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Bayesian Analysis of Power Function Distribution Using Different Loss Functions

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Abstract

Power function distribution is a flexible lifetime distribution that may offer a good fit to some failure data sets. In this paper, we obtain Bayesian estimators of the shape parameter of Power function distribution. For the Posterior distribution of this parameter, we consider Exponential Prior, Pareto Prior, Chi-Square Prior, Quasi Prior and Extension of Jeffrey`s Prior. The three loss functions taken up are Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and Precautionary Loss Function (PLF). The performance of an estimator is assessed on the basis of its relative Posterior risk. Monte Carlo Simulations are used to compare the performance of the estimators. It is discovered that the PLF produces the least Posterior risk when Exponential and Pareto Priors are used. SELF is the best when Chi-Square, Quasi and Extension of Jeffrey`s Priors are used.

Keywords: *Power function distribution, Bayesian estimation, Loss function*

1. Introduction

The Power function distribution is a flexible life time distribution model that may offer a good fit to some sets of failure data. Theoretically Power function distribution is the inverse of Pareto distribution. Meniconi and Barry [19] discussed the application of Power function distribution. They proved that the Power function distribution is the best distribution to check the reliability of any electrical component. They used Exponential distribution, Lognornal distribution and Weibull distribution and showed from reliability and hazard function that Power function distribution is the best distribution.

The probability distribution of Power function distribution is

$$f(x) = \delta\, x^{\delta-1}; \qquad 0 < x < 1 \qquad (1)$$

With shape parameter δ, the interval (0, 1)

Hoffman [12] first time used this distribution to model the attenuation of wireless signals traversing multiple paths. Rider [24] derived Distributions of the product and quotients of the Order Statistics from a Power function distribution. Moments of Order Statistics for a Power function Distribution were calculated by Malik [18]. Lwin [17] discussed Bayesian estimation for the scale parameter of the Pareto Distribution using a Power function prior. Ahsanullah and Kabir [3] discussed the Estimation of the location and scale parameters of a Power function distribution. Cohen and Whitten [8] used the moment and Modified Moment Estimators for the Weibull Distribution. Samia and Mohammad [26] used five modifications of moments to estimate the parameters of the Pareto Distribution. Lalitha and Anand [16] used Modified Maximum Likelihood to estimate the scale parameter of the Rayleigh Distribution. Rafiq *et al.* [22] discussed the parameters of the Gamma Distribution. Kang and Young (1997) estimated the parameters of a Pareto Distribution by Jackknife and Bootstrap Methods. Rafiq *et al.* [23] discussed the method of Fractional Moments to estimate the parameters of Weibull Distribution. Lin

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and Yang [39] investigated and derived the statistical model of spatial-chromatic distribution of images. Through extensive evaluation of large image databases, they discovered that a two-parameter Nakagami distribution well suits the purpose. Abdi and Kaveh [2] have shown that this distribution is useful for modeling multipath faded envelope in wireless channels and also estimated the shape parameter of the distribution. Zhang [43] introduced a direct-sum decomposition principle and determined the statistical mapping between the correlated Nakagami process and a set of Gaussian vectors for its generation. A simple general procedure is derived for the generation of correlated Nakagami channels with arbitrary parameters. Cheng and Beaulieu [7] considered the maximum-likelihood estimation of the Nakagami shape parameter m. Two new estimators were proposed and examined. The sample mean and the sample variance of the new estimators were compared with the best reported estimator. The new estimators offered superior performance. Neil [20] estimated the parameters of Weibull Distribution with the help of percentiles. He called it Common Percentile Method. Shankar *et al.,* [29] and Tsui *et al.,* [33] use the Nakagami distribution to model ultrasound data in medical imaging studies. Tsui *et al.,* [34] showed that Nakagami parameter, estimated using ultrasonic back scattered envelopes, compressed by logarithmic computation denoted by m-log is more sensitive than the original Nakagami parameter m calculated using uncompressed envelopes for detecting the variations of scatter concentration in tissues. Kim and Latch man [15] used the Nakagami distribution in their analysis of multimedia. Sarkar *et al.,* [27] investigated the adequacy of this distribution to derive the Geomorphological Instaneous Unit Hydrographs (GIUH) along with two parameter Logistic, two parameter Weibull and two parameter Gamma distributions. They compared the results of Nakagami distribution with other existing approaches and found that this distribution based on GIUH can be good substitute to other existing approaches. Schwartz *et al.,* [28] developed and evaluated analytic and bootstrap bias-corrected maximum likelihood estimators for the shape parameter in the Nakagami distribution. It was found that both “corrective” and “preventive” analytic approaches to eliminating the bias are equally, and extremely, effective and simple to implement. Dey [9] obtained Bayes estimators for the unknown parameter of inverse Rayleigh distribution using squared error and Linex loss function. Kazmi *et al.,* [14] compared class of life time distributions for Bayesian analysis. They studied properties of Bayes estimators of the parameter using different loss functions via simulated and real life data. Feroze [10] discussed the Bayesian analysis of the scale parameter of inverse Gaussian distribution. Feroze and Aslam [11] found the Bayesian estimators of the scale parameter of Error function distribution. Different informative and non-informative Priors were used to derive the corresponding Posterior distribution. Ali *et al.* [5] discussed the scale parameter estimation of the Laplace model using different asymmetric loss functions. Yahgmaei [38] proposed classical and Bayesian approaches for estimating the scale parameter in the inverse Weibull distribution when shape parameter is known. The Bayes estimators for the scale parameter is derived in Inverse Weibull distribution, by considering Quasi, Gamma and uniform Priors under square error, entropy and precautionary loss function. Zaka and Akhter [41] derived the different estimation methods for the parameters of Power function distribution. Zaka and Akhter [40] discussed the different modifications of the parameter estimation methods and proved that the modified estimators appear better than the traditional maximum likelihood, moments and percentile estimators. Zaka and Akhter [43] derived the Bayes estimators using different loss functions for the scale parameter of Nakagami distribution.

In this paper, the posterior distribution for shape parameter δ of the Power function distribution is derived using Exponential, Pareto, Chi-Square as informative Priors and Quasi and Extension of Jeffrey`s as non-informative Priors. The paper is organized in the following sections. In Section 2, includes the derivation of the Posterior distributions under informative and non-informative Priors. Section 3 includes the derivation of Bayes estimators of shape parameter using different loss functions. Section 4 comprises of derivation of Posterior Risks under different loss functions and Priors. Section 5 is based on simulation results and some remarks are given in the last Section 6.

2. Posterior Distributions under the Assumption of Different Priors

In this chapter Bayesian estimators and Posterior risk are found for the scale parameter of Power function distribution under various loss functions using Exponential Prior. A comparison of these estimates is also made through Monte Carlo simulation by comparing the Posterior risk of SELF, QLF and PLF using different sample sizes.

Exponential Prior distribution relating to the shape parameter δ

$$P(\delta) = \gamma\, e^{-\gamma\delta} \qquad ; \quad 0 < \delta < \infty$$

The likelihood function of Power function distribution is

$$L(\delta|\underline{x}) = \delta^{n} \prod_{i=1}^{n} x_i^{\ \delta-1}$$

$$L(\delta|\underline{x}) \propto \delta^{n}\, e^{-\delta \sum_{i=1}^{n} \ln x^{-1}}$$

The Posterior distribution of shape parameter δ using Exponential Prior is

$$P(\delta|\underline{x}) = \frac{P(B)L(\delta|\underline{x})}{\int_0^{\infty} P(\delta)L(\delta|\underline{x})d\delta}$$

$$P(\delta|\underline{x}) = \frac{\delta^{(n+1)-1}\ e^{-\delta(\gamma+\sum_{i=1}^{n}\ln x^{-1})}(\gamma+\sum_{i=1}^{n}\ln x^{-1})^{n+1}}{\Gamma(n+1)} \qquad (2)$$

Now we use Pareto Prior, Chi-Square Prior, Quasi Prior as informative Priors and Extension of Jeffrey`s Prior as non-informative Prior because they are compatible with the parameter δ of the Power function distribution. Similarly Posterior distributions using informative and non-informative Priors for the parameter δ of the Power function distribution are derived below:

Pareto Prior relating to the shape parameter δ is:

$$P(\delta) = c\, a^{c}\, \delta^{c-1} \qquad\qquad 0 < \delta < \infty$$

Then the Posterior distribution of shape parameter δ using Pareto Prior is

$$P(\delta|\underline{x}) = \frac{\delta^{n-c-1}\ e^{-\delta\sum_{i=1}^{n}\ln x^{-1}}(\sum_{i=1}^{n}\ln x^{-1})^{n-c}}{\Gamma(n-c)} \qquad (3)$$

The Chi-Square Prior relating to the shape parameter δ is given as

$$P(\delta) = \frac{\delta^{v-2/2}\, e^{-\delta/2}}{2^{v/2}\,\Gamma(v/2)} \qquad ; \quad 0 < \delta < \infty$$

Similarly the Posterior distribution of shape parameter δ using Chi-Square Prior is

$$P(\delta|\underline{x}) = \frac{(\delta)^{n+(v-2/2)}\ e^{-\delta(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1})}\ (\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1})^{n+v/2}}{\Gamma(n+v/2)} \qquad (4)$$

Quasi Prior relating to the shape parameter δ is given by

$$P(\delta) = 1/\delta^{d} \ ;\ 0 < \delta < \infty,\ 0 < d < \infty$$

Similarly the Posterior distribution of shape parameter δ using Quasi Prior is

$$P(\delta|\underline{x}) = \frac{(\delta)^{n-d}\ e^{-\delta(\sum_{i=1}^{n}\ln x^{-1})}\ (\sum_{i=1}^{n}\ln x^{-1})^{n-d+1}}{\Gamma(n-d+1)} \qquad (5)$$

The Extension of Jeffrey`s Prior relating to the shape parameter δ is given by

$$P(\delta) \propto I(\delta)^{c} \quad ; c \in R^{+} \text{ and } 0 < \delta < \infty$$

$$P(\delta) = k\frac{n^{c}}{\delta^{2c}}$$

Similarly the Posterior distribution of shape parameter δ using Extension of Jeffrey`s Prior is

$$P(\delta|\underline{x}) = \frac{(\delta)^{n\eta-2c} \; e^{-\delta(\sum_{i=1}^{n}\ln x^{-1})} \, (\sum_{i=1}^{n}\ln x^{-1})^{n-2c+1}}{\Gamma(n-2c+1)} \tag{6}$$

3. Bayesian Estimation under Three Loss Functions

In statistics and decision theory a loss function is a function that maps an event into a real number intuitively representing some "cost" associated with the event. Typically it is used for parameter estimation, and the event in question is some function of the difference between estimated and true values for an instance of data. The use of above lemma is made for the derivation of results.

3.1. Squared Error Loss Function (SELF)

The squared error loss function proposed by Legendre (1805) and Gauss (1810) is defined as:

$$L(\delta, \delta_{SELF}) = (\delta - \delta_{SELF})^{2}$$

The derivation of Bayes estimator under SELF using Exponential Prior is given below:

$$\delta_{SELF} = E(\delta|\underline{x})$$

$$E(\delta|\underline{x}) = \int_{0}^{\infty} \delta \frac{\delta^{(n+1)-1} \; e^{-\delta(\gamma+\sum_{i=1}^{n}\ln x^{-1})}(\gamma+\sum_{i=1}^{n}\ln x^{-1})^{n+1}}{\Gamma(n+1)} \, d\delta$$

$$\delta_{SELF} = \frac{(n+1)}{(\Upsilon+\sum_{i=1}^{n}\ln x^{-1})}$$

3.2. Quadratic Loss Function (QLF)

A quadratic loss function is defined as:

$$\eta(x) = C\,(t-x)^{2}$$

for some constant C, the value of the constant makes no difference to a decision, and can be ignored by setting it equal to 1.
The quadratic loss function can also be defined as

$$L(\delta, \delta_{QLF}) = \left(\frac{\delta-\delta_{QLF}}{\delta}\right)^{2}$$

The Bayes estimator under QLF using Exponential Prior is

$$\delta_{QLF} = \frac{E(\delta^{-1}|\underline{x})}{E(\delta^{-2}|\underline{x})}$$

$$E(\delta^{-1}|\underline{x}) = \int_{0}^{\infty} \delta^{-1} P(\delta|\underline{x}) \, d\delta$$

$$E(\delta^{-1}|\underline{x}) = \int_{0}^{\infty} \delta^{-1} \frac{\delta^{(n+1)-1} \; e^{-\delta(\gamma+\sum_{i=1}^{n}\ln x^{-1})}(\gamma+\sum_{i=1}^{n}\ln x^{-1})^{n+1}}{\Gamma(n+1)} \, d\delta$$

$$E(\delta^{-1}|\underline{x}) = \frac{(\gamma+\sum_{i=1}^{n}\ln x^{-1})}{n}$$

Now

$$E(\delta^{-2}|\underline{x}) = \int_0^\infty \delta^{-2}\, P(\delta|\underline{x})d\delta$$

$$E(\delta^{-2}|\underline{x}) = \int_0^\infty \delta^{-2}\, \frac{\delta^{(n+1)-1}\ \ e^{-\delta(\gamma+\sum_{i=1}^{n}\ln x^{-1})}(\gamma+\sum_{i=1}^{n}\ln x^{-1})^{n+1}}{\Gamma(n+1)}\, d\delta$$

$$E(\delta^{-2}|\underline{x}) = \frac{(\gamma+\sum_{i=1}^{n}\ln x^{-1})^2}{n(n-1)}$$

$$\delta_{QLF} = \frac{(n-1)}{(\gamma+\sum_{i=1}^{n}\ln x^{-1})} \qquad (7)$$

3.3. Precautionary Loss Function (PLF)

Norstrom (1996) introduced an asymmetric precautionary loss function (PLF) which can be presented as:

$$L\ (\delta_{PLF}, \delta) = \frac{(\delta_{PLF} - \delta)^2}{\delta}$$

Similarly the Bayes estimator under PLF using Exponential Prior is derived as:

$$\delta_{PLF} = \{E(\delta^2|\underline{x})\}^{1/2}$$

$$\{E(\delta^2|\underline{x})\}^{1/2} = \sqrt{\int_0^\infty \delta^2\, \frac{\delta^{(n+1)-1}\ \ e^{-\delta(\gamma+\sum_{i=1}^{n}\ln x^{-1})}(\gamma+\sum_{i=1}^{n}\ln x^{-1})^{n+1}}{\Gamma(n+1)}\, d\delta}$$

$$\delta_{PLF} = \{E(\delta^2|\underline{x})\}^{1/2} = \sqrt{\frac{(n+2)(n+1)}{(\gamma+\sum_{i=1}^{n}\ln x^{-1})^2}} \qquad (8)$$

4. Posterior Risks under Different Loss Functions

The Posterior risks of the Bayes estimator under different Loss functions using Exponential Prior is:

Using Square Error Loss function (SELF):

$$P(\delta_{SELF}) = E\ (\delta^2|\underline{x}) - \{E(\delta|\underline{x})\}^2$$

$$P(\delta_{SELF}) = \frac{1}{(\gamma+\sum_{i=1}^{n}\ln x^{-1})^2}\{(n+2)(n+1)-(n+1)^2\}$$

Quadratic Loss function (QLF):

$$P(\delta_{QLF}) = 1 - \frac{\{E(\delta^{-1}|\underline{x})\}^2}{E(\delta^{-2}|\underline{x})}$$

$$P(\delta_{QLF}\ ) = 1 - \frac{(\Gamma(n))^2}{\Gamma(n-1)\Gamma(n+1)} \qquad (9)$$

Precautionary Loss Function (PLF):

$$P(\delta_{PLF}) = 2\,\{\delta_{PLF} - E\ (\delta|\underline{x})\}$$

$$P(\delta_{PLF}\ ) = 2\left(\sqrt{\frac{(n+2)(n+1)}{(\gamma+\sum_{i=1}^{n}\ln x^{-1})^2}} - \frac{(n+1)}{(\gamma+\sum_{i=1}^{n}\ln x^{-1})}\right) \qquad (10)$$

Similarly the expressions for Bayes Estimators and Posterior Risks under Pareto Prior, Chi-Square Prior, Quasi Prior and Extension of Jeffrey`s Prior can be derived in a similar manner.

Table 1. Bayes Estimators under Pareto, Chi-Square, Quasi and Extension of Jeffrey`s Priors, using different Loss Functions

Priors	SELF	QLF	PLF
Pareto	$\frac{(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})}$	$\frac{(n-c-2)}{(\sum_{i=1}^{n}\ln x^{-1})^2}$	$\sqrt{\frac{(n-c+1)(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})^2}}$
Chi-Square	$(n-v/2)\left(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1}\right)$	$\frac{\left(n-\frac{v-2}{2}-1\right)}{\left(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1}\right)}$	$\sqrt{\frac{\left(n-\frac{v}{2}+1\right)\left(n-\frac{v}{2}\right)}{(\sum_{i=1}^{n}\ln x^{-1})^2}}$
Quasi	$\frac{(n-d+1)}{(\sum_{i=1}^{n}\ln x^{-1})}$	$\frac{(n-d-1)}{(\sum_{i=1}^{n}\ln x^{-1})}$	$\sqrt{\frac{(n-d+2)(n-d+1)}{(\sum_{i=1}^{n}\ln x^{-1})^2}}$
Extension of Jeffrey`s	$\frac{(n-2c+1)}{(\sum_{i=1}^{n}\ln x^{-1})}$	$\frac{(n-2c-1)}{(\sum_{i=1}^{n}\ln x^{-1})}$	$\sqrt{\frac{(n-2c+2)(n-2c+1)}{(\sum_{i=1}^{n}\ln x^{-1})^2}}$

Table 2. Posterior Risks under Inverse Chi-Square, Jeffrey`s, Extension of Jeffrey`s and Quasi Priors, using Different Loss Functions

Priors	SELF	QLF	PLF
Pareto	$\frac{(n-c+1)(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})^2} - \left\{\frac{(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})}\right\}^2$	$1-\frac{\{\Gamma(n-c-1)\}^2}{\Gamma(n-c)\Gamma(n-c-2)}$	$2\sqrt{\frac{(n-c+1)(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})^2}-\frac{(n-c)}{(\sum_{i=1}^{n}\ln x^{-1})}}$
Chi-Square	$\frac{1}{\Gamma(n+v/2)(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1})^2}\left[\Gamma\left(n+\frac{v}{2}+2\right)-\frac{\left\{\Gamma(n+\frac{v}{2}+1)\right\}^2}{\Gamma(n+\frac{v}{2})}\right]$	$1-\frac{\left\{\Gamma(n+\frac{v}{2}-1)\right\}^2}{\Gamma(n+\frac{v}{2})\Gamma(n+\frac{v}{2}-2)}$	$2\sqrt{\frac{\Gamma\left(n+\frac{v}{2}+2\right)}{\Gamma(n+\frac{v}{2})(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1})^2}-\frac{\Gamma\left(n+\frac{v}{2}+1\right)}{\Gamma\left(n+\frac{v}{2}\right)}(\frac{1}{2}+\sum_{i=1}^{n}\ln x^{-1})}$
Quasi	$\frac{\Gamma(n-d+3)}{\Gamma(n-d+1)(\sum_{i=1}^{n}\ln x^{-1})^2}-\left\{\frac{\Gamma(n-d+2)}{\Gamma(n-d+1)}\right\}^2\frac{1}{(\sum_{i=1}^{n}\ln x^{-1})^2}$	$1-\left\{\frac{(\Gamma(n-d))^2}{\Gamma(n-d+1)\Gamma(n-d-1)}\right\}$	$2\left\{\sqrt{\frac{\Gamma(n-d+3)}{\Gamma(n-d+1)(\sum_{i=1}^{n}\ln x^{-1})^2}}-\frac{\Gamma(n-d+2)}{\Gamma(n-d+1)\sum_{i=1}^{n}\ln x^{-1}}\right\}$
Extension of Jeffrey`s	$\frac{1}{\Gamma(n-2c+1)(\sum_{i=1}^{n}\ln x^{-1})^2}\left[\Gamma(n-2c+3)-\frac{\{\Gamma(n-2c+2)\}^2}{\Gamma(n-2c+1)}\right]$	$1-\left\{\frac{(\Gamma(n-2c))^2}{\Gamma(n-2c-1)\Gamma(n-2c+1)}\right\}$	$2\left\{\sqrt{\frac{\Gamma(n-2c+3)}{\Gamma(n-2c+1)(\sum_{i=1}^{n}\ln x^{-1})^2}}-\frac{\Gamma(n-2c+2)}{\Gamma(n-2c+1)\sum_{i=1}^{n}\ln x^{-1}}\right\}$

5. Simulation Study

Using Easy fit Software, we have generated 5,000 Random numbers from Power function Distribution with different values of Parameter δ. A program has been developed in R language to obtain the Bayesian Estimates and Posterior Risks under 10,000 replications and averages of 10,000 outputs has been presented in the tables below.

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Table 3. Posterior Risk of Exponential Prior using SELF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.40611	1.69900	3.66441	6.40664	0.11891	0.46914	1.05197	1.82929	0.06131	0.24248	0.54152	0.97160
20	0.01866	0.07835	0.16734	0.29181	0.01122	0.04392	0.09954	0.17026	0.00759	0.02988	0.07090	0.12133
40	0.00750	0.03140	0.06781	0.11781	0.00484	0.01894	0.04297	0.07404	0.00342	0.01358	0.03185	0.05435
100	0.00265	0.01111	0.02394	0.04167	0.00179	0.00697	0.01582	0.02733	0.00129	0.00510	0.01194	0.02048
150	0.00172	0.00718	0.01553	0.02705	0.00117	0.00457	0.01034	0.01791	0.00085	0.00335	0.00786	0.01349
250	0.00101	0.00424	0.00913	0.01583	0.00069	0.00271	0.00613	0.01059	0.00050	0.00199	0.00466	0.00802
400	0.00062	0.00262	0.00562	0.00979	0.00043	0.00168	0.00380	0.00657	0.00031	0.00124	0.00290	0.00498

Table 4. Posterior Risk of Exponential Prior using QLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.20000	0.20000	0.20000	0.20000	0.30000	0.30000	0.30000	0.30000	0.40000	0.40000	0.40000	0.40000
20	0.05000	0.05000	0.05000	0.05000	0.07500	0.07500	0.07500	0.07500	0.10000	0.10000	0.10000	0.10000
40	0.02500	0.02500	0.02500	0.02500	0.03750	0.03750	0.03750	0.03750	0.05000	0.05000	0.05000	0.05000
100	0.01000	0.01000	0.01000	0.01000	0.01500	0.01500	0.01500	0.01500	0.02000	0.02000	0.02000	0.02000
150	0.00667	0.00667	0.00667	0.00667	0.01000	0.01000	0.01000	0.01000	0.01333	0.01333	0.01333	0.01333
250	0.00400	0.00400	0.00400	0.00400	0.00600	0.00600	0.00600	0.00600	0.00800	0.00800	0.00800	0.00800
400	0.00250	0.00250	0.00250	0.00250	0.00375	0.00375	0.00375	0.00375	0.00500	0.00500	0.00500	0.00500

Table 5. Posterior Risk using of Exponential Prior PLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.18499	0.37804	0.55558	0.73513	0.07254	0.14390	0.21557	0.28454	0.04315	0.08586	0.12818	0.17166
20	0.01595	0.03267	0.04771	0.06303	0.00993	0.01965	0.02960	0.03870	0.00703	0.01394	0.02149	0.02808
40	0.00699	0.01429	0.02100	0.02768	0.00455	0.00900	0.01356	0.01779	0.00330	0.00658	0.01008	0.01316
100	0.00260	0.00531	0.00780	0.01029	0.00174	0.00343	0.00516	0.00678	0.00127	0.00253	0.00388	0.00508
150	0.00170	0.00348	0.00511	0.00675	0.00115	0.00226	0.00340	0.00448	0.00084	0.00168	0.00257	0.00336

250	0.00101	0.00206	0.00303	0.00399	0.00068	0.00135	0.00203	0.00266	0.00050	0.00100	0.00153	0.00201
400	0.00063	0.00128	0.00188	0.00248	0.00042	0.00084	0.00126	0.00166	0.00031	0.00062	0.00095	0.00125

Table 6. Posterior Risk of Pareto Prior Using SELF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.13156	0.42026	0.81624	1.34948	0.06717	0.22104	0.45190	0.76187	0.04214	0.14237	0.30096	0.52484
20	0.01623	0.06209	0.13018	0.22165	0.01027	0.03772	0.08378	0.14320	0.00716	0.02714	0.05926	0.10643
40	0.00708	0.02820	0.05974	0.10411	0.00467	0.01776	0.03975	0.06768	0.00334	0.01289	0.02869	0.05125
100	0.00260	0.01067	0.02282	0.03946	0.00176	0.00681	0.01533	0.02650	0.00128	0.00499	0.01113	0.01999
150	0.00170	0.00700	0.01503	0.02614	0.00116	0.00449	0.01016	0.01749	0.00084	0.00332	0.00738	0.01328
250	0.00100	0.00416	0.00894	0.01550	0.00069	0.00267	0.00606	0.01045	0.00050	0.00197	0.00441	0.00793
400	0.00062	0.00259	0.00556	0.00967	0.00043	0.00167	0.00377	0.00651	0.00031	0.00123	0.00275	0.00495

Table 7. Posterior Risk of Pareto Prior Using QLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.14286	0.14286	0.14286	0.14286	0.10526	0.10526	0.10526	0.10526	0.08333	0.08333	0.08333	0.08333
20	0.04545	0.04545	0.04545	0.04545	0.03125	0.03125	0.03125	0.03125	0.02381	0.02381	0.02381	0.02381
40	0.02381	0.02381	0.02381	0.02381	0.01613	0.01613	0.01613	0.01613	0.01220	0.01220	0.01220	0.01220
100	0.00980	0.00980	0.00980	0.00980	0.00658	0.00658	0.00658	0.00658	0.00495	0.00495	0.00495	0.00495
150	0.00658	0.00658	0.00658	0.00658	0.00441	0.00441	0.00441	0.00441	0.00331	0.00331	0.00331	0.00331
250	0.00397	0.00397	0.00397	0.00397	0.00265	0.00265	0.00265	0.00265	0.00199	0.00199	0.00199	0.00199
400	0.00249	0.00249	0.00249	0.00249	0.00166	0.00166	0.00166	0.00166	0.00125	0.00125	0.00125	0.00125

Table 8. Posterior Risk of Pareto Prior Using PLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.16331	0.28662	0.39672	0.50807	0.09424	0.16892	0.24093	0.31162	0.06440	0.11782	0.17067	0.22490
20	0.02827	0.05523	0.07990	0.10422	0.01839	0.03520	0.05244	0.06856	0.01331	0.02591	0.03827	0.05128
40	0.01324	0.02641	0.03842	0.05076	0.00879	0.01714	0.02563	0.03346	0.00644	0.01266	0.01888	0.02523
100	0.00509	0.01031	0.01507	0.01982	0.00342	0.00673	0.01009	0.01327	0.00252	0.00499	0.00745	0.00998
150	0.00336	0.00682	0.00999	0.01318	0.00227	0.00446	0.00671	0.00881	0.00167	0.00332	0.00496	0.00665
250	0.00200	0.00407	0.00597	0.00787	0.00136	0.00267	0.00402	0.00528	0.00100	0.00199	0.00297	0.00398
400	0.00124	0.00254	0.00373	0.00491	0.00085	0.00167	0.00250	0.00329	0.00062	0.00124	0.00185	0.00249

Table 9. Posterior Risk of Chi Square Prior Using SELF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.1090538	0.3954049	0.8107759	1.3598053	0.0593121	0.2081091	0.4573932	0.7657788	0.0378343	0.1370435	0.3009123	0.5326040
20	0.0151628	0.0602440	0.1286693	0.2218914	0.0097751	0.0372344	0.0832861	0.1432424	0.0069055	0.0268225	0.0590551	0.1061109
40	0.0068259	0.0280332	0.0594092	0.1034094	0.0045598	0.0175287	0.0396436	0.0683404	0.0032760	0.0127859	0.0284641	0.0508981
100	0.0025640	0.0106305	0.0227984	0.0396063	0.0017421	0.0068008	0.0153221	0.0264677	0.0012638	0.0049862	0.0111613	0.0200508
150	0.0016792	0.0069988	0.0150071	0.0260399	0.0011538	0.0044732	0.0101464	0.0175292	0.0008381	0.0032983	0.0073752	0.0132675
250	0.0009956	0.0041539	0.0089393	0.0155376	0.0006876	0.0026724	0.0060529	0.0104406	0.0005009	0.0019730	0.0044107	0.0079316
0.0027479	0.0049423	0.0025888	0.0055501	0.0096610	0.0042779	0.0016630	0.0037684	0.0065030	0.0031113	0.0012303		

Table 10. Posterior Risk of Chi Square Prior Using QLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.1538462	0.1538462	0.1538462	0.1538462	0.1111111	0.1111111	0.1111111	0.1111111	0.0869565	0.0869565	0.0869565	0.0869565
20	0.0465116	0.0465116	0.0465116	0.0465116	0.0317460	0.0317460	0.0317460	0.0317460	0.0240964	0.0240964	0.0240964	0.0240964
40	0.0240964	0.0240964	0.0240964	0.0240964	0.0162602	0.0162602	0.0162602	0.0162602	0.0122699	0.0122699	0.0122699	0.0122699
100	0.0098522	0.0098522	0.0098522	0.0098522	0.0066007	0.0066007	0.0066007	0.0066007	0.0049628	0.0049628	0.0049628	0.0049628
150	0.0066007	0.0066007	0.0066007	0.0066007	0.0044150	0.0044150	0.0044150	0.0044150	0.0033168	0.0033168	0.0033167	0.0033168
250	0.0039761	0.0039761	0.0039761	0.0039761	0.0026560	0.0026560	0.0026560	0.0026560	0.0019940	0.0019940	0.0019940	0.0019940
400	0.0024907	0.0024907	0.0024907	0.0024907	0.0016625	0.0016625	0.0016625	0.0016625	0.0012477	0.0012477	0.0012477	0.0012477

Table 11. Posterior Risk of Chi Square Prior Using PLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5
5	0.0833548	0.1572303	0.2236121	0.2890551	0.0292551	0.0542917	0.0803220	0.1037656	0.0143056	0.0271074	0.0400629
20	0.0029490	0.0058712	0.0085753	0.0112670	0.0012581	0.0024535	0.0036677	0.0048102	0.0006790	0.0013380	0.0019849
40	0.0006750	0.0013677	0.0019903	0.0026256	0.0002969	0.0005819	0.0008753	0.0011490	0.0001625	0.0003212	0.0004792
100	0.0001026	0.0002089	0.0003059	0.0004031	0.0000458	0.0000905	0.0001359	0.0001786	0.0000253	0.0000503	0.0000752
150	0.0000450	0.0000919	0.0001345	0.0001772	0.0000202	0.0000399	0.0000600	0.0000789	0.0000112	0.0000222	0.0000332
250	0.0000160	0.0000328	0.0000481	0.0000634	0.0000072	0.0000143	0.0000215	0.0000282	0.0000040	0.0000080	0.0000119
400	0.0000062	0.0000128	0.0000187	0.0000001	0.0000028	0.0000056	0.0000084	0.0000110	0.0000016	0.0000031	0.0000046

Table 12. Posterior Risk of Extension of Jeffery Prior Using SELF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.07357	0.23088	0.45226	0.75718	0.04584	0.14605	0.30576	0.51521	0.03145	0.10637	0.22547	0.39533
20	0.01405	0.05332	0.11293	0.19288	0.00926	0.03398	0.07617	0.12908	0.00659	0.02511	0.05531	0.09808
40	0.00658	0.02613	0.05582	0.09595	0.00449	0.01684	0.03776	0.06469	0.00321	0.01238	0.02749	0.04940
100	0.00252	0.01032	0.02215	0.03834	0.00173	0.00667	0.01503	0.02587	0.00126	0.00492	0.01099	0.01970

150	0.00166	0.00688	0.01478	0.02565	0.00115	0.00435	0.01000	0.01727	0.00083	0.00328	0.00730	0.01314
250	0.00099	0.00411	0.00882	0.01538	0.00068	0.00265	0.00601	0.01038	0.00050	0.00196	0.00439	0.00787
400	0.00062	0.00257	0.00551	0.00961	0.00043	0.00166	0.00376	0.00648	0.00031	0.00123	0.00274	0.00493

Table 13. Posterior Risk of Extension of Quasi Prior Using QLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.12500	0.12500	0.12500	0.12500	0.09524	0.09524	0.09524	0.09524	0.07692	0.07692	0.07692	0.07692
20	0.04348	0.04348	0.04348	0.04348	0.03030	0.03030	0.03030	0.03030	0.02326	0.02326	0.02326	0.02326
40	0.02326	0.02326	0.02326	0.02326	0.01587	0.01587	0.01587	0.01587	0.01205	0.01205	0.01205	0.01205
100	0.00971	0.00971	0.00971	0.00971	0.00654	0.00654	0.00654	0.00654	0.00493	0.00493	0.00493	0.00493
150	0.00654	0.00654	0.00654	0.00654	0.00439	0.00439	0.00439	0.00439	0.00330	0.00330	0.00330	0.00330
250	0.00395	0.00395	0.00395	0.00395	0.00265	0.00265	0.00265	0.00265	0.00199	0.00199	0.00199	0.00199
400	0.00248	0.00248	0.00248	0.00248	0.00166	0.00166	0.00166	0.00166	0.00125	0.00125	0.00125	0.00125

Table 14. Posterior Risk of Extension of Quasi Prior Using PLF

n	δ = 0.5	δ = 1	δ = 1.5	δ = 2	δ = 2.5	δ = 3	δ = 3.5	δ = 4	δ = 4.5	δ = 5	δ = 5.5	δ = 6
5	0.11035	0.19220	0.26709	0.34454	0.07281	0.12847	0.18555	0.23997	0.05298	0.09683	0.14037	0.18552
20	0.02566	0.04991	0.07254	0.09480	0.01718	0.03288	0.04917	0.06403	0.01262	0.02462	0.03651	0.04861
40	0.01260	0.02511	0.03668	0.04809	0.00855	0.01655	0.02478	0.03243	0.00628	0.01233	0.01837	0.02462
100	0.00499	0.01009	0.01477	0.01944	0.00338	0.00664	0.00996	0.01307	0.00250	0.00494	0.00738	0.00989
150	0.00331	0.00674	0.00988	0.01301	0.00225	0.00439	0.00664	0.00873	0.00166	0.00330	0.00492	0.00660

250	0.00198	0.00404	0.00592	0.00782	0.00135	0.00265	0.00400	0.00525	0.00100	0.00198	0.00296	0.00396
400	0.00124	0.00253	0.00370	0.00489	0.00084	0.00166	0.00250	0.00328	0.00062	0.00124	0.00185	0.00248

Table 15. Posterior Risk of Extension of Jeffrey`s Prior Using SELF

n	**δ = 0.5**	**δ = 1**	**δ = 1.5**	**δ = 2**	**δ = 2.5**	**δ = 3**	**δ = 3.5**	**δ = 4**	**δ = 4.5**	**δ = 5**	**δ = 5.5**	**δ = 6**
5	0.14143	0.50466	1.03366	1.77374	0.06905	0.24127	0.52286	0.89673	0.04216	0.15389	0.33534	0.59664
20	0.01610	0.06388	0.13725	0.23614	0.01012	0.03827	0.08669	0.14763	0.00704	0.02744	0.06092	0.10845
40	0.00702	0.02856	0.06147	0.10609	0.00469	0.01786	0.04027	0.06916	0.00331	0.01294	0.02884	0.05194
100	0.00259	0.01069	0.02302	0.03990	0.00176	0.00683	0.01542	0.02657	0.00127	0.00500	0.01120	0.02010
150	0.00169	0.00704	0.01517	0.02635	0.00116	0.00450	0.01017	0.01758	0.00084	0.00332	0.00739	0.01331
250	0.00100	0.00417	0.00896	0.01563	0.00069	0.00267	0.00608	0.01049	0.00050	0.00198	0.00442	0.00793
400	0.00062	0.00259	0.00556	0.00971	0.00043	0.00167	0.00378	0.00652	0.00031	0.00123	0.00276	0.00496

Table 16. Posterior Risk of Jeffrey`s Using QLF

n	**δ = 0.5**	**δ = 1**	**δ = 1.5**	**δ = 2**	**δ = 2.5**	**δ = 3**	**δ = 3.5**	**δ = 4**	**δ = 4.5**	**δ = 5**	**δ = 5.5**	**δ = 6**
5	0.15385	0.15385	0.15385	0.15385	0.11111	0.11111	0.11111	0.11111	0.08696	0.08696	0.08696	0.08696
20	0.04651	0.04651	0.04651	0.04651	0.03175	0.03175	0.03175	0.03175	0.02410	0.02410	0.02410	0.02410
40	0.02410	0.02410	0.02410	0.02410	0.01626	0.01626	0.01626	0.01626	0.01227	0.01227	0.01227	0.01227
100	0.00985	0.00985	0.00985	0.00985	0.00660	0.00660	0.00660	0.00660	0.00496	0.00496	0.00496	0.00496
150	0.00660	0.00660	0.00660	0.00660	0.00442	0.00442	0.00442	0.00442	0.00332	0.00332	0.00332	0.00332

250	0.00398	0.00398	0.00398	0.00398	0.00266	0.00266	0.00266	0.00266	0.00199	0.00199	0.00199	0.00199
400	0.00249	0.00249	0.00249	0.00249	0.00166	0.00166	0.00166	0.00166	0.00125	0.00125	0.00125	0.00125

Table 17. Posterior Risk of Jeffrey`s Prior Using PLF

n	**δ = 0.5**	**δ = 1**	**δ = 1.5**	**δ = 2**	**δ = 2.5**	**δ = 3**	**δ = 3.5**	**δ = 4**	**δ = 4.5**	**δ = 5**	**δ = 5.5**	**δ = 6**
5	0.17665	0.32975	0.46983	0.61469	0.09824	0.18209	0.26809	0.35007	0.06586	0.12535	0.18442	0.24566
20	0.02848	0.05668	0.08300	0.10889	0.01840	0.03576	0.05378	0.07021	0.01328	0.02622	0.03904	0.05208
40	0.01326	0.02675	0.03922	0.05152	0.00884	0.01726	0.02591	0.03395	0.00644	0.01272	0.01899	0.02548
100	0.00509	0.01034	0.01517	0.01998	0.00342	0.00675	0.01014	0.01331	0.00252	0.00500	0.00748	0.01003
150	0.00336	0.00685	0.01006	0.01325	0.00227	0.00447	0.00672	0.00884	0.00167	0.00333	0.00496	0.00666
250	0.00200	0.00408	0.00599	0.00791	0.00135	0.00267	0.00402	0.00529	0.00100	0.00199	0.00297	0.00398
400	0.00125	0.00255	0.00373	0.00493	0.00084	0.00167	0.00251	0.00330	0.00062	0.00124	0.00186	0.00249

6. Summary and Conclusions

The salient results of this analysis are as follow.

Sample size

The Posterior risk based on all Priors and for all loss functions, relating to the shape parameter of a Power function distribution, expectedly decreases with the increase in sample size.

SELF

1. Using the Exponential Prior, the Posterior Risk increases with increase in the value of δ. At the same level of δ, the Posterior risk decreases with for a Power function distribution with decrease in the value of δ.
2. When using the Extension of Jeffrey`s, Chi-Square, and Quasi Priors, then for the same scale parameter, the Posterior Risk increases if a Bayesian estimator of a larger δ is required. At the least values of δ, the Posterior Risk decreases.
3. In the case of SELF using Extension of Jeffrey`s Prior after checking the effect of hyper parameter taking values c=2 & c=3, for the fixed scale parameter value, it is found that the Posterior risk increases when a Bayesian estimator of a larger shape

parameter is needed. For the same unknown δ value, the Posterior risk decreases for a Power function distribution with a larger scale parameter.
4. The performance of loss function is dependent on the values of δ jointly. Using all Priors, the Posterior Risk is inversely proportional to the choice of values of the δ

QLF

1. The Posterior Risk using Exponential Prior is independent of the parameter δ, but it tends to increase for larger values of the parameter δ of Power function distribution.
2. The Posterior Risk after checking the effect of hyper parameter using Chi Square, Extension of Jeffrey`s and Quasi Priors is also free of δ, but for the fixed scale parameter, the Posterior risk decreases with increase in δ.

PLF

1. With Inverse Gamma Prior the Posterior Risk increases when δ increases and η is kept constant. In situations when scale parameter increases and δ is held, the Posterior risk decreases.
2. Using Chi Square, Extension of Jeffrey`s and Quasi Priors and after checking the effect of hyper parameter Posterior Risk decreases when δ increases and scale parameter is constant. In situations when η increases, the Posterior risk decreases whatever δ may be.
3. The performance of loss function is dependent on scale parameter and δ jointly.

SELF/QLF/PLF

1. The PLF shows least Posterior Risk when Exponential and Pareto Priors are used. While SELF is best when Extension of Jeffrey`s, Chi Square and Quasi Priors are considered.
2. The combination under PLF using Pareto Prior appears an optimum combination for producing the minimum Posterior risk among all five Priors.

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References

[1] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Power function fading environment", IEEE Transactions on Communication, vol. 43, no. 8, **(1995)**, pp. 2360-2369.
[2] A. Abid, and M. Kaveh, "Performance comparison of three different estimators for the Power function m parameter using Monte Carlo simulation", IEEE Communications Letters, vol. 4, **(2000)**, pp.119- 121.
[3] M. Ahsanullah, and L. Kabir, "Estimation of the location and scale parameters of a Power function distribution by linear functions of order statistics", Communications in Statistics, vol. 5, **(1975)**, pp. 463-467.
[4] M. A. Al-Fawzan, "Algorithms for Estimating the Parameters of the Weibull Distribution", King Abdul-Aziz City for Science and Technology, Riyadh, Saudi Arabia, **(2000)**.
[5] S. Ali, M. Aslam, S. M. A. Kazmi, and N. Abbas, "Scale Parameter Estimation of the Laplace Model Using Different Asymmetric Loss Functions", International Journal of Statistics and Probability, vol. 1, no. 1, **(2012)**, pp.105-127.
[6] F. M. Batah, T. V. Ramanathan, and S. D. Gore, "The Efficiency of Modified Jackknife and Ridge type regression estimators", A comparison, Surveys in Mathematics and its Applications, vol. 3, **(2008)**, pp. 111-122.
[7] J. Cheng, and N. C. Beaulieu, "Maximum likelihood estimation of the Power function m parameter", IEEE Communications Letters, vol. 5, no. 3, **(2001)**, pp. 123-132.
[8] A. C. Cohen, and B. J. Whitten, "Estimation in the three parameters Lognormal Distribution", Journal of the American Statistical Association, vol. 75, **(1982)**, pp. 399-404.
[9] S. Dey, "Bayesian estimation of the parameter and reliability function of an inverse Rayleigh distribution", Malaysian Journal of Mathematical Sciences, vol. 6, no. 1, **(2012)**, pp.113-124.

[10] N. Feroze, "Estimation of scale parameter of inverse gausian distribution under a Bayesian framework using different loss functions", Scientific Journal of Review, vol. 1, no. 3, **(2012)**, pp.39-52.
[11] N. Feroze, and M. Aslam, "A note on Bayesian analysis of error function distribution under different loss functions", International Journal of Probability and Statistics, vol. 1, no. 5, **(2012)**, pp.153-159.
[12] C. W. Hoffman, "The m-Distribution, a general formula of intensity of rapid fading", Statistical Methods in Radio Wave Propagation: Proceedings of a Symposium held June 18-20, **(1958)**, pp. 3-36. Pergamon Press. 2.
[13] S. B. Kang, and C. S. Young, "Estimation of the Parameters in a Pareto Distribution by Jackknife and Bootstrap Methods", Journal of Information and Optimization Science, vol. 18, **(1997)**, pp. 289-300.
[14] A. M. S. Kazmi, M. Aslam, and S. Ali, "Preference of prior for the class of life-time distributions under different loss functions", Pak.J.Statist, vol. 28, no. 4, **(2012)**, pp. 467-487.
[15] K. Kim, and H. A. Latchman, "Statistical traffic modeling of MPEG frames size experiments and analysis", Statistical Methods in Radio Wave Propagation, vol. 29, no. 3, **(2009)**, pp. 456-465.
[16] S. Lalitha, and M. Anand, "Modified Maximum Likelihood Estimators for Rayleigh Distribution", Communication in Statistics: Theory and Method, vol. 25, no. 2, **(1996)**, pp. 389-401.
[17] T. Lwin, "Estimation of the tail of Paretian Law", Skandinaviskaktuarietiidskrift, vol. 55, **(1972)**, pp. 170-178.
[18] H. J. Malik, "Exact moments of Order Statistics for a Power function distribution", Skandinaviskaktuarietiidskrift, vol. 50, (**1967**), pp. 64-69.
[19] M. Meniconi, and M. D. Barry, "The Power function distribution: A useful and simple Distribution to assess Electrical Component Reliability", MicroelectronicsReliability, vol. 36, no. 9, **(1995)**, pp. 1207-1212.
[20] M. Neil, "Estimation of Weibull parameters from common percentiles", Journal of applied Statistics, vol. 32, **(2005)**, pp. 17-24.
[21] B. S. Pablo, and E. R. Bruce, "Model Parameter Estimation using Least Squares", Water Research, vol. 26, no. 6, **(1992)**, pp. 789-796.
[22] Rafiq, "Estimation of parameters of the Gamma Distribution by the method of fractional moments", Pak. J. Statist, vol. 12, no. 3, **(1996)**, pp. 265-274.
[23] Rafiq, "Estimation of parameters of the WeibullDistribution by the method of Fractional Moments", Pak. J. Statist, vol. 15, no. 2, **(1999)**, pp. 91-96.
[24] P. R. Rider, "Distribution of the product, quotient of maximum values in samples from a Power function population", Journal of the American Statistical Association, vol. 59, **(1964)**, pp. 877-880.
[25] E. W. Ronald and H. M. Raymond, "Probability and Statistics for Engineers and Scientists (2nded)", McMillan Publishing Co., Inc., NewYork, **(1978)**.
[26] A. S. Samia and M. M. Mohammad, "Modified Moment Estimators for three parameters Pareto Distribution", Institute of Statistical Studies and Research, Cairo University, vol. 28, **(1993)**, pp. 2.
[27] S. Sarkar, N. Goel, and B. Mathur, "Adequacy of Nakagami-m distribution function to derive GIUH", Journal of Hydrologic Engineering, vol. 14, no. 10, **(2009)**, pp. 1070-1079.
[28] J. Schwartz, T. R. Gowdin, and E. D. Giles, "Improved maximum likelihood estimation of the shape parameter in the Nakagami distribution", Mathematics Subjects Classification, vol. 62, **(2011)**, pp. 30-36.
[29] K. A. Shanker, "Chromium toxicity in plants. Environment International", vol. 31, **(2005)**, pp. 739-753.
[30] Singh, "An almost unbiased ridge estimator", The Indian Journal of Statistics, vol. 48, no. 2, **(1986)**, pp. 342-346.
[31] R. Smolikova, P. M. Wachowiak, and M. J. Zurada, "An information-theoretic approach to estimating ultrasound backscatter characteristics", Computers in Biology and Medicine, vol. 34, **(2004)**, pp. 355-270.
[32] U. Srivastava, "Bayesian estimation of shift point in Poisson model under asymmetric loss function", Pak. J. Stat. Oper. Res, vol. 8, no.1, **(2012)**, pp. 31-42.
[33] H. Tsui, C. Chang, N. Huang, and K. Chang, "Use of Nakagami statistics and empirical mode decomposition for ultrasound tissue characterization by a nonfocused transducer", Ultrasound in Medicine & Biology, vol. 35, no. 12, **(2009)**, pp. 2055-2068.
[34] P. Tsui, C. Huang and S. Wang, "Use of Nakagami distribution and logistics compression in ultrasonic tissue characterization", Journal of Medical and Biological Engineering, vol. 26, no. 2, **(2006)**, pp. 69-73.
[35] H. D. Vinod and M. Ullah, "Recent Advances in Regression Methods", New York M. Dekker, **(1981)**.
[36] D. D. Wackerly, W. Mendenhall and R. L. Scheaffer, "Mathematical Statistics with Applications", vol. 7, Thomson, **(2008)**.
[37] G. Wimmer and V. Witkovsky, "Proper rounding of the measurements results under the assumption of Uniform distribution", Measurement Science Review, vol. 2, **(2002)**, pp.1-7.
[38] F. Yahgmaei, M. Babanezhad and S. O. Moghadam, "Bayesian estimation of the scale parameter of inverse Weibull distribution under the asymmetric loss functions", Journal of Probability and Statistics, **(2013)**.
[39] D. T. Yang and J. Y. Lin, "Food Availability, Entitlement and the Chinese Femine of 1959-61", Economic Journal, vol. 110, **(2000)**, pp. 136-58.

[40] A. Zaka and A. S. Akhter, "A note on Modified Estimators for the Parameters of the Power Function Distribution", International Journal of Advanced Science and Technology, vol. 59, **(2013)**, pp. 71-84.
[41] A. Zaka and A. S. Akhter, "Methods for estimating the parameters of the Power Function Distribution", Pakistan Journal of Statistics and Operational Research, vol. 9, no. 2, **(2013)**, pp. 213-224.
[42] A. Zaka and A. S. Akhter, "Bayesian Approach in Estimation of Scale Parameter of Nakagami Distribution", Pakistan Journal of Statistics and Operational Research, vol. 10, no. 2, **(2014)**, pp. 217-227.
[43] Q. T. Zhang, "A note on the estimation of Nakagami-m fading parameter", IEEE Communication Letters, vol. 6, no. 6, **(2000)**, pp. 237-238.

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