

## Multisensor Information Fusion State Estimator for Systems with Random Sensor Errors

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### Abstract

*In this paper, a multisensor distributed information fusion state estimator for discrete time stochastic linear systems with random sensor errors is presented. Based on state-space model, the white noise estimator and the observation predictor are applied in this algorithm. Modern time series analysis method and Gevers-Wouters(G-W) algorithm are also used in this paper. The algorithm can deal with the filtering, smoothing and prediction problems via a unified method. In order to improve the estimation accuracy, the multisensor distributed information fusion method is adopted, which calculates the weighting parameters with the forms of matrix, diagonal matrices and scalars respectively, in the sense of linear minimum variance. Among those three kinds of fusion methods, the method weighted by matrix has the highest accuracy but more computation, while the one weighted by scalar has the lowest accuracy but less computation. A simulation example for a typical tracking system with 3-sensor shows the correctness, validity and no obvious difference among three kinds of the fusion algorithms.*

**Keywords:** *distributed information fusion; the linear minimum variance; the state estimator; random sensor errors; Gevers-Wouters algorithm*

### 1. Introduction

The state estimation for stochastic systems with unknown inputs, disturbances and biases can be widely applied to control, communications, signal processing, and fault diagnosis. If we cannot detect and isolate the disturbances and biases effectively, they may cause the loss of personnel and production. If the size and symbol error show certain systematicness or according to certain rules of change, this kind of error is called the system bias [1]. The system deviation has great effect on the observation results. So the system bias should be as far as possible to eliminate or restrict to a minimum. Systems with system bias and sensor errors exist extensively in the field of control, communication and signal processing. The estimation problem with system bias and sensor errors, the original literature presented using two sections of Kalman filtering technique to deal with the estimation problem of the unknown constant bias [2]. Then, the algorithm is extended to the filtering problem of random deviation. Separating stochastic bias two-stage decoupled wiener filtering using modern time sequence analysis method is presented [3].

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By the noise signal to seek the true signal or a state valuation is called estimator or filtering. Multisensor information fusion in the military has become one of the most active research methods. In last decades, the application field of information fusion technology has increased widely. Its field includes guidance, GPS, defense, medical, integrated navigation, target tracking, robot technology, communications and signal processing [4]. One of the purposes of the fusion estimation is to combine the local estimation to obtain a fused estimation. And the estimation accuracy of the fused estimation is higher than any of the single local sensor. The other purpose of the fusion estimation is to fuse the local observation equations in order to obtain a fused observation equation. Further we use the fused observation equation to get a fused estimation. Also, the fusion estimation precision valuation is higher than that of any single sensor [5]. Since the 1970s, the estimation precision of the single sensor is difficult to meet the demand in many fields. For example, in the field of remote guidance weapon and the typical track system needs to improve the estimation precision. The estimation with the single sensor cannot meet the need. Therefore, the multisensor information fusion technology is presented. And it causes the extensive concern of the academia. Also, it receives wide attention in the field of engineering application. For the sake of improving the accuracy of estimation for the system state and signal, it is essential to use multiple sensor information fusion technology to weight much sensor information [6-8]. As the multisensors can provide more information in time and space, the research of the state estimation and input estimation for the system has important meaning both in theory and engineering practice [9-13].

In this paper, state estimators weighted by matrix, diagonal matrices and scalars for discrete time stochastic linear systems with random sensor errors are presented. The random sensor errors are considered firstly. They can deal with the fused filtering, smoothing and prediction estimation in a unified way. The algorithm is under the linear minimum variance sense, the optimal information fusion criterion is weighted by matrix, diagonal matrices and scalars. Matrix, diagonal matrices, and scalars weight the accuracy of above three kinds of weighted fusion filtering from high to low. But the computational burden is on the contrary. Fusion filtering weighted by matrix has a large computational burden, and weighted by scalars with minimal computational burden, and it is suitable for real-time applications. A simulation example for the typical track system with 3-sensor shows the correctness and validity of training. And simulation results show no significant difference between the three kinds of distributed fusion algorithm. So a fast information fusion estimation algorithm is presented.

The main structure of this paper is as follows: Problem formulation is given in Section 2. The Local optimal state estimator is obtained in Section 3. In Section 4 distributed information fusion optimal state estimator is presented. A simulation example with 3-sensor is given in Section 5. In Section 6 the conclusions of this paper are given.

## 2. Problem Formulation

Consider the multisensor discrete time time-invariant stochastic linear system with process errors and measurement errors

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{\Gamma}\mathbf{w}(t) \quad (1)$$

$$\mathbf{z}_j(t) = \mathbf{H}_j\mathbf{x}(t) + \mathbf{v}_j(t) + \mathbf{e}_j(t) \quad (2)$$

$$\mathbf{e}_j(t+1) = \mathbf{e}_j(t) + \boldsymbol{\xi}_j(t), \quad j = 1, \dots, K \quad (3)$$

where  $x(t) \in \mathbf{R}^n$  is the state of the system,  $y_j(t) \in \mathbf{R}^{m_j}$  is the measurement of the  $j$ th sensor subsystem,  $A, \Gamma, H$  is the suitable dimensional matrix respectively,  $e_j(t) \in \mathbf{R}^{m_j}$  is sensor errors.  $\xi_j(t)$ ,  $j = 1, 2, \dots, K$  are independence white noises with zero mean, and they and  $w(t) \in \mathbf{R}^r$ ,  $v_j(t) \in \mathbf{R}^{m_j}$ ,  $j = 1, \dots, K$  are independence. And the subscript  $j$  is the  $j$ th sensor of all the sensor,  $K$  is the number of sensor in the system.

**Assumption 1**  $w(t) \in \mathbf{R}^r$  and  $v_j(t) \in \mathbf{R}^{m_j}$ ,  $j = 1, \dots, K$  are correlated white noises with zero mean:

$$E \left\{ \begin{bmatrix} w(t) \\ v_j(t) \end{bmatrix} \begin{bmatrix} w^T(k) & v_i^T(k) \end{bmatrix} \right\} = \begin{bmatrix} Q_w & S_i \\ S_j^T & Q_{v_j} \end{bmatrix} \delta_{ik}, \begin{bmatrix} Q_w & S_i \\ S_j^T & Q_{v_j} \end{bmatrix} > 0, \quad j = 1, \dots, K \quad (4)$$

where the superscript  $T$  is the transpose,  $E$  is the expectation,  $\delta_{ii} = 1$ ,  $\delta_{ik} = 0 (i \neq k)$ .

**Assumption 2** The system in this paper is completely observable, that is:

$$\text{rank } \Omega_i = n, \quad \Omega_i = [H_i^T, (H_i \Phi)^T, \dots, (H_i \Phi^{\beta_i-1})^T]^T \quad (5)$$

where  $\beta_i$  is the  $i$ th sensor system observability index. The system is completely stable.

**Assumption 3** The initial time  $t_0 = -\infty$ .

State estimation problem is based on the measurement  $(z_i(t+N), z_i(t+N-1), \dots)$ , to obtain the linear minimum variance state  $\hat{x}_j(t|t+N)$ ,  $j = 1, 2, \dots, K$ . For  $N = 0$ ,  $N > 0$  or  $N < 0$ , we named it as state filtering, smoothing or predictor. Further distributed optimal information fusion state estimation  $\hat{x}_0(t|t+N)$  is obtained, it consists of weighted local state estimators.

### 3. Local Optimal State Estimator

For the system (1)~(3), we can get the following augmentation system

$$\alpha_j(t+1) = \bar{A}_j \alpha_j(t) + \bar{\Gamma}_j \bar{w}_j(t) \quad (6)$$

$$z_j(t) = \bar{H}_j \alpha_j(t) + v_j(t), \quad j = 1, \dots, K \quad (7)$$

where

$$\alpha_j(t) = \begin{bmatrix} x(t) \\ e_j(t) \end{bmatrix}, \quad \bar{w}_j(t) = \begin{bmatrix} w(t) \\ \xi_j(t) \end{bmatrix}, \quad \bar{A}_j = \begin{bmatrix} A & 0_{n \times m_j} \\ 0_{m_j \times n} & I_{n_d} \end{bmatrix}, \quad \bar{\Gamma}_j = \begin{bmatrix} \Gamma & 0_{n \times m_j} \\ 0_{m_j \times n} & I_{n_d} \end{bmatrix}, \quad \bar{H}_j = [H_j \quad I_{m_j}] \quad (8)$$

The state  $x(t)$  is the firstly  $n$ th components for the system (6) and (7), so the original problem converts to the distributed fusion state estimator problem for the system (7) and (8). The system noise  $\bar{w}_j(t)$  and observation noise  $v_j(t)$  are correlated, and

$$E \left\{ \begin{bmatrix} \bar{w}_j(t) \\ v_j(t) \end{bmatrix} \begin{bmatrix} \bar{w}_i^T(k) & v_i^T(k) \end{bmatrix} \right\} = \begin{bmatrix} Q_{\bar{w}_j} & S_i \\ S_j^T & Q_{v_j} \end{bmatrix} \delta_{ik} \quad (9)$$

where  $Q_{\bar{w}_j} = Q_{\bar{w}_j} = \text{diag}(Q_w, Q_{\xi_j})$ ,  $Q_{\bar{w}_j} = \text{diag}(Q_w, 0)$ ,  $S_j = [S_j^T, 0]^T$ ,  $j \neq i$ .

From (6) and (7) having

$$z_j(t) = \bar{H}_j (I_n - q^{-1} \bar{A}_j)^{-1} \bar{\Gamma}_j q^{-1} \bar{w}_j(t) + v_j(t) \quad (10)$$

where  $q^{-1}$  is the lag operator for the unit, introducing left decomposition

$$\bar{H}_j(I_n - q^{-1}\bar{\Phi}_j)^{-1}\bar{\Gamma}_jq^{-1} = A_\alpha^{(j-1)}(q^{-1})B_\alpha^{(j)}(q^{-1}) \quad (11)$$

where  $A_\alpha^{(j)}(q^{-1})$  and  $B_\alpha^{(j)}(q^{-1})$  is matrix polynomial, it is shaped as  $X^{(j)}(q^{-1}) = x_0^{(j)} + x_1^{(j)}q^{-1} + \dots + x_{n_j}^{(j)}q^{-n_j}$ , and  $A_{\alpha 0}^{(j)} = I_{m_j}$ ,  $B_{\beta 0}^{(j)} = 0$ , letting  $X^{(j-1)}(q^{-1}) = (X^{(j)}(q^{-1}))^{-1}$ .

Substituting (11) into (10) yields

$$A_\alpha^{(j-1)}(q^{-1})z_j(t) = B_\alpha^{(j)}(q^{-1})\bar{w}_j(t) + A_\alpha^{(j-1)}(q^{-1})v_j(t) \quad (12)$$

For  $(A_\alpha^{(j)}(q^{-1}), B_\alpha^{(j)}(q^{-1}))$  is left and Assumption 1, yielding the ARMA innovation model

$$A_\alpha^{(j-1)}(q^{-1})z_i(t) = D_\alpha^{(j)}(q^{-1})\varepsilon_i(t) \quad (13)$$

where innovation  $\varepsilon_j(t) \in R^{m_j}$  is white noises with zero mean, and its covariance is  $\mathcal{Q}_{\varepsilon_j}$ .  $D_\alpha^{(j)}(q^{-1})$  is stable,  $D_0^{(j)} = I_m$ , and having

$$D_\alpha^{(j)}(q^{-1})\varepsilon_j(t) = B_\alpha^{(j)}(q^{-1})\bar{w}_j(t) + A_\alpha^{(j-1)}(q^{-1})v_j(t) \quad (14)$$

$D_\alpha^{(j)}(q^{-1})$  and  $\mathcal{Q}_{\varepsilon_j}$  are computed through G-W algorithm.

**Lemma 1**[14] For the system (6) and (7) under the Assumption 1-3, the state  $\alpha^{(t)}$  has the non recursive expression

$$\alpha(t) = \sum_{i=0}^{\beta-1} \Omega_i^{(j)}[z_j(t+i)] - \sum_{r=0}^{i-1} \bar{H}_j\bar{\Phi}_j^{i-1-r}\bar{\Gamma}_j\bar{w}_j(t+r) - v_j(t+i) \quad (15)$$

where  $\bar{\Phi}_j^i = 0 (i < 0)$  and  $r \geq 0$ . Defining  $\Omega_i^{(j)} (n \times m)$  as

$$\Omega_j^+ = (\Omega_j^T \Omega_j)^{-1} \Omega_j^T = [\Omega_0^{(j)}, \Omega_1^{(j)}, \dots, \Omega_{\beta-1}^{(j)}] \quad (16)$$

where  $\Omega_j$  is defined by (5). Non recursive optimal state estimator can be calculated as

$$\hat{\alpha}_j(t|t+N) = \sum_{i=0}^{\beta-1} \Omega_i^{(j)}[\hat{z}_j(t+i|t+N)] - \sum_{r=0}^{i-1} \bar{H}_j\bar{\Phi}_j^{i-1-r}\bar{\Gamma}_j\hat{w}_j(t+r|t+N) - \hat{v}_j(t+i|t+N) \quad (17)$$

**Lemma 2**[14] For the system (6) and (7) under the Assumption 1-4, White noise innovation filtering and State filtering respectively are as follows

$$\hat{\theta}_j(t|t+N) = L_N^{(j)\theta}(q^{-1})\varepsilon_j(t+N), \quad \theta = w, v_j \quad (18)$$

$$\hat{\theta}_j(t|t+N) = L_N^{(j)\theta}(q^{-1})\tilde{A}^{(j)}(q^{-1})\tilde{D}^{(j-1)}(q^{-1})z_j(t+N), \quad \theta = w, v_j \quad (19)$$

having pseudo exchange

$$D_\alpha^{(j-1)}(q^{-1})A_\alpha^{(j)}(q^{-1}) = \tilde{A}^{(j)}(q^{-1})\tilde{D}^{(j-1)}(q^{-1}) \quad (20)$$

measurement predictor can be calculated

$$\hat{z}_j(t|t+N) = J_{-N}^{(j)}(q^{-1})\tilde{D}^{(j-1)}(q^{-1})z_j(t+N) \quad (21)$$

where  $J_i^{(j)}(q^{-1})$  is obtained by Diophantine equation

$$\tilde{D}^{(j)}(q^{-1}) = E_i^{(j)}(q^{-1})\tilde{A}^{(j)}(q^{-1}) + q^{-i}J_i^{(j)}(q^{-1}), i > 0 \quad (22)$$

$$E_i^{(j)}(q^{-1}) = I_m + E_1^{(j)}q^{-1} + \dots + E_{i-1}^{(j)}q^{-(i-1)} \quad (23)$$

$$\mathbf{J}_i^{(j)}(q^{-1}) = \tilde{\mathbf{D}}^{(j)}(q^{-1})q^i, i \geq 0 \quad (24)$$

$$\mathbf{J}_i^{(j)}(q^{-1}) = \mathbf{J}_0^{(j)} + \mathbf{J}_1^{(j)}q^{-1} + \dots + \mathbf{J}_{n_{ji}}^{(j)}q^{-n_{ji}}, \quad n_{ji} = \max(\tilde{n}_{aj} - 1, \tilde{n}_{dj} - i) \quad (25)$$

Defining

$$L_N^{(j)\theta}(q^{-1}) = \sum_{i=0}^N \Lambda_k^{(j)\theta} \mathcal{Q}_{\varepsilon_j}^{-1} q^{i-N}, N \geq 0 \quad (26)$$

$$L_N^{(j)\theta}(q^{-1}) = 0, N < 0$$

measurement predictor error is shown as

$$\tilde{\mathbf{z}}_j(t | t + N) = \mathbf{E}_{-N}^{(j)}(q^{-1})\boldsymbol{\varepsilon}_j(t) \quad (27)$$

**Lemma 3**[14] The following formula is established

$$\mathbf{E}[\bar{\mathbf{w}}_j(t)\boldsymbol{\varepsilon}_j^T(k)] = \mathbf{A}_{k-t}^{(j)w}, \quad \mathbf{A}_p^{(j)w} = \mathcal{Q}_{\bar{\mathbf{w}}_j} \mathbf{F}_p^{(j)\top} + \mathbf{S}_j \mathbf{G}_p^{(j)\top}, \quad \mathbf{E}[\mathbf{v}_j(t)\boldsymbol{\varepsilon}_j^T(k)] = \mathbf{A}_{k-t}^{(j)v}, \quad \mathbf{A}_p^{(j)v} = \mathcal{Q}_{\mathbf{v}_j} \mathbf{G}_p^{(j)\top} + \mathbf{S}_j^T \mathbf{F}_p^{(j)\top} \quad (28)$$

The  $\mathbf{F}_j^{(i)}$  and  $\mathbf{G}_j^{(i)}$  can be computed as

$$\mathbf{F}_j^{(i)} = -D_1^{(i)}\mathbf{F}_{j-1}^{(i)} - \dots - D_{n_{di}}^{(i)}\mathbf{F}_{j-n_{di}}^{(i)} + \mathbf{B}_j \quad (29)$$

$$\mathbf{G}_j^{(i)} = -D_1^{(i)}\mathbf{G}_{j-1}^{(i)} - \dots - D_{n_{di}}^{(i)}\mathbf{G}_{j-n_{di}}^{(i)} + \mathbf{A}_j \quad (30)$$

**Theorem 1** For the system (6) and (7) under the Assumption 1-3, asymptotically stability state estimator in a unified form is as follows

$$\hat{\mathbf{a}}_j(t | t + N) = \mathbf{K}_N^{(j)}(q^{-1})\tilde{\mathbf{D}}^{(j)-1}(q^{-1})\mathbf{y}_j(t + N) \quad (31)$$

Defining polynomial matrix  $\mathbf{K}_N^{(j)}(q^{-1})$ :

$$\mathbf{K}_N^{(j)}(q^{-1}) = \sum_{i=0}^{\beta_j-1} \Omega_i^{(j)} [\mathbf{J}_{i-N}^{(j)}(q^{-1}) - \sum_{r=0}^{i-1} \mathbf{H}_j \bar{\boldsymbol{\Phi}}^{i-1-r} \bar{\mathbf{F}} \mathbf{L}_{N-r}^{(j)w}(q^{-1}) \tilde{\mathbf{A}}^{(j)}(q^{-1}) - \mathbf{L}_{N-i}^{(j)v}(q^{-1}) \tilde{\mathbf{A}}^{(j)}(q^{-1})] \quad (32)$$

Letting  $\phi^i = 0 (i < 0)$  and  $r \geq 0$ . It has ARMA recursive form

$$\det \tilde{\mathbf{D}}^{(j)}(q^{-1}) \hat{\mathbf{a}}_j(t | t + N) = \mathbf{K}_N^{(j)}(q^{-1}) \text{adj} \tilde{\mathbf{D}}^{(j)}(q^{-1}) \mathbf{y}_j(t | t + N) \quad (33)$$

**Proof:** Substituting (19) and (21) into (17) yields (31) and (32). Then Substituting  $\tilde{\mathbf{D}}^{(j)-1}(q^{-1}) = \text{adj} \tilde{\mathbf{D}}^{(j)}(q^{-1}) / \det \tilde{\mathbf{D}}^{(j)}(q^{-1})$  into (31) yields (33). Due to  $\det \tilde{\mathbf{D}}^{(j)}(q^{-1}) = \det \mathbf{D}^{(j)}(q^{-1})$ , the stability of  $\mathbf{D}^{(j)}(q^{-1})$  leads to  $\tilde{\mathbf{D}}^{(j)}(q^{-1})$  is stable. So (31) and (33) are asymptotically stable. The proof is completed.

**Theorem 2** For system (6) and (7) under the Assumption 1-3, estimation error  $\tilde{\mathbf{a}}_j(t | t + N) = \mathbf{a}(t) - \hat{\mathbf{a}}_j(t | t + N)$  has the following expressions

$$\tilde{\mathbf{a}}_j(t | t + N) = \sum_{i=0}^{\beta_j-2} \Omega_i^{(j)w} \bar{\mathbf{w}}_j(t + i) - \sum_{i=0}^{\beta_j-1} \Omega_i^{(j)v} \mathbf{v}_j(t + i) + \sum_{i=0}^{n_0} \Omega_i^{(j)e} \boldsymbol{\varepsilon}_j(t + i), \quad N \geq 0 \quad (34)$$

Where defining  $n_0 = \max(\beta_j - 1, N)$ ,  $\Omega_j^{(j)e}$  is obtained by the merger of similar coefficient matrix.

$$\tilde{\mathbf{a}}_i(t | t + N) = \sum_{j=0}^{\beta_i-2} \Omega_j^{(i)w} \bar{\mathbf{w}}_i(t + j) - \sum_{j=0}^{\beta_i-1} \Omega_j^{(i)v} \mathbf{v}_i(t + j) + \sum_{j=0}^{n_1} \Omega_j^{(i)y} \boldsymbol{\varepsilon}_i(t + N + 1 + j), \quad N < 0 \quad (35)$$

Where defining  $n_1 = (\beta_i - 2 - N)$ ,  $\Omega_j^{(i)y}$  is obtained by the merger of similar coefficient matrix. (34) and (35) can be expressed as a unified form

$$\tilde{\alpha}_j(t | t + N) = \sum_{i=0}^{n_0} [\Omega_i^{(j)w} \bar{w}_j(t + i) - \Omega_i^{(j)} v_j(t + i) + \Omega_i^{(j)e} \varepsilon_j(t + i)], \quad N \geq 0 \quad (36)$$

$$\tilde{\alpha}_i(t | t + N) = \sum_{j=0}^{n_1} [\Omega_j^{(i)w} \bar{w}_i(t + j) - \Omega_j^{(i)} v_i(t + j) + \Omega_j^{(i)y} \varepsilon_i(t + N + 1 + j)], \quad N < 0 \quad (37)$$

And letting  $\Omega_i^{(j)w} = 0 (i > \beta_j - 2)$ ,  $\Omega_i^{(j)} = 0 (i > \beta_j - 1)$ .

Proof: (15) minus (17), and Substituting (18) and (27) into it, then mergers similar items, (36) and (37) are obtained. The proof is completed.

**Theorem 3** For the system (6) and (7) under the Assumption 1-3, estimation error covariance  $P_j(N) = E[\tilde{\alpha}_j(t | t + N) \tilde{\alpha}_j^T(t | t + N)]$  is given as

$$P_j(N) = \sum_{r=0}^{n_0} \sum_{s=0}^{n_0} \begin{bmatrix} \Omega_r^{(j)w} & -\Omega_r^{(j)} & \Omega_r^{(j)e} \end{bmatrix} \begin{bmatrix} Q_{\bar{w}_{ij}} \delta_{rs} & \bar{S}_j \delta_{rs} & \Lambda_{s-r}^{(j)\bar{w}_j} \\ \bar{S}_j^T \delta_{rs} & Q_{v_j} \delta_{rs} & \Lambda_{s-r}^{(j)v_j} \\ \Lambda_{r-s}^{(j)\bar{w}_j T} & \Lambda_{r-s}^{(j)v_j T} & Q_{\varepsilon_j} \delta_{rs} \end{bmatrix} \begin{bmatrix} \Omega_s^{(j)w T} \\ -\Omega_s^{(j) T} \\ \Omega_s^{(j)e T} \end{bmatrix}, \quad N \geq 0 \quad (38)$$

$$P_j(N) = \sum_{r=0}^{n_1} \sum_{s=0}^{n_1} \begin{bmatrix} \Omega_r^{(j)w} & -\Omega_r^{(j)} & \Omega_r^{(j)y} \end{bmatrix} \begin{bmatrix} Q_{\bar{w}_{ij}} \delta_{rs} & \bar{S}_j \delta_{rs} & \Lambda_{N+1+s-r}^{(j)\bar{w}_j} \\ \bar{S}_j^T \delta_{rs} & Q_{v_j} \delta_{rs} & \Lambda_{N+1+s-r}^{(j)v_j} \\ \Lambda_{N+1+r-s}^{(j)\bar{w}_j T} & \Lambda_{N+1+r-s}^{(j)v_j T} & Q_{\varepsilon_j} \delta_{rs} \end{bmatrix} \begin{bmatrix} \Omega_s^{(j)w T} \\ -\Omega_s^{(j) T} \\ \Omega_s^{(j)y T} \end{bmatrix}, \quad N < 0 \quad (39)$$

the cross covariance  $P_{ij}(N) = E[\tilde{\alpha}_i(t | t + N) \tilde{\alpha}_j^T(t | t + N)]$  between any two local sensor is computed by

$$P_{ij}(N) = \sum_{r=0}^{n_0} \sum_{s=0}^{n_0} \begin{bmatrix} \Omega_r^{(i)w} & -\Omega_r^{(i)} & \Omega_r^{(i)e} \end{bmatrix} \begin{bmatrix} Q_{\bar{w}_{ij}} \delta_{rs} & \bar{S}_j \delta_{rs} & \Lambda_{s-r}^{(j)\bar{w}_j} \\ \bar{S}_j^T \delta_{rs} & Q_{v_j} \delta_{rs} & \Sigma_{ij}^{ve}(r, s) \\ \Lambda_{r-s}^{(i)\bar{w}_j T} & \Sigma_{ji}^{vT}(r, s) & \Sigma_{ij}^{ee}(r, s) \end{bmatrix} \begin{bmatrix} \Omega_s^{(j)w T} \\ -\Omega_s^{(j) T} \\ \Omega_s^{(j)e T} \end{bmatrix}, \quad N \geq 0 \quad (40)$$

$$P_{ij}(N) = \sum_{r=0}^{n_1} \sum_{s=0}^{n_1} \begin{bmatrix} \Omega_r^{(i)w} & -\Omega_r^{(i)} & \Omega_r^{(i)y} \end{bmatrix} \begin{bmatrix} Q_{\bar{w}_{ij}} \delta_{rs} & \bar{S}_j \delta_{rs} & \Lambda_{N+1+r-s}^{(j)\bar{w}_j} \\ \bar{S}_j^T \delta_{rs} & Q_{v_j} \delta_{rs} & \Sigma_{ij}^{ve}(r, N+1+s) \\ \Lambda_{N+1+r-s}^{(i)\bar{w}_j T} & \Sigma_{ji}^{vT}(r, N+1+r) & \Sigma_{ij}^{ee}(N+1+r, N+1+s) \end{bmatrix} \begin{bmatrix} \Omega_s^{(j)w T} \\ -\Omega_s^{(j) T} \\ \Omega_s^{(j)y T} \end{bmatrix}, \quad N < 0 \quad (41)$$

$$\Sigma_{ji}^{vT}(r, s) = \bar{S}_j^T F_{s-r}^{(j)T} + Q_{v_j}^T G_{s-r}^{(j)T} \quad (42)$$

$$\Sigma_{ij}^{ve}(r, s) = \bar{S}_j^T F_{s-r}^{(j)T} + Q_{v_j} G_{s-r}^{(j)T} \quad (43)$$

$$\Sigma_{ij}^{ee}(r, s) = \sum_{k=0}^{\infty} F_k^{(i)} Q_{\bar{w}_{ij}} F_{k+s-r}^{(j)T} + \sum_{k=0}^{\infty} F_k^{(i)} \bar{S}_j G_{k+s-r}^{(j)T} + \sum_{k=0}^{\infty} G_k^{(i)} S_j^T F_{k+s-r}^{(j)T} + \sum_{k=0}^{\infty} G_k^{(i)} Q_{v_j} G_{k+s-r}^{(j)T} \quad (44)$$

Proof: (36) and (37) have the simplified expression

$$\tilde{\alpha}_j(t | t + N) = \sum_{r=0}^{n_0} \begin{bmatrix} \Omega_r^{(j)w} & -\Omega_r^{(j)} & \Omega_r^{(j)e} \end{bmatrix} \begin{bmatrix} w(t+r) \\ v_j(t+r) \\ \varepsilon_j(t+r) \end{bmatrix}, \quad N \geq 0 \quad (45)$$

$$\tilde{\alpha}_i(t | t + N) = \sum_{r=0}^{n_1} \begin{bmatrix} \Omega_r^{(i)w} & -\Omega_r^{(i)} & \Omega_r^{(i)y} \end{bmatrix} \begin{bmatrix} w(t+r) \\ v_i(t+r) \\ \varepsilon_i(t+N+1+r) \end{bmatrix}, \quad N < 0 \quad (46)$$

Thus by (9) and (18), with the Assumption 1, (38)~(44) is obtained. The proof is completed. The proof is completed.

#### 4. Information Fusion Optimal State Estimator

**Theorem 4** The optimal fused state filtering  $\hat{\alpha}_0(t | t + N)$  in the linear minimum variance sense is given as [14, 15]

$$\hat{\alpha}_0(t|t+N) = \sum_{j=1}^K \alpha_j(N) \hat{\alpha}_j(t|t+N) \quad (47)$$

The optimal fusion weight according to the different weighted fusion algorithm has different calculation formula is as follows.

When weighted criteria are weighted by matrices, the weighted coefficients are

$$\alpha(N) = [\alpha_1(N), \dots, \alpha_K(N)] = (e^T P^{-1}(N) e)^{-1} e^T P^{-1}(N) \quad (48)$$

Where  $P(N) = (P_{ij})_{nL \times nL}$ ,  $i, j = 1, 2, \dots, L$ , and  $e^T = [I_m \quad I_m \quad \dots \quad I_m]$ ,  $P_{ij}$  can be calculated by (40) ~ (44).

The optimal fused variance matrix is given as

$$P_0(N) = (e^T P^{-1}(N) e)^{-1}, \text{ and } \text{tr} P_0 \leq \text{tr} P_j, \quad j = 1, 2, \dots, K \quad (49)$$

When weighted criteria are weighted by diagonal matrices, the weighted coefficients  $A_j = \text{diag}(a_{jk})$ ,  $k = 1, \dots, n$ , while  $a_{jk}$  are given by

$$a_j = [a_{1j}, a_{2j}, \dots, a_{Lj}], \quad j = 1, \dots, n \quad (50)$$

$$a_j = (e^T P^{jj}(t|t)^{-1} e)^{-1} e^T P^{jj}(t|t)^{-1}, \quad j = 1, \dots, n \quad (51)$$

where  $e = [1 \quad \dots \quad 1]^T$  is a  $L \times 1$  row vector, and  $L \times L$  matrices is defined as  $P^{jj}(t|t) = (P_{lk}^{jj}(t|t))$ ,  $l, k = 1, 2, \dots, K$ ,  $P_{lk}^{jj}(t|t)$  is the  $j$ th row and  $j$ th column diagonal element of  $P_{lk}(t|t)$ .  $P_{lk}(t|t)$  is computed by Theorem 3.

When weighted criteria are weighted by scalars, the weighted coefficients are

$$[\alpha_1, \dots, \alpha_K] = \frac{e^T P_{\text{tr}(\alpha)}^{-1} e}{e^T P_{\text{tr}(\alpha)}^{-1} e} \quad (52)$$

where  $P_{\text{tr}} = (\text{tr} P_{ij})_{K \times K}$  is a  $L \times L$  matrix.

The optimal fused variance matrix is given as

$$P_0(N) = \sum_{i,j=1}^L a_i a_j P_{ij}(N) \text{ and } \text{tr} P_0 \leq \text{tr} P_j, \quad j = 1, 2, \dots, K \quad (53)$$

## 5. Simulation Example

Consider 2-sensor discrete-time linear time-invariant stochastic tracking system (1)~(3), where  $T = 0.8$  is the sampled period,  $A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ ,  $\Gamma = \begin{bmatrix} T^2/2 & T \\ T & 1 \end{bmatrix}$ ,  $H_1 = [1 \quad 0]$ ,  $H_2 = [1 \quad 0]$ ,  $H_3 = [1 \quad 0]$ ,  $F_1 = 0.4$ ,  $F_2 = 0.5$ ,  $F_3 = 0.6$ . And  $\xi_j(t)$  is white noises with zero mean,  $\sigma_{\xi_1}^2 = 0.81$ ,  $\sigma_{\xi_2}^2 = 3$ ,  $\sigma_{\xi_3}^2 = 6$ . And  $w(t)$  is white noises with zero mean and its covariance is  $\sigma_w^2 = 1$ .  $v_j(t)$  and  $w(t)$  are correlative noise.

From (2) and (3), we have

$$z_j(t) = M_j x(t) + v_j(t) \quad (54)$$

where

$$M_j = M_0 \Phi - F_j M_0, \quad v_j(t) = H_0 \Gamma w(t) + \xi_j(t) \quad (55)$$

State estimation problem is based on the measurement  $(z_j(t+N), z_j(t+N-1), \dots)$ , to obtain the linear minimum variance fusion state  $\hat{x}_0(t|t+1)$ .

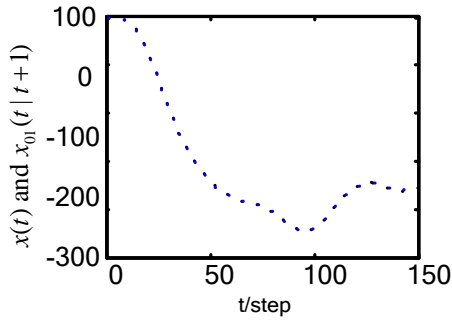


Fig.1 The position and Fusion state smooth weighted by matrix

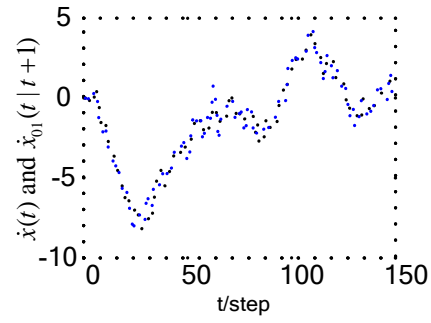


Fig.2 The velocity and Fusion state smooth weighted by matrix

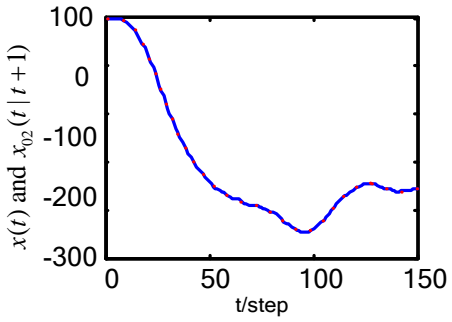


Fig.3 The position and Fusion state smooth weighted by diagonal matrices

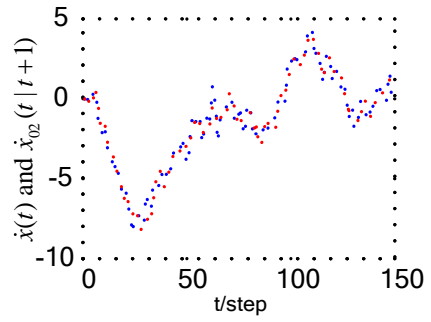


Fig.4 The velocity and Fusion state smooth weighted by diagonal matrices

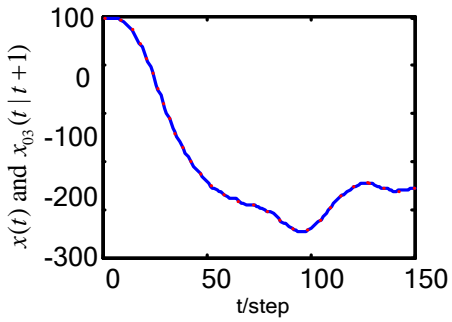


Fig.5 The position and Fusion state smooth weighted by scalars

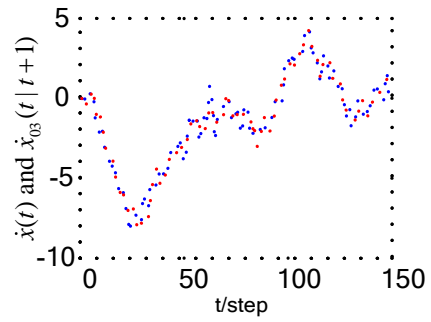


Fig.6 The velocity and Fusion state smooth weighted by scalars

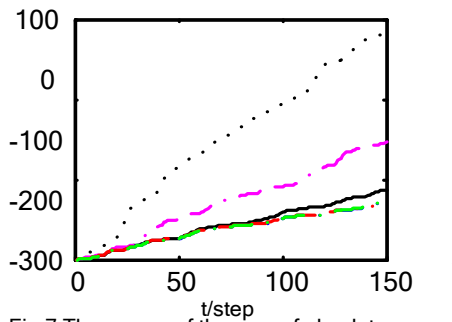


Fig.7 The curves of the sum of absolute error curve for local and fusion filters of the position

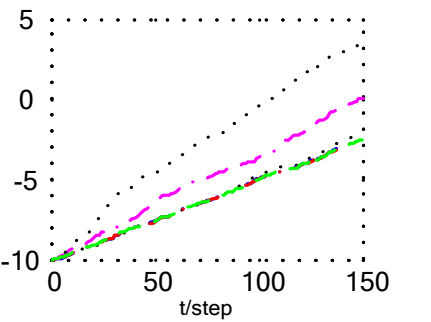
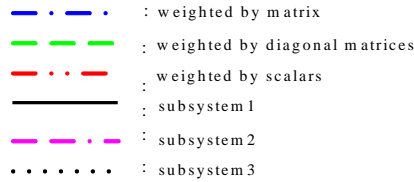


Fig.8 The curves of the sum of absolute error curve for local and fusion filters of the velocity





The simulation results are shown in Fig.1-8. Figure 1 to 6 gives the fusion smoothing weighted by matrix, diagonal matrices and scalars. Figure 7 and Figure 8 are the absolute error curve for the state smoothing and fusion state of position and velocity weighted by matrix, diagonal matrices and scalars. In the figure, the accuracy of the fusion state filtering is higher than any of the single sensor. Simulation results show no significant difference between the three kinds of distributed fusion algorithm, and the three fusion curves almost coincide. But the scalar weighting fusion predictor can significantly reduce the computational burden, and provides a fast information fusion estimation algorithm.

## 6. Conclusions

In this paper, distributed information fusion state estimators for discrete-time stochastic linear systems with sensor errors are presented. Information fusion rule, which adopted in this paper, is weighted by matrix, diagonal matrices and scalars. They can deal with the fused filtering, smoothing and prediction estimation in a unified form. For the sake of calculating the information fusion weighted value, formulas of computation for the variance and cross-covariance matrices are presented. The estimation accuracy for the system is greatly improved compared with the single local sensor. Matrix, diagonal matrices, and scalars weight the accuracy of above three kinds of weighted fusion filtering from high to low. But the computational burden is on the contrary. Fusion filtering weighted by matrix has a large computational burden, and weighted by scalars with minimal computational burden, and it is suitable for real-time applications. The simulation example shows its validity. The algorithm presented in this paper has many advantages. It can solve the fused filtering, smoothing and prediction in a unified way, and can handle self-tuning fusion estimation based on ARMA innovation model.

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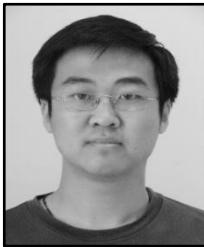
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