Bessel Function Self-Feedback Chaotic Neural Network Model and Applications

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Abstract

In this paper, a new chaotic neural network model is proposed, we introduce a Bessel function as self-feedback term in this model, Compared with other chaotic neural network model, owing to the Bessel function is a nonlinear function with good nature, and it has stronger function approximation ability, so that the novel chaotic network model has stronger traversal search ability. When it is applied to solve combinatorial optimization problems, the simulation results show that the network has better ability to avoid network convergence to local minima if the appropriate coefficient of expansion and the network has been taken, so the efficiency of network optimization capability is improved.

Keywords: chaotic neural network; Bessel function; self-feedback; TSP problem

1. Introduction

In recent years, transiently chaotic neural networks (TCNN) with its complex nonlinear dynamical behavior successfully and efficiently have solved the traveling salesman problem (TSP), and have been extensively research [1-6]. But in the reference [1], Chen-Aihara chaotic neural network is introduced through the simulated annealing mechanism , the network showed both transient and stable chaotic search behavior can converge at one point, so as to effectively overcome the shortcomings of the Hopfield network , greatly increase the accuracy and speed of the network to solve the traveling salesman problem and proves Chen-Aihara chaotic neural network can reach asymptotic stability, convergence of network provides a theoretical basis. Chaotic dynamics due to feedback is related to the different nature of the self-feedback function that can produce different chaotic traversal search, Xu Yaoqun, Sun Ming, Yang xueling put forward a nonlinear wavelet functions and trigonometric functions from transient chaotic neural feedback network model [7-9], such a model with a more sophisticated search capabilities in chaos chaotic search process. In this paper, based Bessel function well localized nature, by improving self- feedback term Chen-Aihara network , construct a Bessel function of the nonlinear self-feedback transiently chaotic neural network, using a unified framework for the construction of a theory Kwok network energy function and prove that the proposed network model under given conditions can be achieved asymptotically stable, continuous function optimization through simulation studies 10 cities TSP problem and show that, Bessel function self-feedback nonlinear transient chaotic neural network after selecting the appropriate scaling factor has a stronger ability to avoid network into a local minimum point.

2. Chaotic Neural Network of Bessel Function Self-feedback

2.1. Bessel Function

In the standard second-order ordinary differential equations in the following, $y''(x) + p(x)y'(x) + q(x)y(x) = 0$, let $p(x) = \frac{1}{x}$, $q(x) = 1 - \frac{v^2}{x^2}$, it come to v-order Bessel equation $x^2 y''(x) + xy'(x) + (x^2 - v^2)y(x) = 0$,it can lead to v-order Bessel function in the process of solving method using power series:

$$J_v(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!\,\Gamma(k+v+1)} \left(\frac{x}{2}\right)^{2k+v} \tag{1}$$

If let v =-1, 0 , 1, then Bessel function of the image shown in Figure 1.
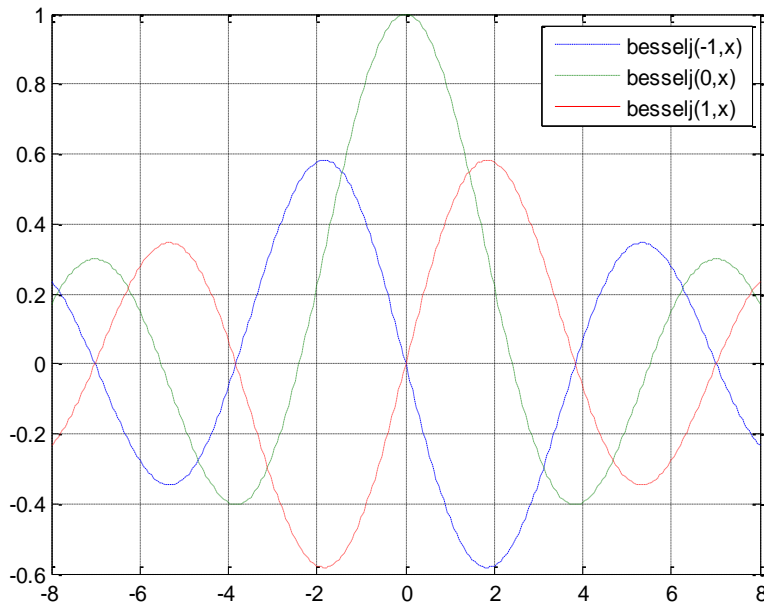
Figure 1. Figure of the Function When v =-1, 0, 1

It is obvious when v =-1or1, Bessel function is an odd function, and when v =0, Bessel function is an even function. These three Bessel function was shock attenuation and amplitude is less than or equal to 1.

2.2. Chaotic Neuron Model of Bessel Function Self-feedback

Select v=1, we construct a self-feedback term chaotic neural network chaotic neuron model in the following:

$$x(t) = \frac{1}{1 + \exp(- y(t)/\varepsilon)} \tag{2}$$

$$y(t+1) = ky(t) - z(t)g(x(t) - I_0) \tag{3}$$

$$g(x) = J_1(cx) \tag{4}$$

$$z(t+1) = (1-\beta)z(t) \tag{5}$$

where $x(t)$ is the output of the neuron at the time of t; $y(t)$ is the internal state of the neuron at the time of t; I_0 is a positive parameter; k is a damping factor of nerve membrane($0 \le k \le 1$); ε is steepness parameter of the activation function; $z(t)$ is the self-feedback connection weight; β is the damping factor of $z(t)$; c is the scaling factor.

In order to make the neuron behave transient chaotic behavior, the parameters are set as follows: $y(1) = 0.825, k = 0.9, z(1) = 0.85, \varepsilon = 0.02, I_0 = 0.45, \beta = 0.002,$
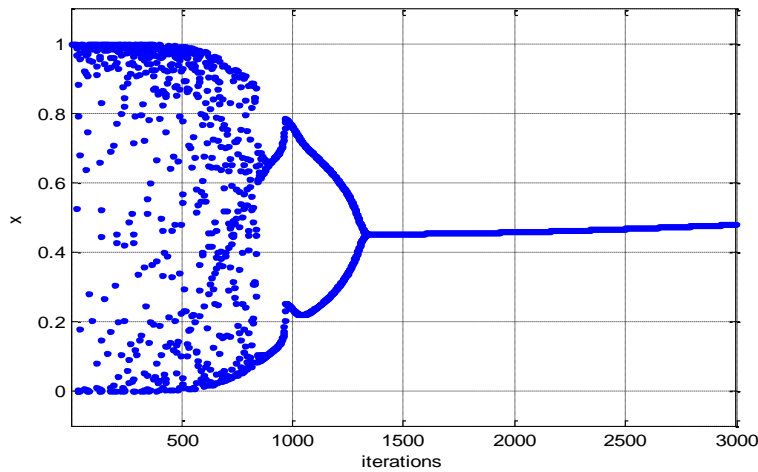
The state bifurcation figures are respectively shown as Figure 2-Figure 4 when $c = 4, c = 5,$ $c = 7$.
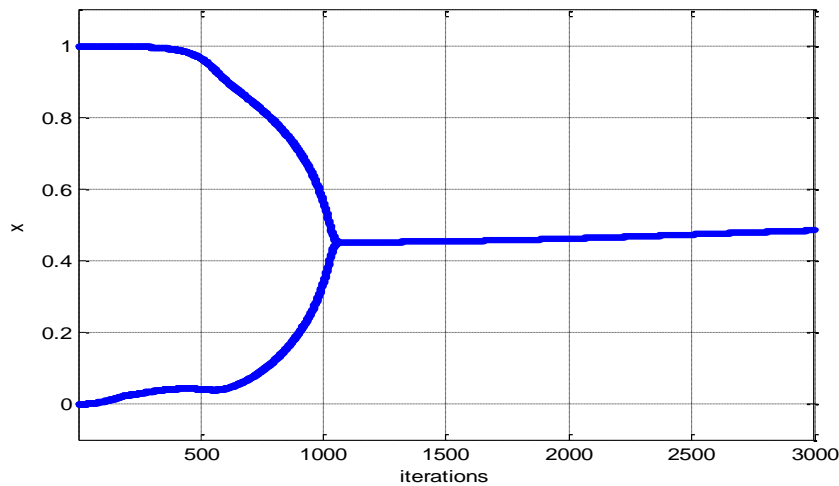
Figure 2. State Bifurcation Figure of the Neuron When *c* =4
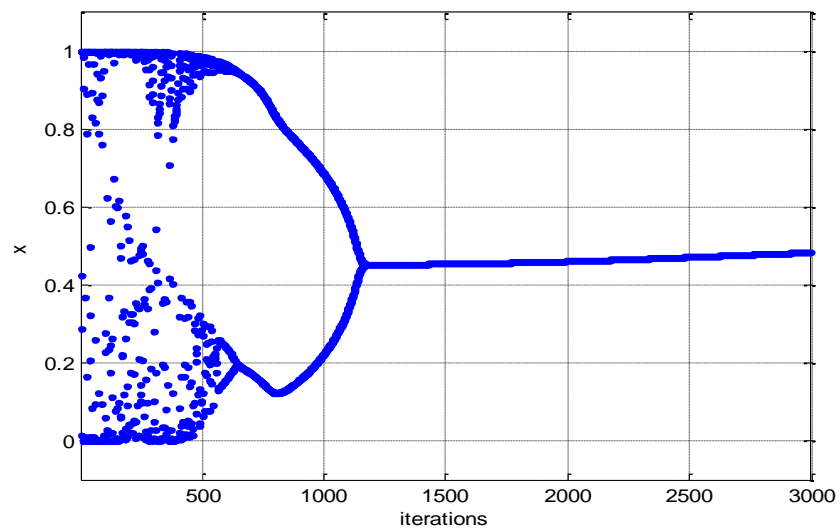
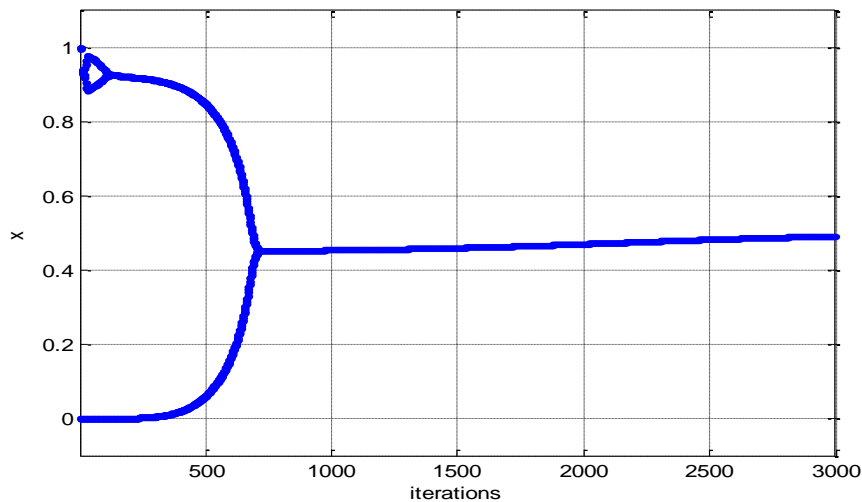**Figure 3. State Bifurcation Figure of the Neuron When *c* =5**

Figure 4. State Bifurcation Figure of the Neuron When *c* =7

For easy comparison, we take the same parameters:

$y(1) = 0.825, k = 0.9, z(1) = 0.85, \varepsilon = 0.02, I_0 = 0.45, \beta = 0.002.$

The state bifurcation Figures of the Chen-Aihara's network is shown as Figure 5.

Figure 5. State Bifurcation Figure of the Chen-Aihara's Neuron

We can draw following conclusions by above figures:

(1) Our neural network model has transient behavior of chaotic dynamics. When $z(t)$ decay continuously, the network will gradually become stable equilibrium point through a chaotic bifurcation process, it is easy to know, in the other parameters remain unchanged, when stretching coefficient is about c=7 ,chaos has the most sophisticated search capabilities.

(2) Comparison with the chaotic neuron model of Chen-Aihara, when the chaotic neuron model to obtain the appropriate scaling factor, Bessel function of the feedback we have created has a more complex chaotic search behavior.

2.3. Chaotic Neuron Model of Bessel Function Self-feedback

Let

$$x_i(t) = \frac{1}{1 + \exp(-y_i(t)/\varepsilon)} \tag{6}$$

$$y_i(t+1) = k y_i(t) + \alpha \left[\sum_{\substack{j=1 \\ j \neq i}}^{n} w_{ij} x_j(t) + I_i \right] - z_i(t) g(x_i(t) - I_0) \quad (i = 1, \cdots, n) \tag{7}$$

$$g(x) = J_1(cx) \tag{8}$$

$$z_i(t+1) = (1 - \beta) z_i(t) \tag{9}$$

Where $x_i(t)$ is the output of neuron i ; $y_i(t)$ is the internal state for neuron i ; $z_i(t)$ self-feedback connection weight; β is the simulated annealing parameter; w_{ij} is the connection weight from neuron j to neuron i , $w_{ij} = w_{ji}$, $w_{ii} = 0$; I_i is the input bias of neuron i ; ε is the steepness parameters of the activation function; k is the damping factor of the nerve membrane, $0 \leq k \leq 1$; I_0 is a positive parameter; $g(x_i(t) - I_0)$ is feedback term; c is the scale factor of the Bessel function.

When $g(x) = x$, the feedback term is linear, the model is a classic Chen-Aihara chaotic neural network, this paper presents a continuous Bessel function with $v = 1$ as nonlinear self-feedback term. Obviously self-feedback term is non-linear. Since the introduction of nonlinear feedback term Bessel function of chaotic neural networks to make new dynamic behavior, compared to linear self-feedback term, the chaotic neural network, Bessel functions chaotic neural networks has more complex dynamics behavior, the internal state of the network with a more complex chaotic search behavior.

2.4. Network Energy Function and Stability Analysis

According to the unified theoretical framework, the energy function network model is as follows

$$E = E_{Hop} + H \tag{10}$$

$$E_{Hop} = -\frac{1}{2} \sum_{i,j}^{n} w_{i,j} x_i x_j - \sum_{i}^{n} I_i x_i + \frac{\varepsilon}{\tau} \sum_{i}^{n} \int_{0}^{x_i} \ln \frac{x}{1-x} dx \tag{11}$$

$$H = \lambda \sum_{i}^{n} \int_{0}^{x_i} z_i g(x - I_0) dx = \lambda \sum_{i}^{n} z_i x_i g(\overline{x}_i - I_0) \tag{12}$$

Where E_{Hop} is the Hopfield network energy function, H is additional energy term, energy function for the network model, $W = W^T$ and $w_{ii} = 0$

Stability is an important characteristic of nonlinear systems, stability analysis can provide a theoretical basis for the convergence of the network. Analysis energy function is an important method of nonlinear stability analysis of complex systems.

Since $J_1'(x) = J_0(x) - J_1(x)/x < 1$, then $J_1'(cx) = c J_0(cx) - J_1(cx)/x < c$

The network with given conditions can be achieved asymptotically stable by the literature [7] derivation process.

Theorem 1 The sufficient condition of Bessel functions asymptotically stable is: there is a minimum eigenvalue of the connection weight matrix (diagonal elements $w_{ii}^{'} = -(\overline{\theta}_i z_i c)/\alpha$) that satisfied:

$$-\lambda^{'} < \frac{2\varepsilon}{\alpha}[4(1-\theta_i) + 4k\theta_i] = \frac{8\varepsilon}{\alpha}[1-(1-k)\theta_i] \tag{13}$$

3. Bessel Function Self-feedback Chaotic Neural Network Applications

In order to verify the validity of transiently chaotic neural networks with the Bessel functions feedback term, which are used in the function optimization and combinatorial optimization problems.

3.1. Application to Function Optimization

Select the following optimization function [10]

$$f(x_1, x_2) = (x_1 - 0.7)^2[(x_2 + 0.6)^2 + 0.1] + (x_2 - 0.5)^2[(x_1 + 0.4)^2 + 0.15] \tag{14}$$

Minimum value of function is 0, the minimum point of function is (0.7, 0.5), local minimum point is (0.6, 0.4) and (0.6, 0.5).

Let ε =0.04, k =1, α =0.05, I_0 =0.45, β =0.002, $y(1)$ =0.825, $y(2)$ =0.825, $z_1(1) = z_2(1)$ =0.85, c =7. When using Bessel function self-feedback transiently chaotic neural network model to solving optimization function, the evolution of the energy function shown in Figure 6.
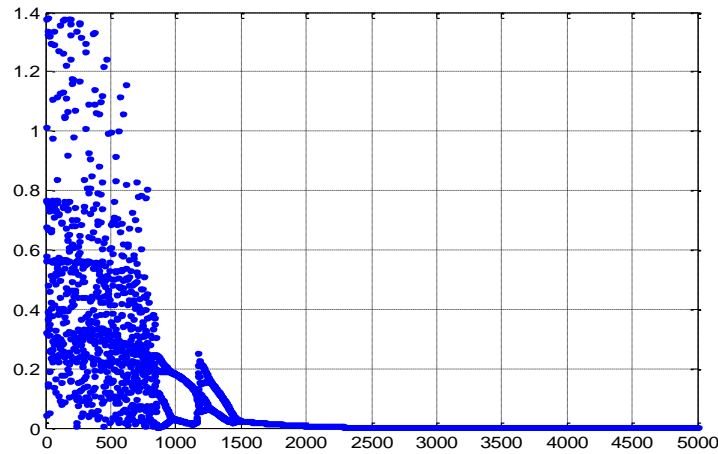
Figure 6. State Bifurcation Figure of our Neuron Network

When our network is running 7000 times, the value of energy function is 1.367386×10^{-11} and x_1 =0.699997 x_2 = 0.499999, the time evolution figure of x_1 and x_2 is shown by Figure 7.
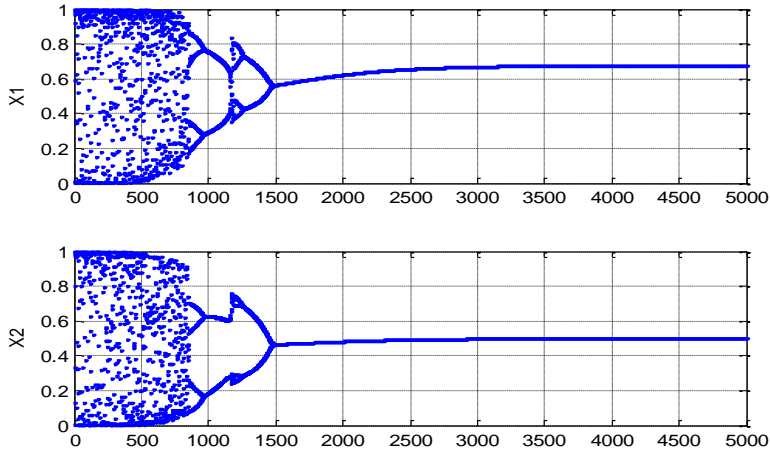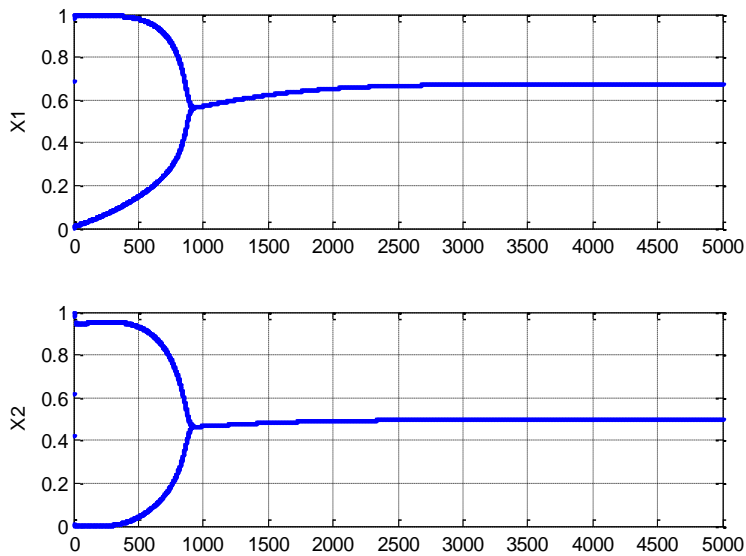
Figure 7. Evolution Figure of x_1 and x_2 in our Neural Network

In order to compare the Bessel function self-feedback chaotic neural network and Chen-Aihara chaos neural network model, the parameters are set as follows:

$$\varepsilon = 0.04, k = 1, \alpha = 0.05, I_0 = 0.45, \beta = 0.002, y(1) = 0.825, y(2) = 0.825, z_1(1) = z_2(1) = 0.85.$$

When Chen-Aihara chaos neural network is running 7000 times, the value of energy function is 2.430467×10^{-12} and $x_1 = 0.699998$ $x_2 = 0.499999$, the time evolution figure of x_1 and x_2 is shown by Figure 8.

Figure 8. Evolution Figure of x_1 and x_2 in Chen-Aihara Chaos Neural Network

As seen from above, for the continuous functions optimal solution problem, two kinds of neural network model are basically the same solving ability, the chaotic neural network model with Bessel functions the feedback has better search capabilities of chaos, it has a stronger ability to avoid local optima caught.

### 3.2. Application to Combination Optimization (TSP)

The Traveling Salesman Problem(TSP) is a classically combinational optimization problem and is a NP-hard problem, Symmetric TSP problem with n cities possible have $(n-1)!/2$ paths, Find an effective way to solve this problem is the target of many scholars over the years. TSP can be described as follows: To confirm a shortest path and need to visit every city only once when known N cities and the distance between two cities. This paper will self-feedback Bessel function transiently chaotic neural network model applied to solve the 10 cities TSP problem, simulation results show that this neural network model has a good ability to solve TSP problems.

The energy function takes the follow form [10]:

$$E = \frac{A}{2}\sum_{i=1}^{n}(\sum_{i=1}^{n}x_{ij}-1)^2 + \frac{B}{2}\sum_{j=1}^{n}(\sum_{i=1}^{n}x_{ij}-1)^2 + \frac{D}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}d_{ik}x_{ij}(x_{k,j+1}+x_{k,j-1}) \qquad (15)$$

Assume d_{xy} to be the distance between city x and y. Because the symmetry of the determinant, coefficient A=B, and a global shortest value of E expresses a shortest effective path. This paper adopts the following 10-city unitary coordinates: (0.4, 0.4439); (0.2439, 0.1463); (0.1707, 0.2293); (0.2293, 0.716); (0.5171,0.9414); (0.8732, 0.6536); (0.6878, 0.5219); (0.8488, 0.3609); (0.6683, 0.2536); (0.6195, 0.2634).

The optimal distance from the feedback from the Bessel function transiently chaotic neural network model simulation results is 2.6776. Figure 9 shows the optimal path.
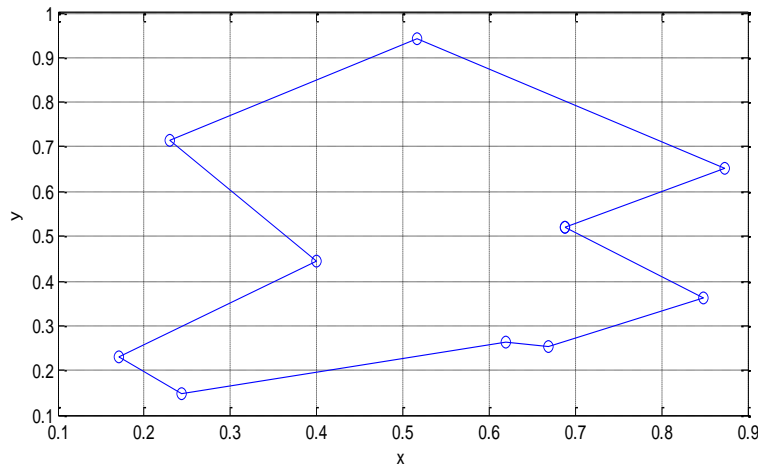
Figure 9. The Optimal Path of 10-city TSP

Select the energy function (15), the parameters of the network are set as follows:

$A = B = 1$, $D = 2.5$, $\alpha = 0.08$, $k = 0.9$, $I_0 = 0.45$, $\varepsilon = 0.04$, $z(1) = 0.8$

The simulation results of 100 different internal conditions with different β and c are summarized in Table 1. the column 'NLP','NOP','RLP','RGM' and ANI respectively represents the number of legitimate path, the number of optimal path, the rate of legitimate path, the rate of global minima, the average number of iterations.

Table 1. Simulation Results with Different *c* and *β*

c	β	NLP	NOP	RLP	RGM	ANI
4	0.002	88	88	88%	88%	512
	0.003	85	85	85%	85%	477
	0.004	82	81	82%	81%	432
5	0.002	97	94	97%	94%	697
	0.003	95	93	95%	93%	631
	0.004	94	91	94%	91%	546
6	0.002	100	100	100%	100%	845
	0.003	99	98	99%	98%	763
	0.004	99	97	99%	97%	712
7	0.002	100	99	100%	99%	1056
	0.003	99	96	99%	96%	943
	0.004	99	95	99%	95%	851
8	0.002	100	55	100%	55%	1385
	0.003	100	52	100%	52%	1342
	0.004	100	50	100%	50%	1261

As seen from Table 1:

(1) When stretching factor is 6 or 7 and simulated annealing parameters is β =0.002, Bessel function self-feedback transiently chaotic neural network model to solve the 10 cities in the traveling salesman problem with the best effect, almost 100 percent to get the optimal path.

(2) The value of the scale factor and simulated annealing parameters have a significant impact on the efficiency of the network model to solve: the smaller the coefficient of expansion and contraction, the more chaotic search process is relatively simple algorithms easier to get the minimum value; Conversely, the greater the stretching coefficient, chaos the more complicated the search process, the algorithm has stronger to get rid of local optimum value reaches the global optimum.

(3) Simulated annealing parameters in the vicinity of 0.003 for optimizing network capacity is not affected, but have a more significant impact on the convergence speed of the network, when the annealing parameters away from 0.003, the network optimization capability significantly decreased.

4. Conclusion

This paper presents a new self-feedback neural network, the feedback chaotic neural network with Bessel functions. Lyapunov stability analysis is performed by using a unified theoretical framework, the results show that the neural network model under the given conditions can reached asymptotically stable, when you select the proper stretching Department, the solving process of optimization problems and the traveling salesman problem show that this neural network model has more to avoid falling into local minima network capacity, which can more effectively improve optimization capabilities of the network.

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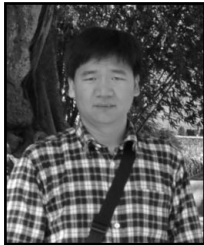
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