot{\theta}_1) & c_{13}+c_{23} & \begin{matrix} -m_3 s_3 s_2 \\ cos(\theta_2)(\dot{\theta}_1) \\ -m_3 s_3(\dot{\theta}_1) \end{matrix} & \begin{matrix} -2m_3 s_3(\dot{\theta}_1) \\ -m_3 s_3(\dot{\theta}_2) \end{matrix} & \begin{matrix} (1/2) \\ (c_{13}+c_{23}) \end{matrix} \\ \begin{matrix} (1/2) \\ (c_{11}+c_{21}) \end{matrix} & \begin{matrix} 2m_3 s_3 cos(\theta_2)(\dot{\theta}_1) \\ -2m_3 s_2(\dot{\theta}_1) \\ +2m_2 s_2(\dot{\theta}_1) \end{matrix} & \begin{matrix} 2m_3 s_3(\dot{\theta}_1+\dot{\theta}_2) \\ -2m_3 s_2 cos(\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} 2m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_2) \\ +(1^2/4) \\ (c_{11}+c_{21}) \end{matrix} & \begin{matrix} m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_2) \end{matrix} & 0 \\ 0 & \begin{matrix} (1/2)(c_{12}+c_{22})+ \\ 2m_3 s_3 cos(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} 2m_3 s_3 \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} (1^2/4) \\ (c_{12}+c_{22}) \end{matrix} & 0 \\ 0 & 0 & (1/2)(c_{13}-c_{23}) & 0 & 0 & \begin{matrix} (1^2/4) \\ (c_{13}+c_{23}) \end{matrix} \end{bmatrix}$$

$$\boldsymbol{G}\left(\underline{\boldsymbol{q}}\right) = \tag{17}$$

$$\begin{bmatrix} -m_1 g - m_2 g + k_{11}(s_1+(1/2)\theta_1 - s_{01}) + k_{21}(s_1-(1/2)\theta_1 - s_{01}) - m_3 g \\ -m_2 g cos(\theta_1) + k_{12}(s_2+(1/2)\theta_2 - s_{02}) + k_{22}(s_2-(1/2)\theta_2 - s_{02}) - m_3 g cos(\theta_1) \\ -m_3 g cos(\theta_1+\theta_2) + k_{13}(s_3+(1/2)\theta_3 - s_{03}) + k_{23}(s_3-(1/2)\theta_3 - s_{03}) \\ m_2 s_2 g sin(\theta_1) + m_3 s_3 g sin(\theta_1+\theta_2) + m_3 s_2 g sin(\theta_1) + k_{11}(s_1+(1/2)\theta_1 - s_{01})(1/2) \\ +k_{21}(s_1-(1/2)\theta_1 - s_{01})(-1/2) \\ m_3 s_3 g sin(\theta_1+\theta_2) + k_{12}(s_2+(1/2)\theta_2 - s_{02})(1/2) + k_{22}(s_2-(1/2)\theta_2 - s_{02})(-1/2) \\ k_{13}(s_3+(1/2)\theta_3 - s_{03})(1/2) + k_{23}(s_3-(1/2)\theta_3 - s_{03})(-1/2) \end{bmatrix}$$

B. Variable structure Controller

Consider a nonlinear single input dynamic system is defined by [6]:

$$x^{(n)} = f(\vec{x}) + b(\vec{x})u \tag{18}$$

Where u is the vector of control input, $x^{(n)}$ is the n^{th} derivation of x, $x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known *sign* function. The main goal to design this controller is train to the desired state; $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d{}^{(n-1)}]^T$, and trucking error vector is defined by [6]:

$$\widetilde{x} = x - x_d = [\widetilde{x}, \dots, \widetilde{x}^{(n-1)}]^T \tag{19}$$

A time-varying sliding surface $s(x,t)$ in the state space R^n is given by [6]:

$$\boldsymbol{s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\,\tilde{x} = 0} \tag{20}$$

where λ is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows [6]:

$$\boldsymbol{s(x,t)} = (\frac{d}{dt} + \boldsymbol{\lambda})^{\boldsymbol{n-1}} \left(\int_0^t \tilde{x}\,\boldsymbol{dt}\right) = \boldsymbol{0} \tag{21}$$

The main target in this methodology is kept the sliding surface slope $\boldsymbol{s(x,t)}$ near to the zero. Therefore, one of the common strategies is to find input $\boldsymbol{U}$ outside of $\boldsymbol{s(x,t)}$ [6].

$$\frac{1}{2}\frac{d}{dt}s^2(x,t) \le -\zeta|s(x,t)| \tag{22}$$

where ζ is positive constant.

$$\text{If } \mathbf{S(0)>0} \rightarrow \frac{\mathrm{d}}{\mathrm{dt}}\mathbf{S(t)} \le -\boldsymbol{\zeta} \tag{23}$$

To eliminate the derivative term, it is used an integral term from t=0 to t=$\boldsymbol{t_{reach}}$

$$\int_{t=0}^{t=t_{reach}} \frac{d}{dt} S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S\,(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0) \tag{24}$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S($t_{reach} = 0$) defined as;

$$0 - S(0) \le -\eta(t_{reach}) \rightarrow t_{reach} \le \frac{S(0)}{\zeta} \tag{25}$$

and

$$if\ S(0) < 0 \rightarrow 0 - S(0) \le -\eta(t_{reach}) \rightarrow S(0) \le -\zeta(t_{reach}) \rightarrow t_{reach} \le \frac{|S(0)|}{\eta} \tag{26}$$

Equation (26) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$ since the trajectories are outside of $S(t)$.

$$if\ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0 \tag{27}$$

suppose S is defined as

$$s(x,t) = (\frac{d}{dt} + \lambda) \quad \tilde{x} = (\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) + \lambda(\mathrm{x} - \mathrm{x}_{\mathrm{d}}) \tag{28}$$

The derivation of S, namely, $\dot{S}$ can be calculated as the following;

$$\dot{S} = (\ddot{\mathrm{x}} - \ddot{\mathrm{x}}_{\mathrm{d}}) + \lambda(\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) \tag{29}$$

suppose the second order system is defined as;

$$\ddot{x} = f + u \rightarrow \dot{S} = f + U - \ddot{x}_d + \lambda(\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) \tag{30}$$

Where $\boldsymbol{f}$ is the dynamic uncertain, and also since $S = 0 \; and \; \dot{S} = 0$, to have the best approximation ,$\widehat{\boldsymbol{U}}$ is defined as

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{31}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law [52-53]:

$$\boldsymbol{U}_{\boldsymbol{dis}} = \widehat{\boldsymbol{U}} - \boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t}) \cdot \mathbf{sgn}(\boldsymbol{s}) \tag{32}$$

where the switching function $\mathbf{sgn}(\mathbf{S})$ is defined as [1, 6]

$$\boldsymbol{sgn}(\boldsymbol{s}) = \begin{cases} \mathbf{1} & \boldsymbol{s} > 0 \\ -\mathbf{1} & \boldsymbol{s} < 0 \\ \mathbf{0} & \boldsymbol{s} = \mathbf{0} \end{cases} \tag{33}$$

and the $\boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t})$ is the positive constant. Suppose by (22) the following equation can be written as,

$$\frac{\mathbf{1}}{\mathbf{2}}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{s}^2(\boldsymbol{x}, \boldsymbol{t}) = \dot{\mathbf{S}} \cdot \mathbf{S} = \left[\boldsymbol{f} - \widehat{\boldsymbol{f}} - \boldsymbol{K}\mathbf{sgn}(\boldsymbol{s})\right] \cdot \boldsymbol{S} = \left(\boldsymbol{f} - \widehat{\boldsymbol{f}}\right) \cdot \boldsymbol{S} - \boldsymbol{K}|\boldsymbol{S}| \tag{34}$$

and if the equation (26) instead of (25) the sliding surface can be calculated as

$$\boldsymbol{s}(\boldsymbol{x}, \boldsymbol{t}) = \left(\frac{\boldsymbol{d}}{\boldsymbol{dt}} + \boldsymbol{\lambda}\right)^{\mathbf{2}} \left(\int_{\mathbf{0}}^{\boldsymbol{t}} \widetilde{\boldsymbol{x}} \, \boldsymbol{dt}\right) = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) - \boldsymbol{\lambda}^{\mathbf{2}}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{35}$$

in this method the approximation of $\boldsymbol{U}$ is computed as [6]

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}^{\mathbf{2}}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{36}$$

Based on above discussion, the variable structure control law for a multi degrees of freedom robot manipulator is written as [1, 6]:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{\boldsymbol{eq}} + \boldsymbol{\tau}_{\boldsymbol{dis}} \tag{37}$$

Where, the model-based component $\boldsymbol{\tau}_{\boldsymbol{eq}}$ is the nominal dynamics of systems calculated as follows [1]:

$$\boldsymbol{\tau}_{\boldsymbol{eq}} = \left[\boldsymbol{M}^{-\mathbf{1}}(\boldsymbol{B} + \boldsymbol{C} + \boldsymbol{G}) + \dot{\boldsymbol{S}}\right]\boldsymbol{M} \tag{38}$$

and $\boldsymbol{\tau}_{\boldsymbol{dis}}$ is computed as [1];

$$\boldsymbol{\tau}_{\boldsymbol{dis}} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{39}$$

By (39) and (38) the variable structure control of robot manipulator is calculated as;

$$\boldsymbol{\tau} = \left[\boldsymbol{M}^{-\mathbf{1}}(\boldsymbol{B} + \boldsymbol{C} + \boldsymbol{G}) + \dot{\boldsymbol{S}}\right]\boldsymbol{M} + \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{40}$$

The lyapunov formulation can be written as follows,

$$\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{S}^{\boldsymbol{T}}.\boldsymbol{M}.\boldsymbol{S} \tag{41}$$

the derivation of V can be determined as,

$$\dot{V} = \frac{1}{2} S^T . \dot{M} . S + S^T M\dot{S} \quad (42)$$

the dynamic equation of robot manipulator can be written based on the sliding surface as

$$M\dot{S} = -VS + M\dot{S} + B + C + G \quad (43)$$

it is assumed that

$$S^T(\dot{M} - 2B + C + G)S = 0 \quad (44)$$

by substituting (43) in (44)

$$\dot{V} = \frac{1}{2} S^T \dot{M} S - S^T B + CS + S^T(M\dot{S} + B + CS + G) = S^T(M\dot{S} + B + CS + G) \quad (45)$$

suppose the control input is written as follows

$$\hat{U} = \widehat{U_{Nonlinear}} + \widehat{U_{dis}} = [\widehat{M^{-1}}(B + C + G) + \dot{S}]\hat{M} + K.sgn(S) + B + CS + G \quad (46)$$

by replacing the equation (49) in (41)

$$\dot{V} = S^T(M\dot{S} + B + C + G - \hat{M}\dot{S} - \widehat{B + CS} + G - Ksgn(S) = S^T\left(\tilde{M}\dot{S} + \widetilde{B + CS} + G - Ksgn(S)\right) \quad (47)$$

and

$$\left|\tilde{M}\dot{S} + \widetilde{B + CS} + G\right| \leq \left|\tilde{M}\dot{S}\right| + \left|\widetilde{B + CS} + G\right| \quad (48)$$

the Lemma equation in robot arm system can be written as follows

$$K_u = \left[\left|\tilde{M}\dot{S}\right| + |B + CS + G| + \eta\right]_i, i = 1, 2, 3, 4, \ldots \quad (49)$$

and finally;

$$\dot{V} \leq -\sum_{i=1}^{n} \eta_i |S_i| \quad (50)$$

Figure 2 shows the pure variable structure controller applied to continuum robot.

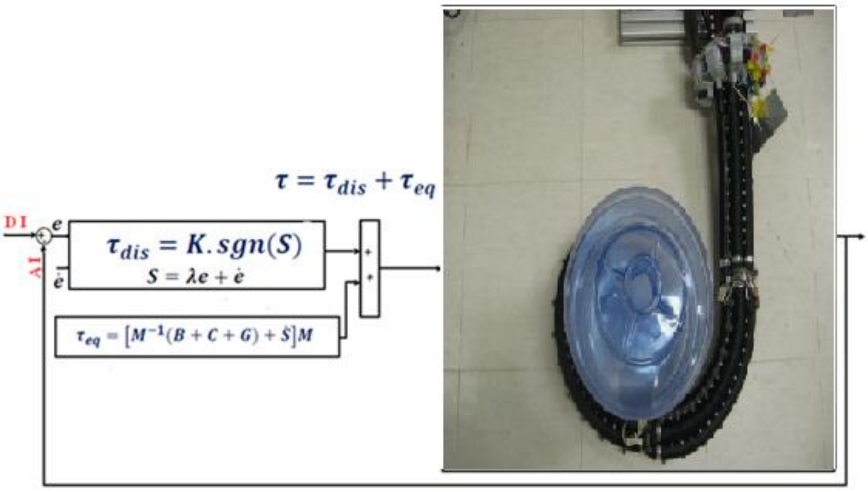

Figure 2. Variable Structure Controller

C. Fuzzy Logic Methodology

Based on foundation of fuzzy logic methodology; fuzzy logic controller has played important rule to design nonlinear controller for nonlinear and uncertain systems [13-14]. However the application area for fuzzy control is really wide, the basic form for all command types of controllers consists of; Input fuzzification (binary-to-fuzzy [B/F] conversion), Fuzzy rule base (knowledge base), Inference engine and Output defuzzification (fuzzy-to-binary [F/B] conversion). Figure 3 shows the fuzzy controller part.

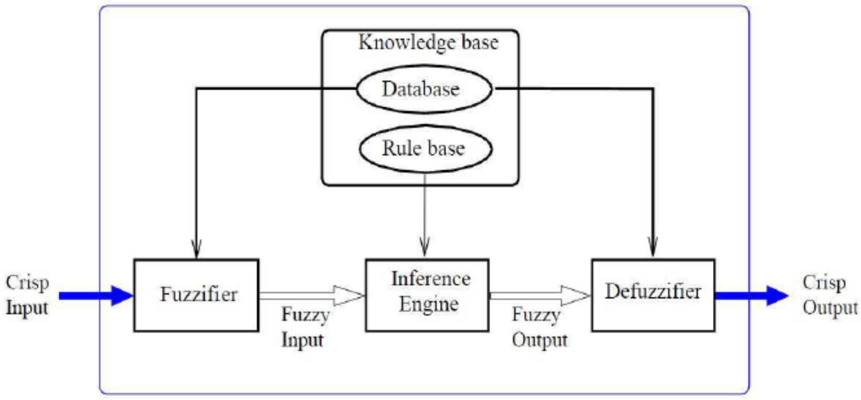

Figure 3. Fuzzy Controller Part

The fuzzy inference engine offers a mechanism for transferring the rule base in fuzzy set which it is divided into two most important methods, namely, Mamdani method and Sugeno method. Mamdani method is one of the common fuzzy inference systems and he designed one of the first fuzzy controllers to control of system engine. Mamdani's fuzzy inference system is divided into four major steps: fuzzification, rule evaluation, aggregation of the rule outputs and defuzzification. Michio Sugeno use a singleton as a membership function of the rule consequent part. The following definition shows the Mamdani and Sugeno fuzzy rule base [10]

$$\boldsymbol{if\ x\ is\ A\ and\ y\ is\ B\ then\ z\ is\ C\ 'mamdani'} \quad \textbf{(51)}$$
$$\boldsymbol{if\ x\ is\ A\ and\ y\ is\ B\ then\ z\ is\ f(x,y)'sugeno'}$$

When x and y have crisp values fuzzification calculates the membership degrees for antecedent part. Rule evaluation focuses on fuzzy operation (AND/OR) in the antecedent of the fuzzy rules. The aggregation is used to calculate the output fuzzy set and several methodologies can be used in fuzzy logic controller aggregation, namely, Max-Min aggregation, Sum-Min aggregation, Max-bounded product, Max-drastic product, Max-bounded sum, Max-algebraic sum and Min-max. Defuzzification is the last step in the fuzzy inference system which it is used to transform fuzzy set to crisp set. Consequently defuzzification's input is the aggregate output and the defuzzification's output is a crisp number. Centre of gravity method (COG) and Centre of area method (COA) are two most common defuzzification methods.

3. Methodology

Based on (38) in VSC controller, if we have uncertainty in systems or when nonlinearity term in (38) are unknown, a fuzzy logic can be used to approximate them as

$$f(x) = \sum_{l=1}^{M} \theta^l \mathcal{E}^l(x) = \theta^T \mathcal{E}(x) \quad \textbf{(52)}$$

Where
$\theta = (\theta^1, \dots, \theta^M)^T, \mathcal{E}(x) = (\mathcal{E}^1(x), \dots, \mathcal{E}^M(x))^T, and\ \mathcal{E}^l(x) =$
$: \prod_{i=1}^{n} \frac{\mu_{A_i^l}(x_i)}{} \sum_{l=1}^{M} (\prod_{i=1}^{n} \mu_{A_i^l}(x_i))$. $\theta^1, \dots, \theta^M$ are adjustable parameters in (52). $\mu_{A_1^1}(x_1), \dots, \mu_{A_n^m}(x_n)$ are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

$$f(x) = \frac{\sum_{l=1}^{M} \theta^l \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l} \right)^2 \right) \right]}{\sum_{l=1}^{M} \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l} \right)^2 \right) \right]} \quad \textbf{(53)}$$

Where $\theta^l, \alpha_i^l\ and\ \delta_i^l$ are all adjustable parameters. From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust θ^l in (53). We define $f^\wedge(x|\theta)$ as the approximator of the real function$f(x)$.

$$f^\wedge(x|\theta) = \theta^T \varepsilon(x) \quad \textbf{(54)}$$

We define θ^* as the values for the minimum error:

$$\theta^* = \arg\min_{\theta \in \Omega} \left[\sup_{x \in U} | f^\wedge(x|\theta) - g(x)| \right] \quad \textbf{(55)}$$

Where Ω is a constraint set for θ. For specific x , $sup_{x \in U} |f^\wedge(x|\theta^*) - f(x)|$ is the minimum approximation error we can get.

We used the first type of fuzzy systems (52) to estimate the nonlinear system (38) the fuzzy formulation can be write as below;

$$f(x|\theta) = \theta^T \varepsilon(x) = \frac{\sum_{l=1}^{n} \theta^l \left[\mu_{A^l}(x)\right]}{\sum_{l=1}^{n}[\mu_{A^l}(x)]} \quad \textbf{(56)}$$

Where $\theta^1, \dots, \theta^n$ are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of $\theta - \theta^*$. The SISO fuzzy system is define as

$$f(x) = \ominus^T \varepsilon(x) \quad \textbf{(57)}$$

Where

$$\ominus^T = (\theta_1, \dots, \theta_m)^T = \begin{bmatrix} \theta_1^1, \theta_1^2, \dots, \theta_1^M \\ \theta_2^1, \theta_2^2, \dots, \theta_2^M \\ \vdots \\ \theta_m^1, \theta_m^2, \dots, \theta_m^M \end{bmatrix} \quad \textbf{(58)}$$

$\varepsilon(x) = (\varepsilon^1(x), \dots, \varepsilon^M(x))^T$, $\varepsilon^1(x) = \prod_{i=1}^{n} \mu_{A_i^l}(x_i) / \sum_{l=1}^{M} (\prod_{i=1}^{n} \mu_{A_i^l}(x_i))$, and $\mu_{A_i^l}(x_i)$ is defined in (56). To reduce the number of fuzzy rules, we divide the fuzzy system in to three parts:

$$F^1(q, \dot{q}) = \ominus^{1^T} \varepsilon(q, \dot{q}) = \left[\theta_1^{1^T} \varepsilon(q, \dot{q}), \dots, \theta_m^{1^T} \varepsilon(q, \dot{q})\ \right]^T \quad \textbf{(59)}$$

$$F^2(q, \ddot{q}_r) = \ominus^{2^T} \varepsilon(q, \ddot{q}_r) = \left[\theta_1^{2^T} \varepsilon(q, \ddot{q}_r), \dots, \theta_m^{2^T} \varepsilon(q, \ddot{q}_r)\ \right]^T \quad \textbf{(60)}$$

$$F^3(q, \ddot{q}) = \ominus^{3^T} \varepsilon(q, \ddot{q}) = \left[\theta_1^{3^T} \varepsilon(q, \dot{q}), \dots, \theta_m^{3^T} \varepsilon(q, \ddot{q})\ \right]^T \quad \textbf{(61)}$$

The control security input is given by

$$\tau = M \quad \ddot{q}_r + B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q) + F^1(q, \dot{q}) + F^2(q, \ddot{q}_r) + F^3(q, \ddot{q}) - K_p e - K_v \dot{e} \quad \textbf{(62)}$$

Where $M^\wedge$,$\boldsymbol{B(q)\dot{q}\dot{q}}, \boldsymbol{C(q)\dot{q}^2}, \boldsymbol{g(q)}$ are the estimations of $M(q)$.

Based on variable structure formulation (28) and PD linear methodology (5);

$$S_{New} = (\dot{e} + \lambda e) \quad \textbf{(63)}$$

And $U_{switc□}$ is obtained by

$$\mathrm{U_{switch}} = \mathrm{K}(\vec{x}, t) \cdot \mathrm{sgn}(\mathrm{S_{New}}) = \mathrm{K}(\vec{x}, t) \cdot \mathrm{sgn}\ (K(\dot{e} + \lambda e)) \quad \textbf{(64)}$$

The Lyapunov function in this design is defined as

$$V=\frac{1}{2}S^{T}MS+\frac{1}{2}\sum_{J=1}^{M}\frac{1}{\gamma_{sj}}\phi^{T}.\phi_{j} \quad \textbf{(65)}$$

where γ_{sj} is a positive coefficient, $\boldsymbol{\phi=\theta^{*}-\theta,\theta^{*}}$ is minimum error and θ is adjustable parameter. Since $\dot{M}-2V$ is skew-symetric matrix;

$$S^{T}M\dot{S}+\frac{1}{2}S^{T}\dot{M}S=S^{T}(M\dot{S}+VS) \quad \textbf{(66)}$$

If the dynamic formulation of robot manipulator defined by

$$\tau=M(q)\ddot{q}+V(q,\dot{q})\dot{q}+G(q) \quad \textbf{(67)}$$

the controller formulation is defined by

$$\tau=\widehat{M}\ddot{q}_{r}+\widehat{V}\dot{q}_{r}+\widehat{G}-\lambda S-K \quad \textbf{(68)}$$

According to (58) and (59)

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}+G(q)=\widehat{M}\ddot{q}_{r}+\widehat{V}\dot{q}_{r}+\widehat{G}-\lambda S-K \quad \textbf{(69)}$$

Since $\boldsymbol{\dot{q}_{r}=\dot{q}-S}$ and $\boldsymbol{\ddot{q}_{r}=\ddot{q}-\dot{S}}$

$$M\dot{S}+(V+\lambda)S=\Delta f-K \quad \textbf{(70)}$$

$$\boldsymbol{M\dot{S}=\Delta f-K-VS-\lambda S}$$

The derivation of V is defined

$$\dot{V}=S^{T}M\dot{S}+\frac{1}{2}S^{T}\dot{M}S+\sum_{J=1}^{M}\frac{1}{\gamma_{sj}}\phi^{T}.\dot{\phi}_{j} \quad \textbf{(71)}$$

$$\boldsymbol{\dot{V}=S^{T}(M\dot{S}+VS)+\sum_{J=1}^{M}\frac{1}{\gamma_{sj}}\phi^{T}.\dot{\phi}_{j}}$$

Based on (61) and (62)

$$\dot{V}=S^{T}(\Delta f-K-VS-\lambda S+VS)+\sum_{J=1}^{M}\frac{1}{\gamma_{sj}}\phi^{T}.\dot{\phi}_{j} \quad \textbf{(72)}$$

where $\Delta f=[M(q)\ddot{q}+V(q,\dot{q})\dot{q}+G(q)]-\sum_{l=1}^{M}\theta^{T}\zeta(x)$

$$\boldsymbol{\dot{V}=\sum_{J=1}^{M}[S_{j}(\Delta f_{j}-K_{j})]-S^{T}\lambda S+\sum_{J=1}^{M}\frac{1}{\gamma_{sj}}\phi^{T}.\dot{\phi}_{j}}$$

suppose K_j is defined as follows

$$K_j = \frac{\sum_{l=1}^{M} \theta_j^l [\mu_A(S_j)]}{\sum_{l=1}^{M} [\mu_A(S_j)]} = \theta_j^T \zeta_j(S_j) \quad (73)$$

Where $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \ldots\ldots, \zeta_j^M(S_j)]^T$

$$\zeta_j^1(S_j) = \frac{\mu_{(A)}{}_j^l (S_j)}{\sum_i \mu_{(A)}{}_j^l (S_j)} \quad (74)$$

where $\mu_{(xi)}$ is membership function.

The fuzzy system is defined as

$$f(x) = \tau_{fuzzy} = \sum_{l=1}^{M} \theta^T \zeta(x) = \psi(e, \dot{e}) \quad (75)$$

where $\theta = (\theta^1, \theta^2, \theta^3, \ldots\ldots, \theta^M)$ is adjustable parameter in (65) according to (62), (63) and (65);

$$\dot{V} = \sum_{j=1}^{M} \left[S_j(\Delta f_j - \theta^T \zeta(S_j)\right] - S^T \lambda S + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T . \dot{\phi}_j \quad (76)$$

Based on $\boldsymbol{\phi} = \boldsymbol{\theta}^* - \boldsymbol{\theta} \rightarrow \boldsymbol{\theta} = \boldsymbol{\theta}^* - \boldsymbol{\phi}$

$$\dot{V} = \sum_{j=1}^{M} \left[S_j(\Delta f_j - \theta^{*T} \zeta(S_j) + \phi^T \zeta(S_j)\right] - S^T \lambda S + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T . \dot{\phi}_j \quad (77)$$

$$\dot{\boldsymbol{V}} = \sum_{j=1}^{M} \left[\boldsymbol{S_j}(\boldsymbol{\Delta f_j} - (\boldsymbol{\theta}^*)^T \boldsymbol{\zeta}(\boldsymbol{S_j})\right] - \boldsymbol{S^T \lambda S} + \sum_{j=1}^{M} \frac{\mathbf{1}}{\boldsymbol{\gamma_{sj}}} \boldsymbol{\phi_j^T} [\boldsymbol{\gamma_{sj}} . \boldsymbol{S_j} . \boldsymbol{\zeta_j}(\boldsymbol{S_j}) + \dot{\boldsymbol{\phi}}_{\boldsymbol{j}}])$$

where $\dot{\boldsymbol{\theta}}_{\boldsymbol{j}} = \boldsymbol{\gamma_{sj} S_j \zeta_j}(\boldsymbol{S_j})$ is adaption law, $\dot{\emptyset}_j = -\dot{\boldsymbol{\theta}}_{\boldsymbol{j}} = -\boldsymbol{\gamma_{sj} S_j \zeta_j}(\boldsymbol{S_j})$

$\dot{\boldsymbol{V}}$ is considered by

$$\dot{V} = \sum_{j=1}^{m} [S_j \, \Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right)] - S^T \lambda S \quad (78)$$

The minimum error is defined by

$$e_{mj} = \Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right) \quad (79)$$

Therefore $\dot{\boldsymbol{V}}$ is computed as

$$\begin{aligned} \dot{V} &= \sum_{j=1}^{m} [S_j \, e_{mj}] - S^T \lambda S \\ &\le \sum_{j=1}^{m} |S_j||e_{mj}| - S^T \lambda S \\ &= \sum_{j=1}^{m} |S_j||e_{mj}| - \lambda_j {S_j}^2 \end{aligned} \quad (80)$$

$$= \sum_{j=1}^{m} |S_j| \left(|e_{mj}| - \lambda_j S_j\right) \tag{81}$$

For continuous function $g(x)$, and suppose $\varepsilon > 0$ it is defined the fuzzy logic system in form of

$$Sup_{x \in U} |f(x) - g(x)| < \epsilon \tag{82}$$

the minimum approximation error (e_{mj}) is very small.

$$if \ \lambda_j = \alpha \quad that \quad \alpha |S_j| > e_{mj} \ (S_j \neq 0) \qquad then \quad \dot{V} < 0 \ for \ (S_j \neq 0) \tag{83}$$

This method has two main controller's coefficients, K_p and K_V. To tune and optimize these parameters mathematical formulation is used

$$U_{model-base} = U_{fuzzy} + U_{VSC} \tag{84}$$

$$\mathrm{U_{model-base}} = \mathrm{U_{fuzzy}} + \mathrm{U_{VSC}} = \left[\boldsymbol{M^{-1}}(\boldsymbol{B} + \boldsymbol{C} + \boldsymbol{G}) + \dot{\boldsymbol{S}}\right]\boldsymbol{M} + \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) + \frac{\sum_{l=1}^{M} \theta^{l} \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]}{\sum_{l=1}^{M} \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]} \tag{85}$$

The most important different between VSC and FVSC is the uncertainty. In VSC the uncertainty is d = G+F + f. The variable structure gain must be bigger than its upper bound. It is not an easy job because this term includes tracking errors e_1 and$\dot{q}_1$. While in FVSC, the uncertainty η is the fuzzy approximation error for $G + F + f$.

$$G + F + f = \frac{\sum_{l=1}^{M} \theta^{l} \left[\prod_{i=1}^{n} exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]}{\sum_{l=1}^{M} \left[\prod_{i=1}^{n} exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]} \tag{86}$$

It is usually is smaller than $G + F + f$; and the upper bound of it is easy to be estimated.

4. Results and Discussion

In this section, we use a benchmark model, OCTARM VI robot manipulator, to evaluate our control algorithms [22]. We compare the following controllers: classical PD controller, PD fuzzy controller and model base fuzzy variable structure controller which is proposed in this paper. The simulation was implemented in MATLAB/SIMULINK environment.

Close loop response of tracking result without any disturbance: Figure 4 illustrates the tracking performance in three types of controller; linear PD controller, linear PD controller based on fuzzy logic estimator and proposed controller.

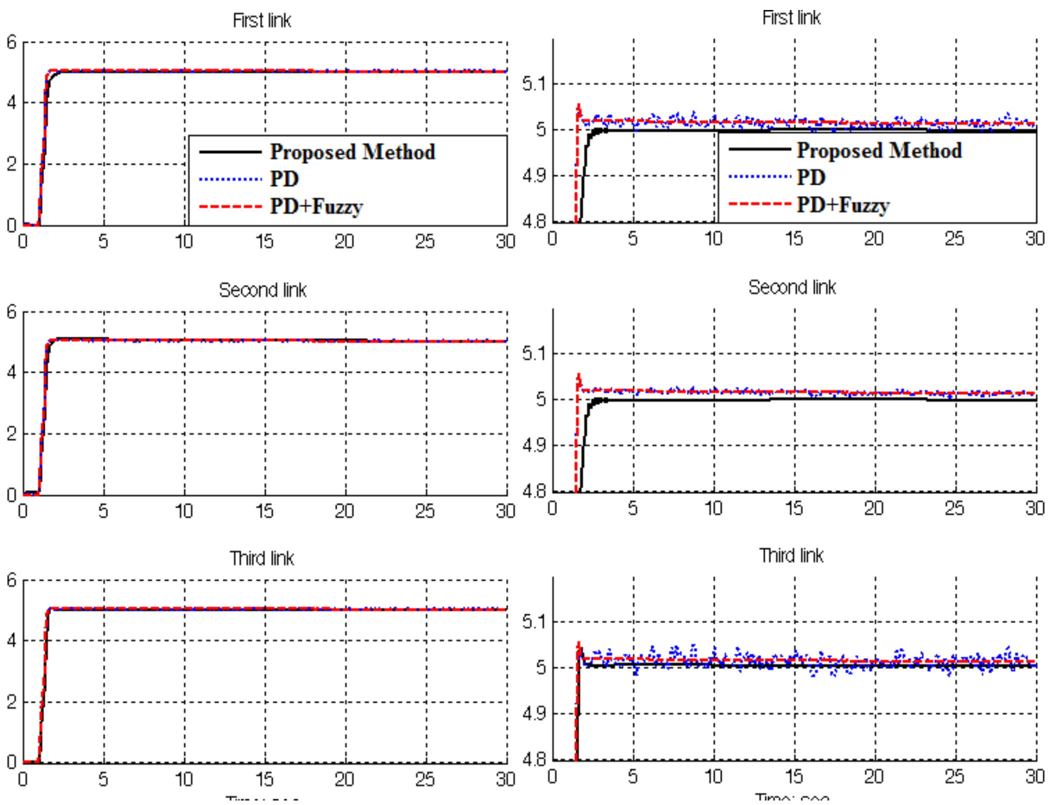

Figure 4. Linear PD, PDFLC and Proposed Method Trajectory Following without Disturbance

Based on Figure 4; pure PD controller has oscillation in first and three links, because continuum robot manipulator is a highly nonlinear controller and control of this system by linear method is very difficult. Based on above graph, however PD-FUZZY controller is a nonlinear methodology but it has difficulty to control this plant because it is a model base controller.

Close loop response of trajectory following in presence of load disturbance: Figure 5 demonstrates the power disturbance elimination in three types of controller in presence of disturbance for robot manipulator. The disturbance rejection is used to test the robustness comparisons of these three methodologies.

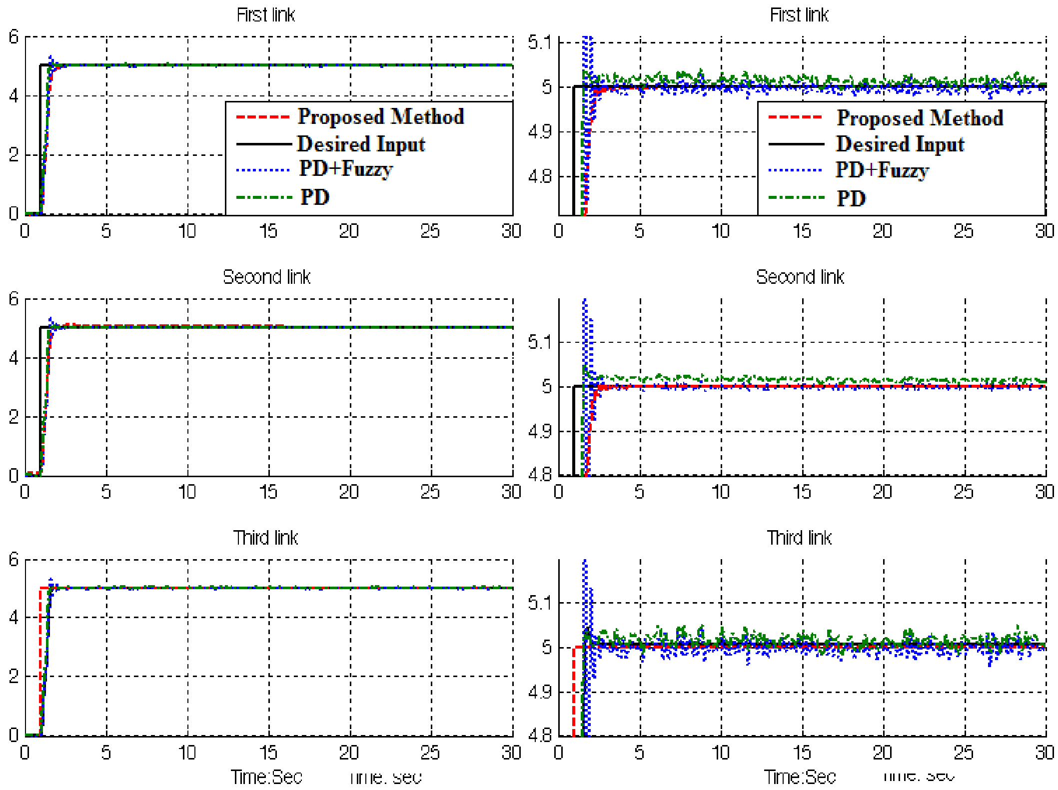

Figure 5. Linear PD, PD-FLC and Proposed Method Trajectory Following with Disturbance

Based on Figure 5; by comparison with the PD, PD-FLC and proposed model base FVSC is more stable and robust and this method is a chattering free design.

5. Conclusion

This research focuses on basic concepts of continuum robot manipulator (*e.g.*, OCTARM VI robot manipulator) and model base robust soft computing control methodology. The dynamic parameters of this system are highly nonlinear. To control of this system nonlinear control methodology variable structure controller (VSC) is introduced. VSC is a significant nonlinear controller under condition of partly uncertain dynamic parameters of system. This controller is used to control of highly nonlinear systems especially for robot manipulators, because this controller is a robust and stable. Conversely, pure VSC is used in many applications; it has two important drawbacks namely; chattering phenomenon, and nonlinear equivalent dynamic formulation in uncertain dynamic parameter. In this research the chattering phenomenon and equivalent part problems can resolve by using artificial intelligence methodology. Fuzzy logic theory is used to estimate the system dynamic. To estimate the system dynamics and eliminate the chattering, fuzzy variable structure controller is commencing. This methodology is based on applied fuzzy logic in equivalent nonlinear dynamic part to estimate unknown parameters. This controller has acceptable performance in presence of uncertainty (*e.g.*, overshoot=1%, rise time=0.9 second, steady state error = 1.6e-8 and RMS error=4.8e-8).

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Iranian center of Advance Science and Technology (IRAN SSP) is one of the independent research centers specializing in research and training across of Control and Automation, Electrical and Electronic Engineering, and Mechatronics & Robotics in Iran. At IRAN SSP research center, we are united and energized by one mission to discover and develop innovative engineering methodology that solve the most important challenges in field of advance science and technology. The IRAN SSP Center is instead to fill a long standing void in applied engineering by linking the training a development function one side and policy research on the other. This center divided into two main units:

- Education unit
- Research and Development unit

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