

Simulation and Research of Robust Geometry Method Guidance Law

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Abstract

For the issue of engagement geometry, the modern guidance laws can attack the adversarial and high maneuvering targets. Robust geometric in modern guidance laws is based on the classical differential geometry curve theory and Lyapunov stability theory. For the maneuvering target, based on different geometry and Lyapunov stability theory, the ballistic of robust geometry method guidance law in pursuit of different of maneuvering targets are analyzed. Through simulation of proportional navigation law and robust geometric method in the pursuit of the same target with different emission angles, ballistic properties of the two guidance laws are analyzed. Comparing the simulation results of the two kinds of guidance laws, advantages of the robust geometric guidance law are demonstrated as follows: no additional measurement information, easy to perform and different geometry, and the effectiveness of the guidance law for the line of sight rotation inhibition. Simulation results prove that trajectory of robust geometry is straighter, and distribution of required normal overload is reasonable. Robust geometry is a more precision and critical navigation law.

Keywords: *proportional guidance; robust geometry method; different geometry; guidance law*

1. Introduction

Robust control is the research focus in the control group. The previous system inevitably exist all kinds of disturbances and unmodeled dynamics. It may damage the system to work properly. If this problem can be solved, we can explore more effective guidance to the robust method [1-3]. Classical proportional guidance law as the most mature and applied most widely a guidance law, is under the assumption that the missile and target speed as the constant and goal without motor, the optimal guidance law. But in practice, in order to avoid attack, the enemy's mobile target is the distinctive features of mobility. Target maneuver will seriously affect the performance of proportional guidance, which resulted in increased miss distance. For us, the enemy of the maneuvering target motion is difficult to know in advance, only through observation method for maneuvering target motion. The mathematical model of continuous moving targets is objective existence. But in practice, due to the complexity of calculating the model or it is not necessary, therefore, we can put such goals as a maneuvering target. In this paper, we discuss the robust geometric guidance law based on the maneuvering target. Under different launch Angle and pursuing the same goal, we compare the proportional guidance method and robust geometry method ballistic characteristics of two kinds of guidance law[4-6]. The robust geometry method can interrupt maneuvering target, and better performance than the existing algorithm of variable coefficient proportional guidance, and intercept the overload curve in the process of change is more reasonable.

Ballistic flat don't need to get target accurate acceleration and velocity azimuth information, which has strong robustness on target maneuver.

2. Differential Game Guidance Law

Robust geometry method is a kind of guidance law based on modern control theory, involving the basic theory of differential game guidance, Lyapunov theory, the robust control theory and differential geometry curve theory. Differential game guidance law and the proportional guidance law of different requirements accurately know the target acceleration is proportional guidance law. the differential game guidance law does not need to know accurate information of target maneuvering acceleration, only need to know the target maneuver ability, namely the maximum acceleration. As long as the target acceleration is less than its mobility, no matter what kind of motor, it takes all the guarantee performance index can be achieved [7-8], compared to the differential game guidance law and the proportional guidance law with strong robustness.

Proportional navigation applies to homing missile generally. This guidance law means change rate of speed direction of missile is in proportion to change rate of target sight in the cause of flying toward the target. Relative position of missile and target is shown in Figure 1, and motion equations are as follows:

$$\begin{cases} \frac{dr}{dt} = V_t \cos \eta_t - V_m \cos \eta_m \\ r \frac{d\theta}{dt} = V_m \sin \eta_m - V_t \sin \eta_t \\ \theta = \eta_m + \sigma_m \\ \theta = \eta_t + \sigma_t \\ \frac{d\sigma_m}{dt} = K \frac{d\theta}{dt} \end{cases}$$

Where V_m, V_t are speed of missile and target, and m subscript means missile, and t subscript means target, and r is relative distance between missile and target, and θ is angle of target sight, and σ_m, σ_t are angles between missile and target speed vector and reference line, and η_m, η_t are angles between missile and target speed vector and target line.

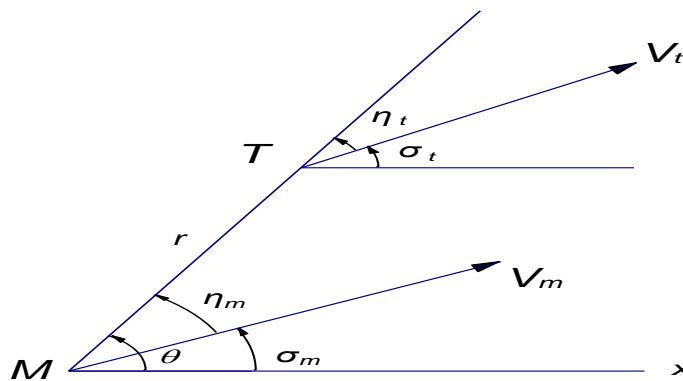


Figure 1. Relative Position of Missile and Target

The issue of engagement geometry is shown in Figure 2 [9-10], $\dot{\omega} = \dot{\theta}$ is sight angle velocity relative to arc length of the missile trajectory, and \dot{r} is velocity of approach relative to arc length of the missile trajectory.

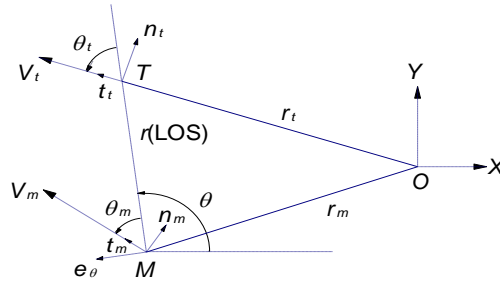


Figure 2. Issue of Engagement Geometry

Taking the derivative of (1) with respect to arc length of the missile trajectory and according to Frenet-Serret formula:

$$\dot{r}_m = \dot{r}_t - r \dot{e}_r \quad (1)$$

Formula of the missile and the target subscript m and t, respectively, for a unit vector; the subscript said along the line of sight direction. Of formula (1) relative to the missile trajectory arc length and partial derivative, and applies the Frenet - Serret formula, you can get

$$\dot{t}_m = \dot{t}_t - r' \dot{e}_r - r \omega \dot{e}_\theta \quad (2)$$

Given a certain things W and by some form of disturbance, if something W a P properties in things W D after disturbance can remain completely still, or within the scope of a degree or to continue, you can say something W P for D perturbation robust character.

The first kind of robustness analysis of the problem is in the things of the disturbance in the form of a known, but the range of the disturbance is analyzed under the condition of unknown, analyze things to maintain some form of nature allowed by the perturbation range of sizes. In the second category of robustness analysis, we know things and W things and P nature of some form of disturbance and the disturbance D range, to give something W is D disturbance and whether still have the exact conclusion P properties [10-12].

3. The Mathematical Deduction of Robust Geometric Guidance Law

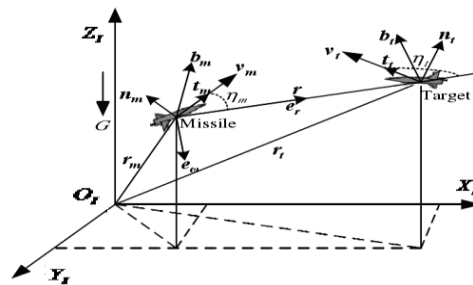


Figure 3. Missile Target Space Position

Assumes that the missile and the target can be as particle, missile speed v_t and target speed v_m is constant, and $v_t/v_m = l < 1$. Among them, the subscript m represents missile, the subscript t represents the target. Assumes that the missile and the target acceleration vector, respectively, and their respective vertical velocity vector, which applied on the missile and target acceleration vector only change the direction of speed without changing the speed, the size of the aerodynamic control of missile and target is such a kind of general conditions. We need to consider the problem of missile interception, as shown in Figure 3.

Because the formula of t as the unit tangent vector; $l = v_t / v_m$ for rate than; ω for the line of sight rotation angular velocity. The direction of the unit vector of e_r and e_ω are defined as shown in Figure 3 [13-15].

Formula (1) the component can be expressed as e_r and e_ω direction

$$r' = (lt_t - t_m) \cdot e_r = l \cos \theta_t - \cos \theta_m \quad (3)$$

$$r\omega = (lt_t - t_m) \cdot (e_\omega \times e_r) = l \sin \theta_t - \cos \theta_m \quad (4)$$

In formula, r is the relative distance between missile and target, θ Angle of sight for the missile and target, $\omega = \theta'$ as compared to the line of sight angular velocity, missile trajectory arc length s . r' on behalf of its relative to the missile trajectory approaching velocity arc length s , the subscript ω represents along the line of sight angular velocity direction of rotation.

The formula (2) relative to the missile trajectory arc length 2, calculate the derivative, you can get

$$k_m n_m = l^2 k_t n_t - (r'' - r\omega^2)e_r - (r\omega' + 2r'\omega)e_\omega \quad (5)$$

The component can be expressed as e_r and e_ω direction

$$r'' - r\omega^2 = l^2 k_t n_t e_r - k_m n_m e_r \quad (6)$$

$$r\omega' + 2r'\omega = l^2 k_t n_t e_\omega - k_m n_m e_\omega \quad (7)$$

From formula (2) to (7) for the kinematics equation of missile and target. Note from type (2) into type (7) in the calculation of the derivative operation symbol "''" is relative to the derivative of missile trajectory arc length s .

$$U = \frac{1}{2} \omega^2 \quad (8)$$

Of formula (8) relative to the missile trajectory arc length s derivative calculation, in practice, on the curvature of the target motion commands (acceleration) and velocity azimuth information is difficult to get. This article will command to the curvature of the target motion and velocity azimuth information about the amount of computation as interference, and assume that $k_t \leq c$, c of them as the constant, says the goal can be one of the biggest mobile curvature command or maximum acceleration, k_m for the curvature of the missile command, then

$$U' \leq -\frac{2r'}{r} \omega^2 + \frac{l^2 c}{r} |\omega| + \frac{k_m \cos \theta_m}{r} \omega \quad (9)$$

$$U' \leq [-A_1 \omega^2 - (A_2 + \text{sgn}(r')) |r'| \omega^2 +$$

$$\begin{aligned}
 & [-A_1 \omega^2 + l^2 C |\omega| - l^2 C \tanh(\frac{\omega}{\varepsilon}) \omega] / r \\
 & m^2 C |\omega| - l^2 C \tanh(\frac{\omega}{\varepsilon}) / r \leq \\
 & \leq [-A_1 \omega^2 + l^2 C k \varepsilon] / r
 \end{aligned} \tag{10}$$

According to the formula (10), k_m for optional

$$k_m = \frac{-A\omega - B \cdot |r'| \cdot \omega - l^2 \cdot c \cdot \tanh(\frac{\omega}{\varepsilon})}{\cos \theta_m} \tag{11}$$

In the formula (11), $A > 0$, $B > 2$ as the proportionality constant; ε for a small number of normal, they shall ensure that the choice of ω has satisfactory dynamic performance. When given to meet the conditions of

$$|\theta_m| < \frac{\pi}{2},$$

indicated by the formula (11) missile curvature command is nonsingular.

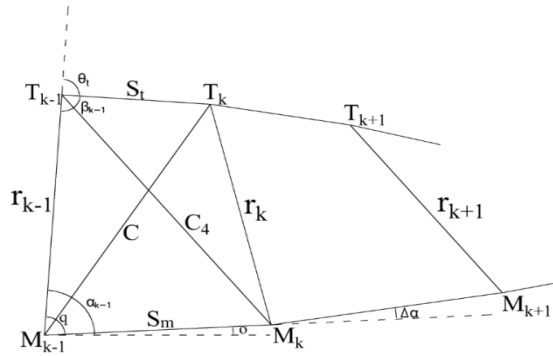


Figure 4. Simulation Principle of Robust Geometry Method

In the derivation of equation (10), the application of the following inequality:

$$|\omega| - \tanh(\frac{\omega}{\varepsilon}) \omega \leq k \varepsilon, k = 0.2785 \tag{12}$$

By the formula (12) can get a conclusion, when $\omega > \omega_0 = m \sqrt{C k \varepsilon / A_1}$ satisfy this condition, U' has a definite matrix. And ascends and A_1 or ε all can make ω_0 full of small, thus ensure the ω tend to a small neighborhood of zero.

4. Robust Geometric Simulation of Guidance Law

In the process of trajectory simulation, the ballistic n points and n equal, and line connects adjacent points in the process of actual simulation, n will be greater than 3, when n is large enough, can think inside the arc line and arc itself is almost perfect. For the ballistic simulation, when we take a small time interval, the missile in the distance traveled by a time

interval compared with the total length of the ballistic enough hours, you can imagine that is nearly consistent with the actual trajectory simulation trajectory.

According to the geometric relationship can be conclude

$$c = T_k M_{k-1} = \sqrt{(x_t(k) - x_m(k-1))^2 + (y_t(k) - y_m(k-1))^2} \quad (13)$$

$$c_4 = M_k T_{k-1} = \sqrt{(x_m(k) - x_t(k-1))^2 + (y_m(k) - y_t(k-1))^2} \quad (14)$$

$$\beta_{k-1} = \arccos\left(\frac{[r_t(k-1)]^2 + s_t^2 - c^2}{2r_t(k-1)s_t}\right) \quad (15)$$

$$\theta_t = \pi - \beta_{k-1} \quad (16)$$

$$\alpha_{k-1} = \arccos\left(\frac{[r(k-1)]^2 + s_m^2 - c_4^2}{2r(k-1)s_m}\right) \quad (17)$$

$$\Delta\alpha = s_m \cdot k_m \quad (18)$$

$$x_m(k+1) = x_m(k) + s_m \cdot \cos(\Delta\alpha) \quad (19)$$

$$y_m(k+1) = y_m(k) + s_m \cdot \sin(\Delta\alpha) \quad (20)$$

Simulation principle is shown in Figure 4, 5, 6.

5. Simulation of Engagement Geometry

Simulating missile's intercepting target using Proportional guidance and robust geometric respectively, and the simulation parameters : the speed of the missile is $V_m = 1000$ m / s, and the speed of the target is $V_t = 400$ m / s, and velocity ratio is $l = 0.4$, and the initial relative heading angle of missile is $\theta_m^{(0)} = 5^\circ$, the initial relative heading angle of target is $\theta_t^{(0)} = 45^\circ$, the initial coordinates for the missile is $r_{m0} = (0,0)$, the initial coordinates for the target is $r_{t0} = (0,10000)$, and units are m .

Supposing Missile and target moving in the same plane, and M_k is location of the missile for the time interval k , and M_{k+1} is location of the missile for the time interval $k+1$, and T_k is location of the target for the time interval k , and $r^{(k)}$ is the distance between missile and target for the time interval k , and S_m is moving distance of missile in one time interval, and x_m, y_m are the three-dimensional coordinates of missile. x_t, y_t are the three-dimensional coordinates of target, and $M_{k-1}M_k$ is moving distance of missile in one time interval (S_m), and $T_{k-1}T_k$ is moving distance of target (S_t), and $M_{k-1}T_{k-1}$ and $M_k B$ are parallel lines.

Supposing

$$M_{k-1}T_k = c, AM_{k-1} = c_1, \beta_{k-1} = \angle AT_{k-1}M_{k-1} = \arccos\left(\frac{[r_t(k-1)]^2 + s_t^2 - c^2}{2r_t(k-1)s_t}\right)$$

$$\Delta q_{k-1} = \angle T_k M_k B \approx \angle T_k M_{k-1} T_{k-1} = \arccos\left(\frac{[r_t(k-1)]^2 + c^2 - s_t^2}{2r_t(k-1)c}\right)$$

$$r(k) = \sqrt{(x_t(k) - x_m(k))^2 + (y_t(k) - y_m(k))^2}$$

$$c = \sqrt{(x_t(k) - x_m(k-1))^2 + (y_t(k) - y_m(k-1))^2}$$

$$c_1 = \frac{r(k-1) \sin(\beta_{k-1})}{\sin(a_{k-1} + \beta_{k-1})}$$

$$c_2 = \frac{r_t(k-1) \sin(a_{k-1})}{\sin(a_{k-1} + \beta_{k-1})}$$

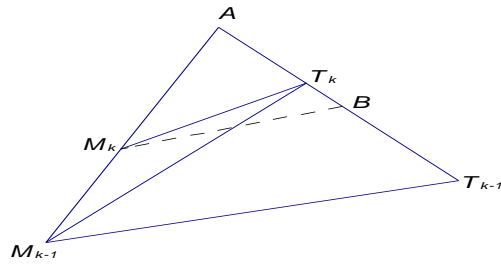


Figure 5. Proportional Guidance

The end of trajectory of proportional navigation is flat relatively, but distribution of the required normal overload is not reasonable enough. And proportional navigation can not against the goal's mobility and interference.

In the process of engagement, from time M_k to time M_{k+1} , change of value of trajectory inclination angle is $d\theta$, and the change rate is $\frac{d\theta}{dt} = v_m \cdot k_m$

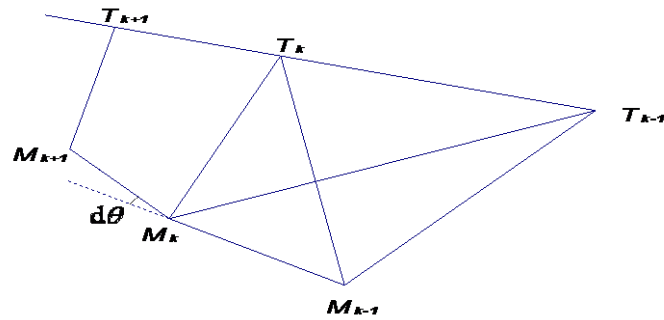


Figure 6. Robust Geometry

Supposing

$$M_k T_{k-1} = c_4, \text{ and } c_4 = \sqrt{(x_t(k-1) - x_m(k))^2 + (y_t(k-1) - y_m(k))^2}$$

Other parameters are the same as simulation of proportional navigation. Simulation result is shown in Figure 6. Compared to Figure 4, it can be seen that trajectory of robust geometry is straighter, and distribution of required normal overload is reasonable. At the time of missile hitting target, attitude angle meets the requirements.

Simulation parameters: the missile's velocity $v_m = 1000\text{m/s}$, the target speed $v_t = 400\text{m/s}$, the missile's initial heading Angle of 45 degrees, the goal of the initial heading Angle of 45

degrees, the missile's initial coordinates (0, 0), target initial coordinates: (0,10000), the unit to m. Target maneuvering curvature command for $k = 0.00048375$, the results of the simulation is shown in Figure 7. So can conclude that the robust geometry method in pursuit of the goal of a curve maneuver flight when first half ballistic method to overload is larger, the second half when the ballistic close to a straight line. When the first half so missiles will be a larger motor, because after the ballistic missile paragraph is generally keep flying state, by inertia rocket engine work only a short period of time, so this kind of ballistic characteristics can give full play to the power of the rocket engine.

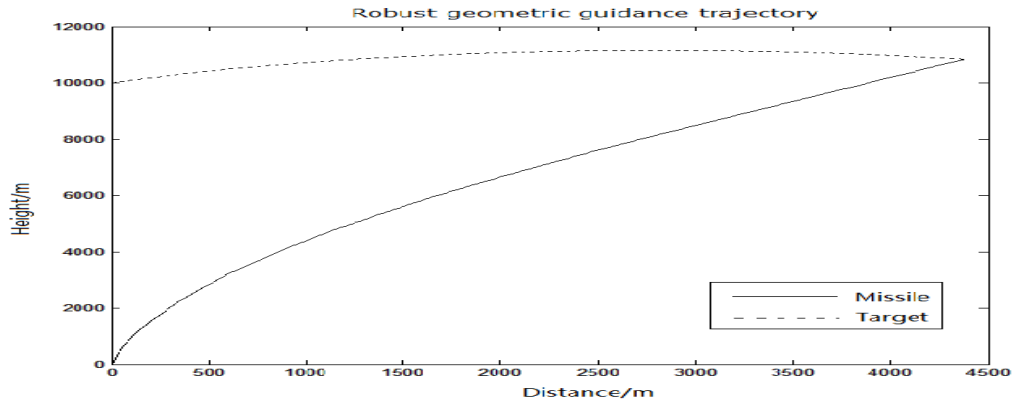


Figure 7. Missile Tracking Trajectory as $k_t = -0.00048375$

Figure 8 represents goal when making its movement tracking trajectory of the missile. By figure can be concluded that the change of the target trajectory after still according to the original speed, front of the ballistic missile trajectory curvature increases significantly, in the middle period of ballistic trajectory smoothly and ease, when the distance of the missile and target is $r < 5000$ m, terminal guidance trajectory began to bend, when close to the target trajectory curvature increases, the increase of the normal overload lead to missile capacity is limited, but robust geometry method under the general guidance trajectory quite gentle. Robust geometry method on the target for its movement, can still maintain a straight trajectory. Don't need to get accurate curvature command and target azimuth information, has strong robustness to the target maneuver.

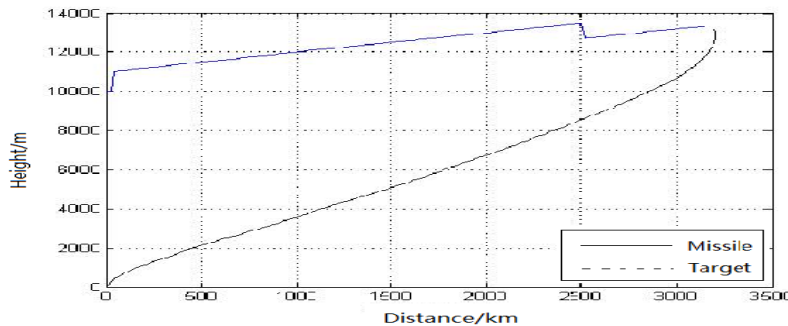


Figure 8. Missile Tracking Trajectory for the Maneuvering Target

Robust geometry method and the proportional guidance method under the condition of invariable in other simulation parameters, using different missile guided by the initial heading Angle (front) ballistic comparison, from Figure 9 to Figure 12 for the results obtained from the simulation.

From the comparison of two kinds of guidance law, the robust geometry method to initial heading Angle change of the ballistic curve of significantly less than the bending degree of the traditional proportional guidance law of ballistic. This shows that the robust geometry method has stronger robustness, can withstand a certain external disturbances. Classical proportional guidance law is simple structure, easy implementation, you can get a relatively flat trajectory, however, hit points need usage to overload by missiles speed and the direction of attack. Robust geometry method only in the beginning the process of the ballistic correction, at the end of the ballistic missile basic is zero, the normal acceleration at the end of the ballistic relatively flat.

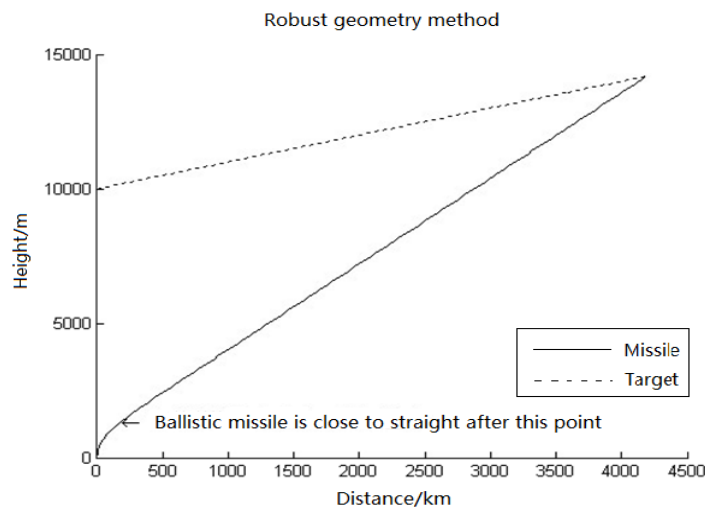


Figure 9. Trajectory of Robust Geometry Method when Initial Heading Angle is 90°

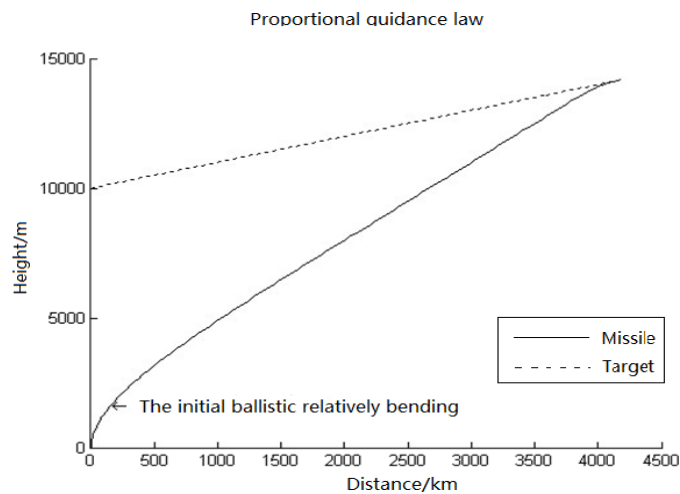


Figure 10. Trajectory of Proportional Guidance when Initial Heading Angle is 90°

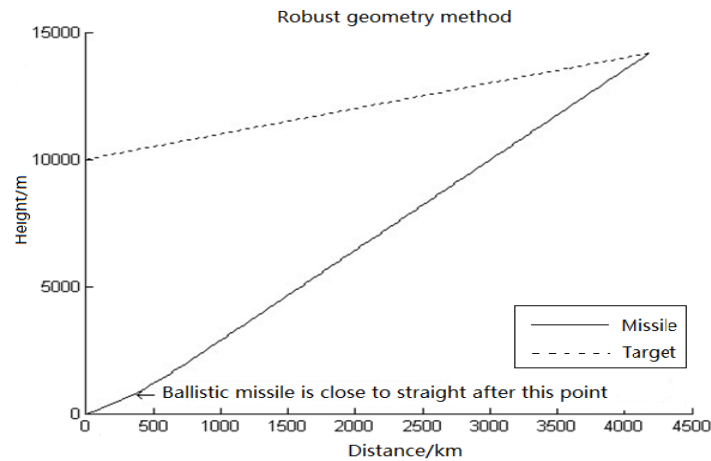


Figure 11. Trajectory of Robust Geometry Method when Initial Heading Angle is 60°

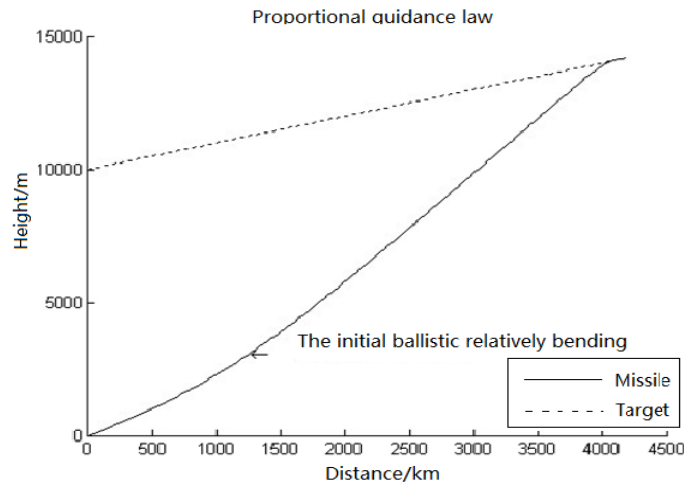


Figure 12. Trajectory of Proportional Guidance when Initial Heading Angle is 60°

6. Conclusion

Although the application of proportional guidance law more extensive, but this method have antagonism target, the attack is still possible to make a big miss distance, so it is necessary for us to further study has higher accuracy and more practical guidance law, so as to adapt to the needs of the development. Robust geometry method, however, it is a kind of can satisfy the requirements of the guidance law, the robust geometry method as a kind of modern guidance law, it will be related to the target azimuth curvature command and speed information of items as a disturbance variable, don't need to get accurate curvature command and target azimuth information, rational distribution of ballistic need normal overload, against a target maneuver and interference ability strong, this kind of method is a kind of guidance law with higher precision and more practical. The proportional guidance and robust geometric which is combined with the classical differential geometry curve theory and Lyapunov

stability theory are analyzed and compared. Simulation results prove that the combination of modern control theory of robust geometric approach is a kind of more high-precision guidance law.

Compared with the proportional navigation, trajectory of robust geometry is straighter, and distribution of required normal overload is reasonable. Robust geometry is a more precision and critical navigation law.

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