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An Improved Square Root Cubature Particle Filter for Navigation

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Abstract

In order to balance filtering accuracy and time cost, this paper introduces an improved square root cubature particle filter algorithm (ISRCPF) navigation method. For the lack of measurement information of the PF, it combines square root cubature Kalman filter (SRCKF) with strong tracking filter (STF), and proposes an improved square root cubature Kalman filter (ISRCKF). The ISRCPF adopting the ISRCKF to develop the proposal distribution and incorporates the latest measurement into updating phase is proposed by introducing the ISRCKF into the particle filter framework. The results show that considering both time cost and filtering accuracy, the ISRCPF is a good choice to improve the particle filter algorithm.

Keywords: *Square Root, Cubature Kalman Filter, Particle Filter, Strong Tracking Filter*

1. Introduction

In modern times, the extended Kalman filter (EKF) is probably the most widely used estimation algorithm for nonlinear systems. Nevertheless, it is only suitable for almost linear on the time scale of the updates [1]. The unscented Kalman filter (UKF) and the cubature Kalman filter (CKF) [2] which reliable for nonlinear systems approximate the state distribution with a weighted set of symmetric points. However, in non-Gaussian distribution system, they are still difficulties for state estimation [3].

The particle filter (PF) is a Monte Carlo estimation algorithm for non-Gaussian systems. However, the traditional PF can't get a good approximation of the posterior probability, especially when the likelihood is located at the tail of the prior probability density or likelihood presents spike as Figure 1 and Figure 2 [4-5].

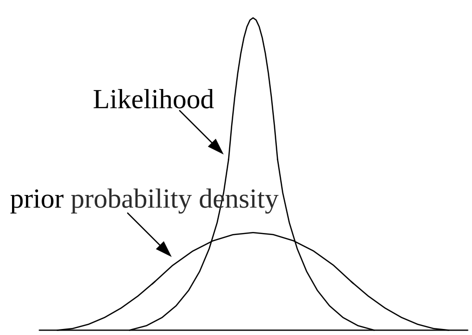

Figure 1. Likelihood Presents Spike

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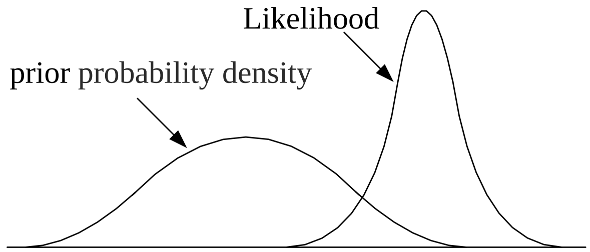


Figure 2. Likelihood is Located at the Tail of Prior Probability Density

For the above issues, a lot of improvements in terms of choice of importance density function have been made by domestic and international literature. References [6-9] proposed the unscented particle filter (UPF), the iterated extended particle filter (IEPF), the extended particle filter (EPF) and the cubature particle filter (CPF), which incorporate the latest observation and have a higher filtering accuracy. However, some unstable or even divergent behavior problem may be lead with numerically ill-conditioned for nonlinear filter. For this problem, square root cubature Kalman filter (SRCKF) [2] improving numerical accuracy and stability is proposed based on the square root filtering ideas.

For the sake of balancing filtering accuracy and time cost, this paper introduces an improved square root cubature particle filter algorithm (ISRCPF) navigation method. Firstly, combining the SRCKF and the strong tracking filter (STF), the strong tracking SRCKF (an improved SRCKF, ISRCKF) is presented. And then the ISRCPF is proposed by introducing the ISRCKF into the particle filter framework. The ISRCPF adopts the ISRCKF to develop the proposal distribution and incorporates the latest measurement into updating phase, thus the filtering accuracy is improved.

2. ISRCPF Algorithm Design

Analyzing the following discrete nonlinear dynamic system with additive noise:

$$\begin{cases} x_k = f(x_{k-1}) + w_{k-1} \\ z_k = h(x_k) + v_k \end{cases} \tag{1}$$

Where, $f(x_{k-1})$ and $h(x_k)$ are one order continuous partial derivatives about state, $w_k \sim N(0, Q_k)$ is random system noise, $v_k \sim N(0, R_k)$ is random observation noise. Initial system states x_0, w_k and v_k are statistically independent.

The SRCKF decreases the computational complexity and ensures the stability of the algorithm by using the least-squares method. However, it may confront robustness when model parameters changing and slow convergence. For this reason, it proposes the strong tracking SRCKF (ISRCKF) by combining the SRCKF with the STF.

The ISRCPF algorithm introduces the ISRCKF into the particle filter framework, and considers the latest measurement at time *k*. It adopts the ISRCKF to develop the proposal distribution can be expressed as follows:

$$q\left(x_k^i \mid x_{k-1}^i, z_k\right) = N\left(\hat{x}_k^i, P_k^i\right) \tag{2}$$

There are the steps of the ISRCPF algorithm:

(1) Initialization.

For $k = 0$, $i = 1, 2, \cdots, N$, particles $x_0^{(i)}$ are chose from $P(x_0)$:

$$\overline{x}_0^{(i)} = \mathrm{E}\left[x_0^{(i)} \right] \tag{3}$$

$$P_0^{(i)} = \mathrm{E}\left[\left(x_0^{(i)} - \overline{x}_0^{(i)} \right)\left(x_0^{(i)} - \overline{x}_0^{(i)} \right)^{\mathrm{T}} \right] \tag{4}$$

(2) Updating.

For $k = 1, 2, \cdots$, $i = 1, 2, \cdots, N$, each particle is updated with ISRCKF.

Time updating:

$$\hat{x}_{k-1|k-1}^{(i)} = \overline{x}_{k-1|k-1}^{(i)} \tag{5}$$

$$X_{i,k-1|k-1} = S_{k-1|k-1}\xi_i + \hat{x}_{k-1|k-1} \tag{6}$$

$$X_{i,k-1|k-1}^{*} = f\left(X_{i,k-1|k-1} \right) \tag{7}$$

$$\hat{x}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} X_{i,k|k-1}^{*} \tag{8}$$

$$S_{k|k-1} = \mathbf{Tria}\left(\left[\xi_{k|k-1}^{*} \quad S_{Q,k-1} \right] \right) \tag{9}$$

Measurement updating:

$$X_{i,k|k-1} = S_{k|k-1}\xi_i + \hat{x}_{k|k-1} \tag{10}$$

$$z_{i,k|k-1} = h\left(X_{i,k|k-1} \right) \tag{11}$$

$$\hat{z}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} z_{i,k|k-1} \tag{12}$$

$$S_{zz,k|k-1} = \mathbf{Tria}\left(\left[\zeta_{k|k-1} \quad S_{R,k} \right] \right) \tag{13}$$

$$P_{xz,k|k-1} = \eta_{k|k-1}\zeta_{k|k-1}^{\mathrm{T}} \tag{14}$$

$$K_k = \left(P_{xz,k|k-1} / S_{zz,k|k-1}^{\mathrm{T}} \right) \Big/ \left(\lambda_k \cdot S_{zz,k|k-1} \right) \tag{15}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(z_k - \hat{z}_{k|k-1} \right) \tag{16}$$

$$S_{k|k} = \mathbf{Tria}\left(\left[\eta_{k|k-1} - K_k\zeta_{k|k-1} \quad K_k S_{R,k} \right] \right) \tag{17}$$

Where, $S_{Q,k-1}$ and $S_{R,k}$ denote a square root factor of Q_{k-1} and R_k respectively. In formula (15), time-varying fading factor λ_k is introduced to online correct gain matrix K_k in order to weaken the influence of the old data on the current filter value.

$$Q_{k-1} = S_{Q,k-1}S_{Q,k-1}^{\mathrm{T}} \tag{18}$$

$$R_k = S_{R,k}S_{R,k}^{\mathrm{T}} \tag{19}$$

$$\xi_{k|k-1}^{*} = \frac{1}{\sqrt{m}}\left[X_{1,k|k-1}^{*} - \hat{x}_{k|k-1} \quad X_{2,k|k-1}^{*} - \hat{x}_{k|k-1} \ \cdots \ X_{m,k|k-1}^{*} - \hat{x}_{k|k-1} \right] \tag{20}$$

$$\zeta_{k|k-1} = \frac{1}{\sqrt{m}}\left[z_{1,k|k-1} - \hat{z}_{k|k-1} \quad z_{2,k|k-1} - \hat{z}_{k|k-1} \ \cdots \ z_{m,k|k-1} - \hat{z}_{k|k-1} \right] \tag{21}$$

$$\eta_{k|k-1} = \frac{1}{\sqrt{m}}\left[X_{1,k|k-1} - \hat{x}_{k|k-1} \quad X_{2,k|k-1} - \hat{x}_{k|k-1} \ \cdots \ X_{m,k|k-1} - \hat{x}_{k|k-1} \right] \tag{22}$$

$$\lambda_k = \frac{\mathrm{tr}\left(C_{0,k} - \beta R_k \right)}{\mathrm{tr}\left(S_{zz,k|k-1}S_{zz,k|k-1}^{\mathrm{T}} \right)} \tag{23}$$

Where $C_{0,k} = \begin{cases} \gamma_k \gamma_k^T & k = 1 \\ \dfrac{\rho C_{0,k-1} + \gamma_k \gamma_k^T}{1+\rho} & k > 1 \end{cases}$, $\rho = 0.95$ is the weakening factor, it can improve the fast tracking capability of the filter.

Re-generate particles:

$$x_k^{(i)} \sim \pi\left[\hat{x}_k^{(i)} \mid x_k^{(i)}, z_k\right] = N\left[\hat{x}_k^{(i)}, P_k^{(i)}\right] \tag{24}$$

(3) Evaluation and normalization.

For $i = 1, 2, \cdots, N$, evaluate the importance weights:

$$w_k^{(i)} = \frac{P\left(z_k \mid \hat{x}_k^{(i)}\right) P\left(\hat{x}_k^{(i)} \mid x_{k-1}^{(i)}\right)}{\pi\left(\hat{x}_k^{(i)} \mid x_k^{(i)}, z_k\right)} \tag{25}$$

Normalize the importance weights:

$$\tilde{w}_k^{(i)} = w_k^{(i)} \Big/ \sum_{j=1}^{N} w_k^{(j)} \tag{26}$$

(4) Resample.

To monitor how bad is the weight degeneration, the so-called effective sample size, N_{eff}, is introduced in [10].

$$N_{eff} = 1 \Big/ \sum_{i=1}^{N} \left(\tilde{w}_k^{(i)}\right)^2 \tag{27}$$

Where $N_{threshold}$ ($N/2$ or $N/3$) is a predefined threshold, and when $N_{eff} \le N_{threshold}$, the resampling procedure is performed, and N new particles $x_k^{(j)}$ are generated, denoted as:

$$\left\{x_k^{(j)}, \tilde{w}_k^{(j)}\right\}, \quad \tilde{w}_k^{(j)} = \frac{1}{N} \tag{28}$$

(5) Output.

State estimating:

$$\hat{x}_k = \sum_{j=1}^{N} \tilde{w}_k^{(j)} x_k^{(j)} \tag{29}$$

Variance estimating:

$$P_k = \sum_{j=1}^{N} \tilde{w}_k^{(j)} \left(x_k^{(j)} - \hat{x}_{k|k}\right)\left(x_k^{(j)} - \hat{x}_{k|k}\right)^{\mathrm{T}} \tag{30}$$

3. Algorithm Verification and Analysis

The performance of the ISRCPF, SRCPF, CPF and PF algorithms is verified by two typical nonlinear non-Gaussian examples.

3.1. Nonlinear Non-Gaussian Time Sequence Estimate

State model:

$$X_{t+1} = 1 + \sin\left(4 \times 10^{-2} \pi t\right) + 0.5 X_t + v_t \tag{31}$$

Observation model:

$$Y_t = \begin{cases} 0.2X_t^2 + n_t, & t \le 30 \\ 0.5X_t - 2 + n_t, & t > 30 \end{cases} \tag{32}$$

Where system noise $v_t \sim \mathrm{Gamma}(3,2)$, and observation noise obey Gaussian distribution $n_t \sim N(0,1e-5)$. Simulation time of each Monte Carlo experiment $t = 100s$, number of particles $N = 200$, $P_0 = 3/4$. 200 times Monte Carlo simulation experiments are carried out, and the minimum square error (MSE) is used to compare the performance of the four algorithms.

$$\mathrm{MSE} = \frac{1}{t}\sum_{k=1}^{t}\left(X_k - X_{k|k}\right)^2 \tag{33}$$

The simulation results are shown in Figure 3 and Figure 4. Figure3 is the average estimation curve of state X. Figure 4 is the MSE of 200 times Monte Carlo simulations.

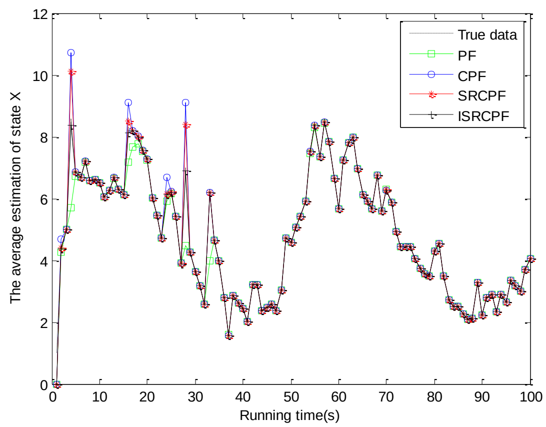


Figure 3. The Average Estimation Curve of State X

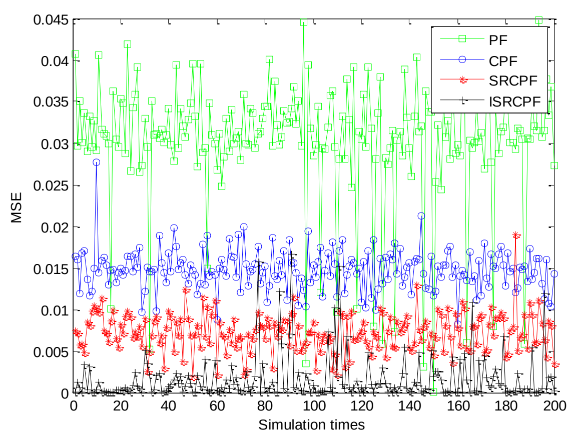


Figure 4. The MSE of 200 Monte Carlo Simulations

As can be seen from Figure 3 and Figure 4, in the nonlinear non-Gaussian system, the filtering accuracy of the ISRCPF, SRCPF and CPF algorithm is higher than the traditional PF. The ISRCPF, SRCKF and CKF three algorithms complete system state estimation through Gaussian approximation of the posterior probability density. However, the ISRCPF can ensure the good tracking capability by introducing the fading factor and weakening factor to online correct the gain matrix. The ISRCPF propagates square root factors of the mean and the covariance, preserves the symmetry and positive semi-definiteness of the covariance and improves numerical accuracy and stability. So filtering accuracy of the ISRCPF is higher than the other algorithms.

Table 1 shows the average running time of each algorithm. Figure 5 gives the MSE mean and MSE covariance. According to Table 1 and Figure 5, the MSE mean of the ISRCPF algorithm is the minimum. Running time of the PF is shortest, and the ISRCPF is shorter, while the SRCPF and CPF are longest.

Table 1. Comparison of Average Running Time

Algorithm	Average Running Time (s)
ISRCPF	7.1240
SRCPF	8.5176
CPF	8.7081
PF	1.6997

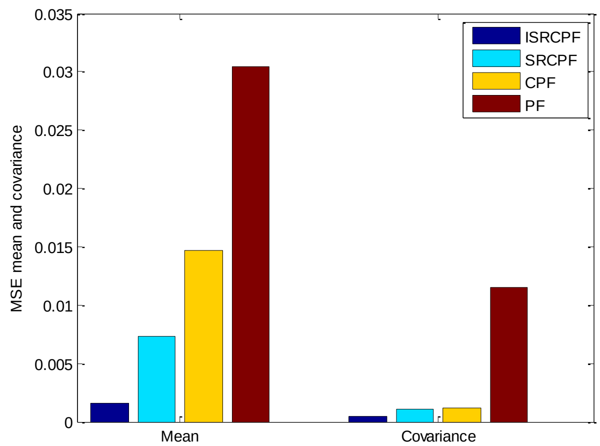


Figure 5. The MSE Mean and MSE Covariance

3.2. Bearing Tracking

Bearing tracking model is a two-dimension nonlinear model and its discrete model is as follows:

$$x_k = \begin{bmatrix} 0.9 & 0 \\ 0 & 1 \end{bmatrix} x_{k-1} + r_{k-1} \tag{34}$$

Where, state $x = \left[x(1) \quad x(2) \right]^{\mathrm{T}} = [s \quad t]^{\mathrm{T}}$, it presents location in s-t plane Cartesian coordinate system, system noise $r_k \sim N(0,Q)$, $Q = 0.01 \times [1 \quad 0.5; 0.5 \quad 1]$. Observer is located at $(\cos(k), \sin(k))$, and it observes the target observation with noise:

$$z_k = \tan^{-1}\left(\frac{t_k - \sin k}{s_k - \cos k}\right) + v_k \tag{35}$$

Where, observation noise is $v_k \sim N(0, R)$, $R = 0.05$.

Simulation parameters are set: Initial state $x_0 = [10 \;\; 2]^T$, initial covariance is $P_0 = [0.01 \;\; 0;0 \;\; 0.01]^T$, simulation time is 200 seconds, 100 Monte Carlo simulation tests are carried out under the same initial conditions using ISRCPF, SRCPF, CPF and PF respectively. Fig.6 and fig.7 give MSE of state for the four filter algorithms. Fig.8 shows MSE mean of $x(1)$ and $x(2)$ across 100 Monte Carlo runs. Through the simulation results, it can be seen that MSE mean of $x(1)$ and $x(2)$ obtained by the ISRCPF algorithm is obviously lower than that of other algorithms, so the filter precision is higher than other algorithms.

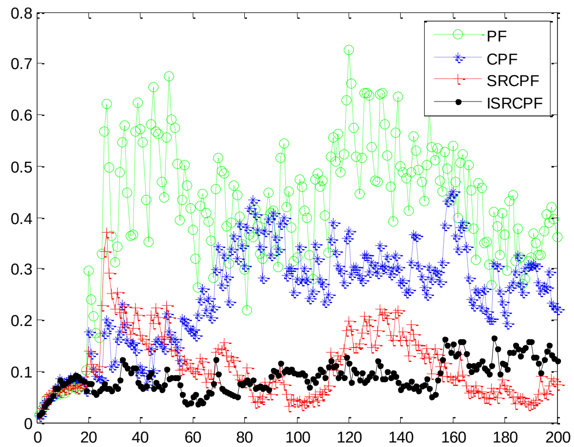


Figure 6. MSE of *x*(1)

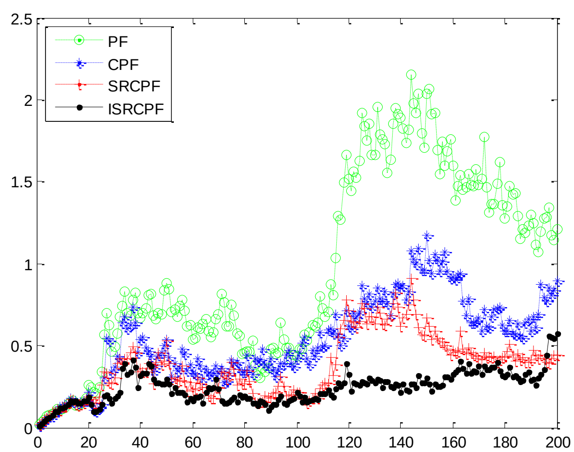
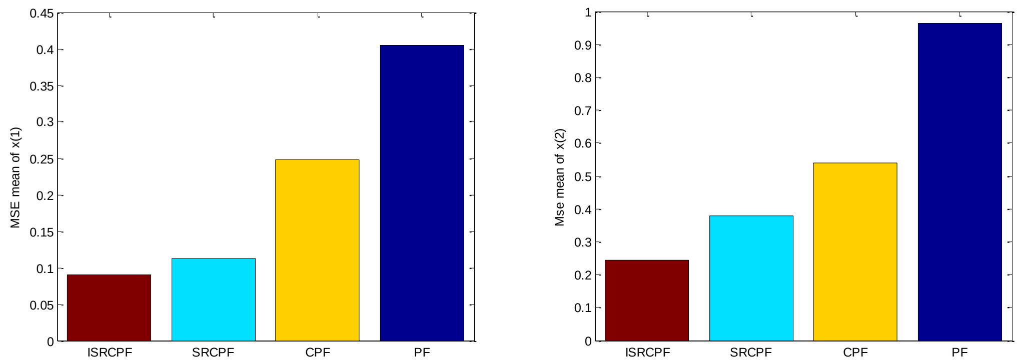


Figure 7. MSE of *x*(2)

According to the above analysis, the ISRCPF algorithm adopts the ISRCKF to develop the proposal distribution. Due to considering the current measurement, the

ISRCPF effectively reduces the estimating error, and therefore its filtering accuracy is higher than the other algorithms.



(a) MSE mean of *x*(1) **(b) MSE mean of *x*(2)**

Figure 8. MSE Mean across 100 Monte Carlo Runs

5. Conclusion

This paper presents a new ISRCPF filtering navigation method. First, an improved strong tracking SRCKF algorithm is proposed, the fading factor and weakening factor is introduced to real-time correct gain matrix and the filtering accuracy is increased. The ISRCPF adopts the ISRCKF to develop the proposal distribution, by incorporating the latest measurement information to prior distribution update phase, it effectively reduces the estimated error. The validation of the ISRCPF algorithm is verified through simulation experiments.

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