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The Research to Regular Sequences of Similar Algorithm Based on Sparse Linear

Di Jin

School of Computer and Information Engineering
Harbin University of Commerce
Harbin, China
jindi800622@sohu.com

Abstract

The main idea of Regular Sequences of Similar Algorithm based on Sparse Linear (HR Algorithm) is to calculation the near optimal solution from the solution of sparse integer programming model. The advantages of HR algorithm are simplifying the problem of huge solution space of integer programming, reducing the number of goal constraints, and turning the problem into a simplified integer programming. The near optimal solution, which is hard to apply accurate method, can be found under limited time by using HR algorithm based on regular sequences.

Keywords: *Sparse linear; Regular Sequences of Similar Algorithm; near optimal solution*

1. Introduction

HR algorithm is to solve the scheduling problem of concurrent open-shop Their process is not bound by the orders, the difference between them is concurrent open-shop can make multiple machines work on a workpiece simultaneously, which has more feasible solution space, therefore most of the concurrent open-shop scheduling problem is NP problem.(non-deterministic polynomial problem)

Mathematical programming method should make n related and quantifiable decisions into the Decision Variables(x1, x2,..., xn), the reasonable measure of the results is presented as a mathematical function of these Decision Variables, the function is called the objective function, restrictions of a scheduling model will be expressed by inequalities among Decision Variables, then it becomes the mathematical programming model, solving the scheduling problem can get the optimal solution. But this method will lead to a very large solution space, if you use computer calculation; calculated time is often unacceptable too, so this method can be used when solving small problems. When the problem is NP problem, the established model is just a representation of the scheduling problem, because it is difficult to give the optimal solution of the problem in limited time.

HR algorithm can help reduce the component reserve, and emolliate the total time of component release, then largely cut the total completion time of each workpiece, while calculating the minimum manufactural time simultaneously. Applying the HR algorithm, the complex scheduling problem of concurrent open-shop problem turns into a normal integer programming based on permutation and combination, which makes the problem become a normally solvable permutation and combination problem.

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In this paper, the executive procedure of the HR algorithm has been researched, and the error, application and practice value of the HR algorithm have bee proved.

2. The Executive Procedure of HR Algorithm

The executive procedure of HR algorithm is to make the integer programming model of scheduling concurrent open-shop model sparse, and then calculate its solutions. After processing the solutions, the machines process the workpiece followed the order which is determined by the rules based on the processed solutions. The order of executive order is as Figure 2.1.

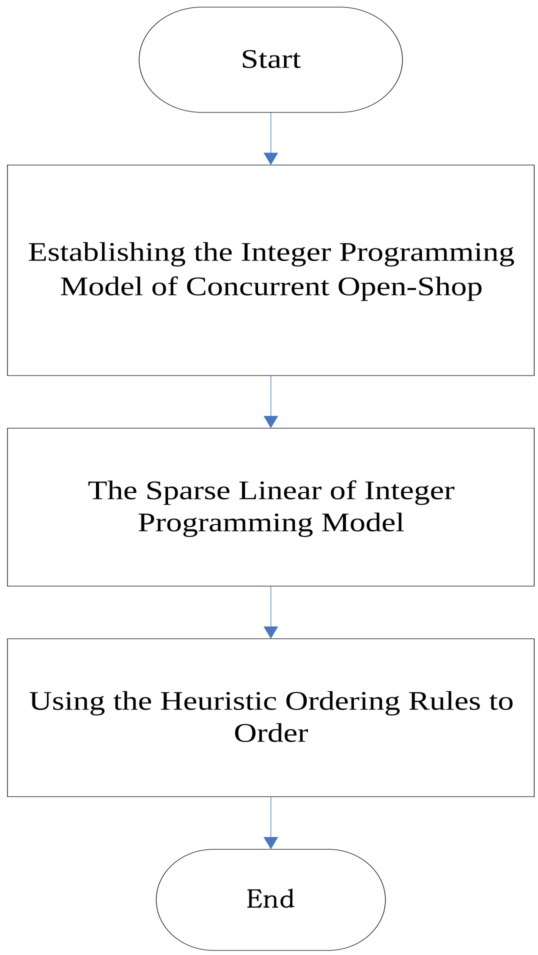

Figure 2.1. Order of Executive

The specify phrases of HR algorithm are as following:

(1)Establishing the Integer Programming Model for a Scheduling Problem.

(2)Making the Built Integer Programming Model sparse

(3)Sbustitution the Process-Time Matrix into Sparse Linear

(4)Processing the Solution of Sparse Linear Model by Ordering Workpiece finished time-Cj from small to large.

(5)Processing the workpiece in each machine-i by the above order, wrokpiece with little completion time will be processed firstly, so the manchines are full of use with no leisure time.

(6)If the machine finishes the processing, that machine is released as waiting condition, and searching the workpiece which features the least completion time in unprocessed workpieces.

(7)To ensure the last workpiece has the less completion time, the less completion- time workpiece should be drawed out from the interrupted mahcines, if in prepared machine, the existence of interrupted machine is ture.

(8)The machine and processing procedure will stop as the processing of all workpiece has been finished.

The phrases (1) to (3) are the pre-processing phrase of algorithm, and the phrases (4) to (7) are ordering phrase. Phrase (2)-the resolution to interrupted machin refers to LPT (Longest processing time first), a practice sample of Gantt chart compasion in Figure 2.2 and Figure 2.3 is LPT rule applied and no LPT rule applied respectively. From the compasion of the scheduling Gantt chart of 4 machines and 3 workpieces in Figures, the LPT rules can reduce the machine waiting time during ordering process. The process figure of ordering part of HR algorithm is as Figure 2.4.

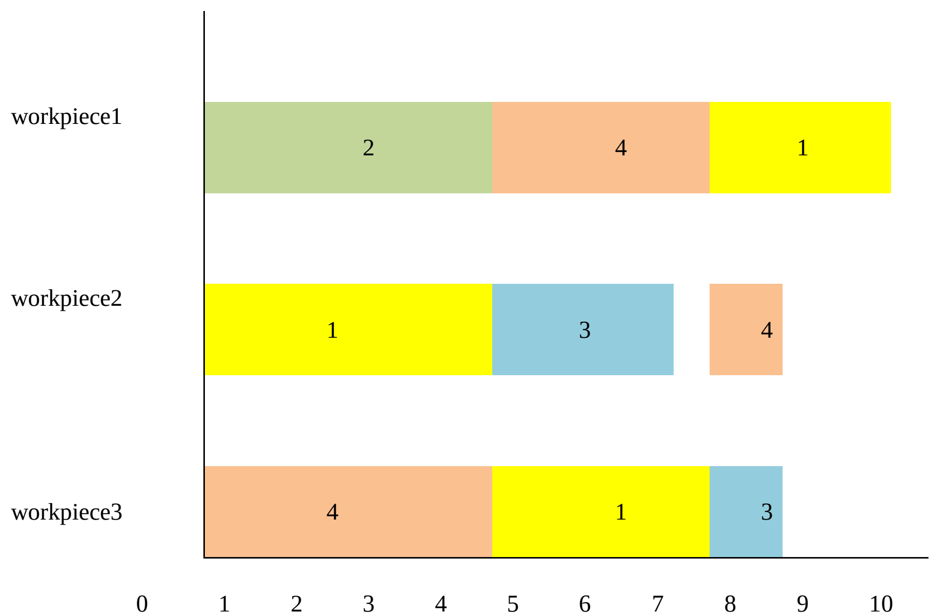

Figure 2.2. The Application of LPT Rules

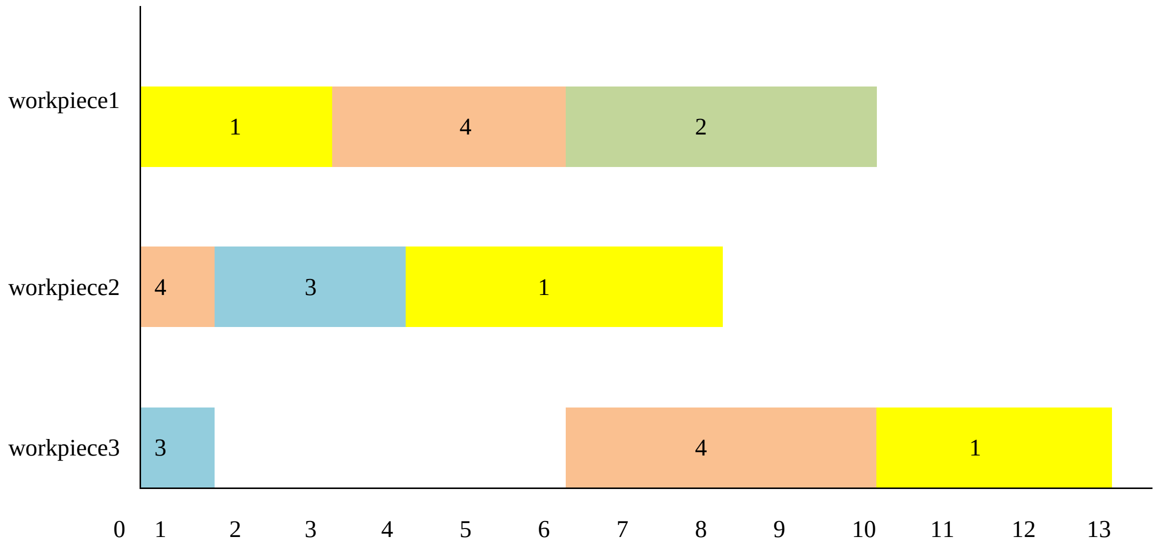

Figure 2.3. The No Application of LPT Rules

3. The Algorithm Error

HR algorithm is an approximate algorithm based on sparse linear, and error, which are allowed to NP problem, exist between the solutions gotten from it and optimal solutions. In this section, the error between minimum manufacture time of HR algorithm solved and total completion time of workpieces will be calculated, and the worst performance of HR algorithm will also count out. In scheduling problem, the error refers to the variances between solutions gotten from HR algorithm and optimal solutions, which can reflect the deviation situation. Its value fluctuate around 1, the closer to 1, the better the algorithmic solution. The ratio of HR algorithm solution and optimal solution under worst condition refers to maximum error, which is usually bigger than 1 or equal to 1. If the maximum error is small and its value is closer to 1, the stability of the algorithm is predicted more reliable and the solution gotten from algorithm is closer to optimal solution.

Variable quantities will be defined. F refers to optimal completion solution of concurrent open-shop' scheduling problem, workpiece total completion time and optimal solution refers to $\sum F$.

Near optimal solution of completion time of scheduling concurrent open-shop by using HR algorithm refers to F^{HR}, completion time and near optimal solution refers to $\sum F^{HR}$.

c_j^H refers completion time of workpiece j under optimal solution.

c_j^0 refers to completion time of workpiece j by using the integer programming.

c_{ij}^0 refers to completion time of workpiece j on machine i by using the integer programming.

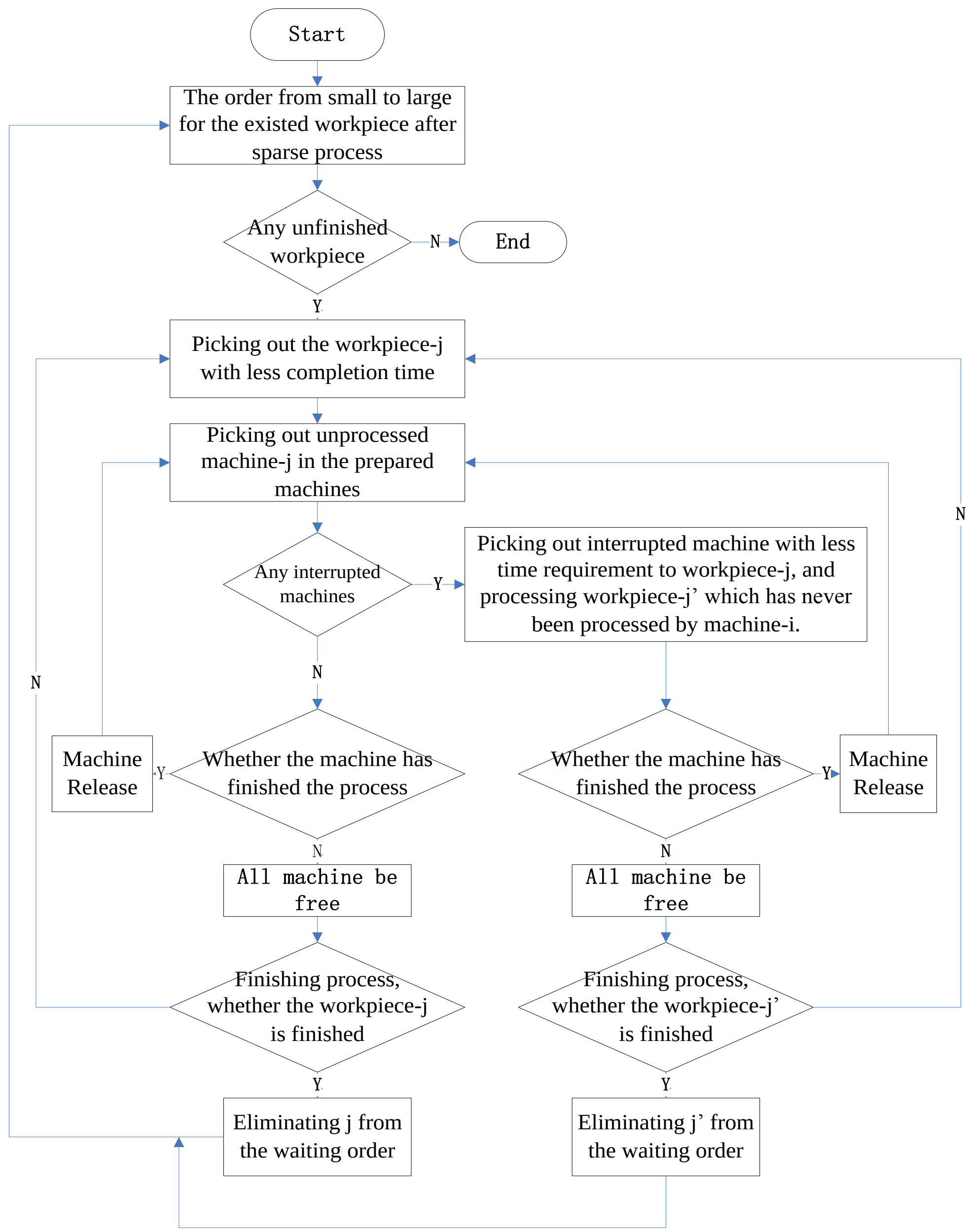

Figure 2.4. The Flow Diagram Ordering Part of HR Algorithm

c_j^H refers to completion time of workpiece j by using the HR algorithm.

c_{ij}^H refers to completion time of workpiece j on machine i by using the HR algorithm.

a_c refers to the maximum error of completion time by using HR algorithm.

a_t refers to the maximum error of total completion time.

The $F^{HR} \leq a_c$ *F and $\sum F^{HR} \leq a_t * \sum F$ are proved from reasoning, the value of a_c and a_t will be supplied. The ratio of worst condition and optimal solution of HR algorithm will be calculated out.

The reasoning calculative process as following:

As machines have no leisure time in HR algorithm, which means the machine is running all the time and the processing can be interrupted, so the formula 3-1 is proved, which says that the completion time of workpiece j on the machine i is equal to all workpiece's total processing time before workpiece j on the machine j. The formula 3-2 can be figured out from the formula 3-1.

$$c_{ij}^{H} = \sum_{k=1}^{j} p_{ik} \qquad \textbf{(3-1)}$$

$$\sum_{k=1}^{j} p_{ik} c_{ik}^{0} \geq \frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^{2} + \sum_{k=1}^{j} p_{ik}^{2}\right] \geq \frac{1}{2}\left(c_{ij}^{H}\right)^{2} \qquad \textbf{(3-2)}$$

From all the $k \leq j$, $i \in \{1, 2, \ldots, m\}$, we can be the inequality:

$$c_{ik}^{0} \leq c_{k}^{0} \leq c_{j}^{0} \qquad \textbf{(3-3)}$$

$$\frac{1}{2}\left(c_{ij}^{H}\right)^{2} \leq \sum_{k=1}^{j} p_{ik} c_{j}^{0} \qquad \textbf{(3-4)}$$

To all the $j \in N, i \in (1 \ldots \mathrm{m})$, we can get following from formula 3-1 and 3-4:

$$c_{j}^{0} \geq \frac{1}{2} c_{ij}^{H} \qquad \textbf{(3-5)}$$

$$c_{j}^{H} = \max_{i=1,2,\ldots m} \left\{c_{ij}^{H}\right\} \qquad \textbf{(3-6)}$$

To all the $j \in N$ we can get from formula 3-6:

$$c_{j}^{0} \geq \frac{1}{2} c_{j}^{H} \qquad \textbf{(3-7)}$$

The inequality has been proved:

$$c_{j}^{0} \leq c_{j} \qquad \textbf{(3-8)}$$

The reasoning process is as following:

For the left inequality of formula 3-2,

$$\sum_{k=1}^{j} p_{ik} c_{ik}^{0} \geq \frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^{2} + \sum_{k=1}^{j} p_{ik}^{2}\right] \qquad \textbf{(3-9)}$$

Value of p_{ik} is fixed, and value of $\sum_{k=1}^{j} p_{ik} c_{ik}^{0}$ is the value of $\frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^{2} + \sum_{k=1}^{j} p_{ik}^{2}\right]$.

So only need to prove:

$$\sum_{k=1}^{j} p_{ik} c_{ik}^{0} - \frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^2 + \sum_{k=1}^{j} p_{ik}^2\right] \geq 0 \quad \textbf{(3-10)}$$

Then the formula 3-8 has been proved. Due to

$$c_j \geq \sum_{k=1}^{j} p_{ik} \quad \textbf{(3-11)}$$

There is:

$$\sum_{k=1}^{j} p_{ik} c_j \geq \sum_{k=1}^{j} p_{ik} \times \sum_{k=1}^{j} p_{ik} = \left(\sum_{k=1}^{j} p_{ik}\right)^2 \quad \textbf{(3-12)}$$

Only need to prove:

$$\left(\sum_{k=1}^{j} p_{ik}\right)^2 - \frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^2 + \sum_{k=1}^{j} p_{ik}^2\right] \geq 0 \quad \textbf{(3-13)}$$

Also:

$$\frac{1}{2}\left[\left(\sum_{k=1}^{j} p_{ik}\right)^2 - \sum_{k=1}^{j} p_{ik}^2\right] \geq 0 \quad \textbf{(3-14)}$$

The formula 3-8 has been proved, and as the p_{ik} is the integer larger than zero. From mathematical induction, formula 3-15 can be proved if $j \geq 1$,

$$\left(\sum_{n=1}^{j} x_n\right)^2 - \sum_{n=1}^{j} x_n^2 \geq 0 \quad \textbf{(3-15)}$$

The inequality formula 3-8 can be proved from inequality formula 3-15 and 3-14, also, from the formula 3-7 and 3-8, the following formula can be induced:

$$c_j \geq \frac{1}{2} c_j^H \quad \textbf{(3-16)}$$

The maximum error of manufacture period of HR algorithm- a_c is 2, due to $F = \max_{j=1,2,\ldots n}\{c_j\}$, $F^{HR} = \max_{j=1,2,\ldots n}\{c_j^H\}$ and 2F$\geq$ FHR gotten from formula 4-16. In HR algorithm, the maximum error of total completion time- a_t is 2m, and according to formula 3-16, 2m $\sum F \geq \sum F^{HR}$ can be obtained, m is the number of machine. The conclusion is that, in HR algorithm, the maximum error of manufacture period is 2, and the maximum error of total completion time is 2m.

4. Conclusion

In this paper, the HR algorithm can be used to solve the scheduling problem of concurrent open-shop; also, the calculation flow and algorithmic ordering process have been presented. According to the calculation and demonstration, the maximum error of manufacture period of HR algorithm- a_c is 2, the maximum error of total completion time - a_t is 2m (m is the number of machine).

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Author

Di Jin, master, lecturer, School of computer and information engineering, Harbin University of Commerce.