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Information Fusion Predictive Control Algorithm for Time-Varying Systems with Unknown Stochastic System Bias

Ming Zhao[1], Yun Li[1], Gang Hao[2]*, Junling Li[1] and Hao Jin[1]

[1]School of Computer and Information Engineering
Harbin University of Commerce
HeiLongJiang, Harbin, 150001, China
[2]Electronic Engineering Institute
Heilongjiang University
Heilongjiang, Harbin, 150080, China
liyunhd@sina.com, haogang@hlju.edu.cn

Abstract

This paper puts forward on a fast distributed information fusion predictive control algorithm for the time-varying system with unknown stochastic system bias. It is based on the distributed fusion estimation algorithms and state-space model. The optimal information fusion rule for this algorithm is weighted by matrices, diagonal matrices and scalars. It can avoid the complicated Diophantine equation, thus obviously reduces the amount of calculation. Via the distributed information fusion algorithm, the comparison of algorithm in this paper with the local sensor, this algorithm improves stability and accuracy for the time-varying system with unknown stochastic system bias. By testing through the three-sensor target tracking control system simulation, this algorithm shows its effectiveness and correctness, and the results of simulation also show no significant difference in error between the three kinds of distributed fusion algorithm. With reduction of calculation using the scalar weighting fusion predictor, the information fusion estimation algorithm presented in this paper also improves the calculation speed and accuracy.

Keywords: *Stochastic Bias; Predictive Control; Distributed Information Fusion*

1. Introduction

With the development of electronic technology and computer application technology, in order to improve the tracking accuracy and the dynamic system state estimation precision, large numbers of multisensory systems with different application background appear [1]. Multisensor information fusion refers to a comprehensive perception data from a lot of sensors. It has many advantages. The first advantage is to produce more reliable data, accurate information. The second is to improve and reflect the characteristics of detecting object accurately. The third is to eliminate the uncertainty of information. The forth is to improve the reliability of sensors and so on. In addition of this, Multisensor information fusion has the following characteristics: information redundancy, information complementary to each other, real-time information and the low-cost of information. Since the 1970s, along with a variety of advanced modern weapons, the demand for the accuracy of the estimation is increasing more and more in this sophisticated system. Therefore, the application of information fusion techniques

* Corresponding Author

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with multisensor fusion to improve the target tracking precision is required and the optimization of sensor structure will move (reinforce) synchronous development. Therefore, the synchronous development of the computing ability of software and hardware meets the computational complexity with large amounts of multisensor information fusion data. This is one of the main development trends of multisensor information fusion technology.

System bias problem for predictive control has been widely concerned recently. Taking account of the existence of system bias and sensor bias in the field of control, communication and signal processing, the estimation problem for systems with system bias, the original literature [2] presented using two sections of Kalman filtering technique handle the estimation problem for having the unknown constant bias. Then, the algorithm is extended to the filtering problem of random deviation in the literature [3-5]. Separating stochastic bias two-stage decoupled wiener filtering using modern time sequence analysis method is presented in literature [6].

In the time of science and technology leading the world, the requirement of automatic control technology is constantly improving the huge, complex and uncertain systems, the limitations of control technology and theory existed are more and more obviously unsuitable. Predictive control technology has been widely applied in engineering field, such as medicine, weather forecasting, prediction of crop yield, chemical and so on since last century [7]. At the same time, the predictive control theory has been greatly concerned with the agriculture, industry and academic field [8], especially in the network control field [9-11]. The distributed predictive control system has been studied. These areas have made some progress. But there are still many problems need to be further solved.

The predictive control algorithm that presented in this paper is based on state-space model. This algorithm is different from the traditional prediction algorithm. It is based on the time-varying system. As the Kalman filter algorithm is used, and it avoids solving the complicated Diophantine functions. Predictive control algorithm is proposed in this paper, its accuracy depends on the predictive accuracy and stability of output. It can only get part of systems information from the single local sensor system. If the external signal interferes the sensor, the accuracy will become worse, severe cases can lead to system collapse [12]. The multisensor information fusion can fuse detection data from a lot of sensors, and generate more accurate estimations than the single data source.

In this paper, we present a distributed information fusion predictive control algorithm. It is based on the distributed fusion estimation algorithms weighted by matrices, diagonal matrices and scalars. This algorithm does not need to solve the complicated Diophantine equation. We can get the state prediction through Kalman filtering, so it can reduce the amount of computation greatly. Moreover, the comparison of algorithm presented in this paper with the local sensor, this algorithm presented improves stability and accuracy for the system with unknown stochastic system bias. Simulation results verify effectiveness and validity of training.

Chapter titles of the article are as follows: Problem formulation is explained in Section 2. The local time-varying subsystem optimal Kalman filtering with unknown stochastic system bias is presented in Section 3. The fusion Kalman filtering weighed by matrices, diagonal matrices and scalars for time-varying system is given in Section 4. Predictive control algorithm based on Kalman filtering and state-space model is obtained in Section 5. In Section 6, a 3-sensor simulation example is given. Summaries for the algorithm are given in the Section 7.

2. Problem Formulation

Taking into account that the multisensor discrete-time time-varying stochastic linear control system with unknown stochastic system bias

$$\boldsymbol{x}(k+1)=\boldsymbol{\Phi}(k)\boldsymbol{x}(k)+\boldsymbol{B}(k)\boldsymbol{U}(k)+\boldsymbol{\Gamma}(k)\boldsymbol{w}(k)+\boldsymbol{D}(k)\boldsymbol{d}(k) \tag{1}$$

$$\boldsymbol{y}_j(k)=\boldsymbol{H}_j(k)\boldsymbol{x}(k)+\boldsymbol{v}_j(k),\quad j=1,\cdots,M \tag{2}$$

$$\boldsymbol{d}(t+1)=\boldsymbol{d}(t)+\omega_j(t) \tag{3}$$

where k is the discrete time, The subscript *j* denotes the *j*th sensor, M denotes the number of sensor, $\boldsymbol{x}(k)\in\boldsymbol{R}^n$ is the state of the system, $\boldsymbol{U}(k)$ is the scalar input, $\boldsymbol{d}(k)\in\boldsymbol{R}^{n_d}$ is the unknown stochastic system bias of the subsystem *j*, $\boldsymbol{y}_j(k)\in\boldsymbol{R}^{mj}$ is the observation of the local system *j*, $\boldsymbol{v}_j(t)\in\boldsymbol{R}^{mj}$ is the measurement noise of the local system, $\boldsymbol{w}(t)\in\boldsymbol{R}^w$ is the process noise, $\boldsymbol{\Phi}(k)$, $\boldsymbol{B}(k)$, $\boldsymbol{\Gamma}(k)$, $\boldsymbol{D}(k)$, $\boldsymbol{H}_j(k)$ is the suitable dimensional matrix respectively.

Assumption 1 $\omega_j(k)$ and $\boldsymbol{w}(k)$ are independence white noises with zero mean, and $\omega_j(k)$ and $\bar{\boldsymbol{v}}_j(k)$ are independence white noises with zero mean, and

$$\mathrm{E}\left\{\omega_j(k)\quad \omega_j^{\mathrm{T}}(k)\right\}=\boldsymbol{Q}_{\omega_j} \tag{4}$$

where $\boldsymbol{w}(t)\in\boldsymbol{R}^w$ and $\bar{\boldsymbol{v}}_j(t)\in\boldsymbol{R}^{mj}$, are correlated white noises , and the mean is zero:

$$\mathrm{E}\left\{\begin{bmatrix}\boldsymbol{w}(k)\\ \boldsymbol{v}_j(k)\end{bmatrix}\begin{bmatrix}\boldsymbol{w}^{\mathrm{T}}(t) & \boldsymbol{v}_i^{\mathrm{T}}(t)\end{bmatrix}\right\}=\begin{bmatrix}\boldsymbol{Q}_w & \boldsymbol{S}\\ \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_{ji}\end{bmatrix}\delta_{kt} \tag{5}$$

where E is the expectation, the superscript T denotes the transpose operation, $\delta_{kk}=1$, $\delta_{kt}=0(k\neq t)$.

Assumption 2 The input of the system is a linear function of measurement, or is the known time series.

Assumption 3 The initial state value $\boldsymbol{x}(0)$ is uncorrelated with $\boldsymbol{w}(k)$ and $\boldsymbol{v}_j(k)$ and

$$\mathrm{E}\boldsymbol{x}(0)=\boldsymbol{x}_0,\ \mathrm{cov}\,\boldsymbol{x}(0)=\boldsymbol{P}_0 \tag{6}$$

where cov is covariance.

Assumption 4 The initial time $k_0=-\infty$.

For the system (1) ~ (3), we can get the following augmentation system

$$\boldsymbol{\alpha}(k+1)=\bar{\boldsymbol{\Phi}}(k)\boldsymbol{\alpha}(k)+\boldsymbol{B}(k)\boldsymbol{U}(k)+\bar{\boldsymbol{\Gamma}}(k)\bar{\boldsymbol{w}}(k) \tag{7}$$

$$\bar{\boldsymbol{y}}_j(k)=\bar{\boldsymbol{H}}_j(k)\boldsymbol{\alpha}(k)+\boldsymbol{v}_j(k),\quad j=1,\cdots,M \tag{8}$$

where

$$\boldsymbol{\alpha}(k)=\begin{bmatrix}\bar{\boldsymbol{x}}(k)\\ \boldsymbol{d}(k)\end{bmatrix},\ \bar{\boldsymbol{w}}(k)=\begin{bmatrix}\boldsymbol{w}(k)\\ \omega_j(k)\end{bmatrix},\ \bar{\boldsymbol{\Phi}}(k)=\begin{bmatrix}\boldsymbol{\Phi}(k) & \boldsymbol{D}(k)\\ 0_{n_d\times n} & I_{n_d}\end{bmatrix},\ \bar{\boldsymbol{\Gamma}}(k)=\begin{bmatrix}\boldsymbol{\Gamma}(k) & 0_{n_d\times n}\\ 0_{n_d\times r} & I_{n_d}\end{bmatrix},$$

$$\bar{\boldsymbol{H}}_j(k)=\begin{bmatrix}\boldsymbol{H}_j(k) & 0_{m_j\times n_d}\end{bmatrix}.$$

Our goal is based on measurement $(\boldsymbol{y}(k),\ \boldsymbol{y}(k-1),\cdots,\ \boldsymbol{y}(0))$ and input $(u(k-1),\cdots,\ u(0))$, to obtain t-step-ahead optimal predictive control algorithm. Fused criteria is weighted by matrices, diagonal matrices and scalars. Block diagram is shown in Figure 1.

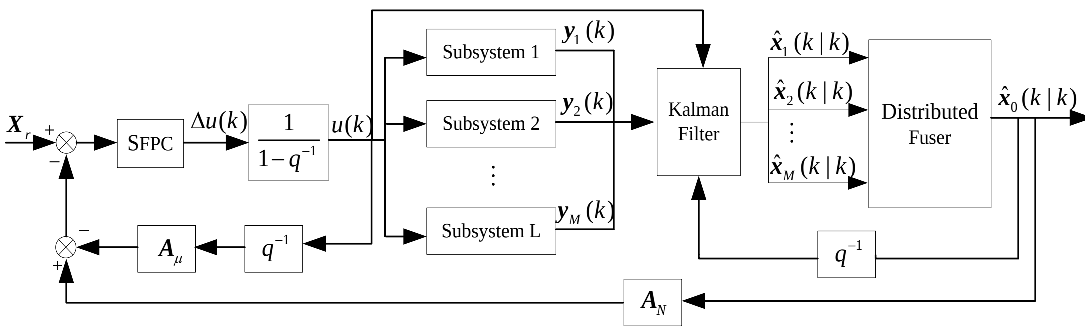

Figure 1. Block Diagram of Predictive Control System with Unknown Stochastic System Bias

3. Optimal Kalman Filter

Lemma 3.1 The augmentation system (7) and (8), under the assumption 1 to 4, the jth sensor subsystem has the optimal recursive Kalman filter equations [13]:

$$\hat{\boldsymbol{\alpha}}(k+1\mid k+1)=\hat{\boldsymbol{\alpha}}(k+1\mid k)+\boldsymbol{K}_f(k+1)\bar{\boldsymbol{\varepsilon}}(k+1) \tag{9}$$

$$\bar{\boldsymbol{\varepsilon}}(k+1)=\boldsymbol{y}(k+1)-\bar{\boldsymbol{H}}(k+1)\hat{\boldsymbol{\alpha}}(k+1\mid k) \tag{10}$$

$$\hat{\boldsymbol{\alpha}}(k+1\mid k)=\bar{\boldsymbol{\Phi}}_\alpha\hat{\boldsymbol{\alpha}}(k\mid k)+\boldsymbol{B}\boldsymbol{U}(k)+\boldsymbol{J}(k)\bar{\boldsymbol{y}}(k) \tag{11}$$

$$\bar{\boldsymbol{\Phi}}_\alpha(k)=\bar{\boldsymbol{\Phi}}(k)-\boldsymbol{J}(k)\bar{\boldsymbol{H}}(k) \tag{12}$$

$$\boldsymbol{J}(k)=\bar{\boldsymbol{\Gamma}}(k)\boldsymbol{S}\boldsymbol{R}_{ij} \tag{13}$$

$$\boldsymbol{K}_f(k)=\boldsymbol{P}(k+1\mid k)\bar{\boldsymbol{H}}^{\mathrm{T}}(k+1)[\bar{\boldsymbol{H}}(k+1)\boldsymbol{P}(k+1\mid k)\bar{\boldsymbol{H}}^{\mathrm{T}}(k+1)+\boldsymbol{R}_{ji}]^{-1} \tag{14}$$

$$\boldsymbol{P}(k+1\mid k)=\bar{\boldsymbol{\Phi}}_\alpha(k)\boldsymbol{P}(k\mid k)\boldsymbol{\Phi}_\alpha^{\mathrm{T}}(k)+\bar{\boldsymbol{\Gamma}}(k)[\boldsymbol{Q}_w-\boldsymbol{S}\boldsymbol{R}_{ij}^{-1}\boldsymbol{S}^{\mathrm{T}}]\bar{\boldsymbol{\Gamma}}^{\mathrm{T}}(k) \tag{15}$$

$$\boldsymbol{P}(k+1\mid k+1)=[\boldsymbol{I}_n-\boldsymbol{K}_f(k+1)\bar{\boldsymbol{H}}(k+1)]\boldsymbol{P}(k+1\mid k) \tag{16}$$

$$\hat{\boldsymbol{x}}(0\mid -1)=\boldsymbol{\mu},\quad \boldsymbol{P}(0\mid -1)=\boldsymbol{P}_0 \tag{17}$$

Lemma 3.2 For the augmentation system (7) and (8), under the assumption 1 to 4, the covariance $\boldsymbol{P}_{ji}=\mathrm{E}[\tilde{\boldsymbol{\alpha}}_i(k\mid k)\tilde{\boldsymbol{\alpha}}_i^{\mathrm{T}}(k\mid k)]$ $(j\neq i)$ among the prediction errors $\tilde{\boldsymbol{\alpha}}_j(k\mid k)=\boldsymbol{\alpha}(k)-$ $\hat{\boldsymbol{\alpha}}_j(k\mid k)$ are given as [14]:

$$\boldsymbol{P}_{ji}(k+1\mid k+1)=\boldsymbol{\Psi}_{fj}(k+1)\boldsymbol{P}_{ji}(k\mid k)\boldsymbol{\Psi}_{fi}^{\mathrm{T}}(k+1)+\boldsymbol{\Psi}_{fj}(k+1)\boldsymbol{K}_{fj}(k)\times[\boldsymbol{R}_{ji}\boldsymbol{J}_{i}^{\mathrm{T}}(k)-\boldsymbol{S}_{j}^{\mathrm{T}}\bar{\boldsymbol{\Gamma}}^{\mathrm{T}}(k)][\boldsymbol{I}_{n}-\boldsymbol{K}_{fi}(k+1)\times \bar{\boldsymbol{H}}_{i}(k+1)]^{\mathrm{T}}+[\boldsymbol{I}_{n}-\boldsymbol{K}_{fj}(k+1)\bar{\boldsymbol{H}}_{j}(k+1)][\boldsymbol{J}_{j}^{\mathrm{T}}(k)\boldsymbol{R}_{ji}-\bar{\boldsymbol{\Gamma}}(k)\boldsymbol{S}_{i}]\boldsymbol{K}_{fi}^{\mathrm{T}}(k)\boldsymbol{\Psi}_{fi}^{\mathrm{T}}(k+1)+[\boldsymbol{I}_{n}-\boldsymbol{K}_{fj}(k+1)\bar{\boldsymbol{H}}_{j}(k+1)]\times [\bar{\boldsymbol{\Gamma}}(k)\boldsymbol{Q}_{w}\bar{\boldsymbol{\Gamma}}^{\mathrm{T}}(k)-\boldsymbol{J}_{j}(k)\boldsymbol{S}_{j}^{\mathrm{T}}\bar{\boldsymbol{\Gamma}}(k)^{\mathrm{T}}-\bar{\boldsymbol{\Gamma}}(k)\boldsymbol{S}_{i}\boldsymbol{J}_{i}^{\mathrm{T}}(k)+\boldsymbol{J}_{j}(k)\boldsymbol{R}_{ji}\boldsymbol{J}_{i}^{\mathrm{T}}(k)][\boldsymbol{I}_{n}-\boldsymbol{K}_{fi}(k+1)\bar{\boldsymbol{H}}_{i}(k+1)]^{\mathrm{T}}+\boldsymbol{K}_{fj}(k+1)\times \boldsymbol{R}_{ji}\boldsymbol{K}_{fi}^{\mathrm{T}}(k+1) \quad (18)$$

and the initial values is $\boldsymbol{P}_{ji}(0\mid 0)=\boldsymbol{P}_0$, having

$$\boldsymbol{\Psi}_{fj}(k+1)=[\boldsymbol{I}_{\boldsymbol{n}}-\boldsymbol{K}_{\boldsymbol{f}}(k+1)\bar{\boldsymbol{H}}_{j}(k+1)]\bar{\boldsymbol{\Phi}}_{\alpha}(k) \quad (19)$$

4. Distributed Fusion Predictive Control Algorithm

Lemma 4.1 The augmentation system (7) and (8), under the assumption 1 to 4, the optimal fused filtering $\hat{\boldsymbol{\alpha}}_0(k\mid k)$ weighted by matrices is given as [14]:

$$\hat{\boldsymbol{\alpha}}_0(k\mid k)=\sum_{j=1}^{M}\boldsymbol{M}_j(k)\hat{\boldsymbol{\alpha}}_j(k\mid k) \quad (20)$$

Under the linear minimum variance optimal information fusion criterion which minimize the performance index, the optimal weighting coefficients $\boldsymbol{M}_j,\ j=1,2,\cdots,L$ are given as follows

$$\left[\boldsymbol{M}_1(k),\cdots,\boldsymbol{M}_l(k)\right]=(e^{\mathrm{T}}\boldsymbol{P}^{-1}(k)e)^{-1}\boldsymbol{e}^{\boldsymbol{T}}\boldsymbol{P}^{-1}(k) \quad (21)$$

Where

$$\boldsymbol{P}(k)=\begin{bmatrix}\boldsymbol{P}_{11}(k) & \boldsymbol{P}_{12}(k) & \cdots & \boldsymbol{P}_{1M}(k)\\ \boldsymbol{P}_{21}(k) & \boldsymbol{P}_{22}(k) & \cdots & \boldsymbol{P}_{2M}(k)\\ \cdots & \cdots & \cdots & \cdots\\ \boldsymbol{P}_{M1}(k) & \boldsymbol{P}_{M2}(k) & \cdots & \boldsymbol{P}_{MM}(k)\end{bmatrix},\ \boldsymbol{e}=\begin{bmatrix}\boldsymbol{I}_m\\ \boldsymbol{I}_m\\ \cdots\\ \boldsymbol{I}_m\end{bmatrix},$$

$$\boldsymbol{P}_{jj}(k)=\boldsymbol{P}_j(k)(j=1,2,\cdots,M) \quad (22)$$

where $\boldsymbol{P}(k)$ and $\boldsymbol{P}_{ji}(k)$ are computed by Lemma 3.1 and 3.2.

The optimal fused variance matrix is given as

$$\boldsymbol{P}_0(k)=(e^{\mathrm{T}}\boldsymbol{P}^{-1}(k)e)^{-1} \quad (23)$$

And

$$\operatorname{tr}\boldsymbol{P}_0\le\operatorname{tr}\boldsymbol{P}_j,\quad j=1,2,\cdots,M \quad (24)$$

Lemma 4.2 For the augmentation system (7) and (8), under the same conditions, the optimal fused Kalman filter $\hat{\boldsymbol{\alpha}}_0(k\mid k)$ weighted by diagonal matrices is given as [14]

$$\hat{\boldsymbol{\alpha}}_0(k\mid k)=\sum_{j=1}^{M}\boldsymbol{A}_j(k)\hat{\boldsymbol{\alpha}}_j(k\mid k) \quad (25)$$

where the optimal weighting coefficients $\boldsymbol{A}_j=\mathrm{diag}(a_{jt}),\ \ t=1,\cdots,n$,while a_{jt} are given by

$$\boldsymbol{a}_j=[a_{1j},a_{2j},\cdots,a_{Mj}],\quad j=1,\cdots,n \quad (26)$$

$$\boldsymbol{a}_j=(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}^{jj}(k\mid k)^{-1}\boldsymbol{e})^{-1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}^{jj}(k\mid k)^{-1},\quad j=1,\cdots,n \quad (27)$$

where $\boldsymbol{e}=[1 \quad \cdots \quad 1]^{\mathrm{T}}$ is a $M\times 1$ row vector, and $M\times M$ matrices is defined as $\boldsymbol{P}^{jj}(k\mid k)=(\boldsymbol{P}_{lt}^{jj}(k\mid k)),\quad l,t=1,2,\cdots,M$, $\boldsymbol{P}_{lk}^{jj}(t\mid t)$ is the *j*th row and *j*th column diagonal element of $\boldsymbol{P}_{lk}(t\mid t)$. $\boldsymbol{P}_{lk}(t\mid t)$ is computed by Lemma 3.2.

Lemma 4.3 For the augmentation system (7) and (8), under the same conditions, the optimal fused Kalman filter $\hat{\boldsymbol{\alpha}}_0(k\mid k)$ weighted by scalars is given as [14]

$$\hat{\boldsymbol{\alpha}}_0(k\mid k)=\sum_{i=1}^{M}\alpha_i(k)\hat{\boldsymbol{\alpha}}_i(k\mid k) \tag{28}$$

Under the linear minimum variance optimal information fusion criterion which minimize the performance index, the optimal weighting coefficients $\alpha_j(k),\ j=1,2,\cdots,M$ are given by

$$[\alpha_1(k),\cdots,\alpha_l(k)]=\frac{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{\mathrm{tr}}^{-1}}{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{\mathrm{tr}}^{-1}\boldsymbol{e}} \tag{29}$$

where we define the $M\times M$ matrix $\boldsymbol{P}_{\mathrm{tr}}=(\mathrm{tr}\,\boldsymbol{P}_{ji})_{M\times M},\ j,i=1,2,\cdots,M$, and $\boldsymbol{P}_{ji}$ can be calculated by (18), and $M\times 1$ row vector $\boldsymbol{e}=[1 \quad \cdots \quad 1]^{\mathrm{T}}$.

The optimal fused variance matrix is given as

$$\boldsymbol{P}_0(N)=\sum_{j,i=1}^{M}\alpha_i(k)\alpha_j(k)\boldsymbol{P}_{ji}(N) \tag{30}$$

and

$$\mathrm{tr}\,\boldsymbol{P}_0\le \mathrm{tr}\,\boldsymbol{P}_j,\quad j=1,2,\cdots,M \tag{31}$$

It is proved in document [14] that the accuracy of above three kinds of weighted fusion filtering from high to low is weighted by matrices, diagonal matrices, scalars. But the computational burden is on the contrary, fusion filtering weighted by matrices has a large computational burden, and weighted by scalars with minimal computational burden, and it is suitable for real-time applications, and the computational burden that weighted by diagonal matrices is between the above two.

5. Predictive Control Algorithm Base on Kalman Filter

Theorem 5.1 For the augmentation system (7) and (8) under the Assumption 1 to 4, the *t*-step ahead optimal predictive control increments is obtained:

$$\Delta\boldsymbol{U}(k)=[(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_e(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)+\Lambda_u)^{-1}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_e\times\{\boldsymbol{\alpha}_r-\boldsymbol{e}^{\mathrm{T}}[\boldsymbol{\Phi}_N\hat{\boldsymbol{\alpha}}(k\mid k)-\boldsymbol{\Phi}_\mu U(k-1)]\} \tag{32}$$

where $\boldsymbol{e}^{\mathrm{T}}=\left[\boldsymbol{e}_j^{\mathrm{T}} \quad \cdots \quad \boldsymbol{e}_j^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\boldsymbol{Q}_e=\mathrm{diag}(\boldsymbol{Q}_1,\cdots,\boldsymbol{Q}_N)$ and $\Lambda_u=\mathrm{diag}(\Lambda_1,\cdots,\Lambda_{N_\mu})$ are unified called the weighted matrix.

We define the controlled state as

$$\hat{\boldsymbol{A}}_0=\boldsymbol{e}^{\mathrm{T}}\left[\hat{\boldsymbol{\alpha}}_0^{\mathrm{T}}(k+1\mid k) \quad \hat{\boldsymbol{\alpha}}_0^{\mathrm{T}}(k+2\mid k) \quad \cdots \quad \hat{\boldsymbol{\alpha}}_0^{\mathrm{T}}(k+t\mid k)\right] \tag{33}$$

And the reference tracking at the time k is defined as

$$\boldsymbol{A}_r = \begin{bmatrix} \boldsymbol{\alpha}_r(k+1) & \boldsymbol{\alpha}_r(k+2) & \cdots & \boldsymbol{\alpha}_r(k+t) \end{bmatrix}^{\mathrm{T}} \tag{34}$$

$$\bar{\boldsymbol{\Phi}}_x(k) = \begin{bmatrix} \bar{\boldsymbol{\Phi}}(k) \\ \bar{\boldsymbol{\Phi}}^2(k) \\ \cdots \\ \bar{\boldsymbol{\Phi}}^N(k) \end{bmatrix}, \boldsymbol{\Delta}\bar{\boldsymbol{U}} = \begin{bmatrix} \Delta\boldsymbol{U}(k) \\ \Delta\boldsymbol{U}(k+1) \\ \cdots \\ \Delta\boldsymbol{U}(k+N_\mu-1) \end{bmatrix}, \bar{\boldsymbol{\Phi}}_\mu(k) = \begin{bmatrix} \boldsymbol{B}(k) \\ (\bar{\boldsymbol{\Phi}}(k)+\boldsymbol{I})\boldsymbol{B}(k) \\ \cdots \\ (\bar{\boldsymbol{\Phi}}^{N-1}(k)+\cdots+\bar{\boldsymbol{\Phi}}(k)+\boldsymbol{I})\boldsymbol{B}(k) \end{bmatrix},$$

$$\bar{\boldsymbol{\Phi}}_N(k) = \begin{bmatrix} \boldsymbol{B}(k) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ (\bar{\boldsymbol{\Phi}}(k)+\boldsymbol{I})\boldsymbol{C} & \boldsymbol{B}(k) & \cdots & \boldsymbol{0} \\ \cdots & \cdots & \cdots & \cdots \\ (\bar{\boldsymbol{\Phi}}^{N-1}(k)+\cdots+\bar{\boldsymbol{\Phi}}(k)+\boldsymbol{I})\boldsymbol{B}(k) & (\bar{\boldsymbol{\Phi}}^{N-2}(k)+\cdots+\bar{\boldsymbol{\Phi}}(k)+\boldsymbol{I})\boldsymbol{B}(k) & \cdots & \boldsymbol{B}(k) \end{bmatrix} \tag{35}$$

And the t-step ahead predictive control is computed as

$$\boldsymbol{U}(k) = \boldsymbol{U}(k-1) + \boldsymbol{e}_1 \Delta\boldsymbol{U}(k) \tag{36}$$

Where $\boldsymbol{e}_1 = [1\ 0 \cdots 0]$, and the filtering $\hat{\boldsymbol{x}}(k \mid k)$ can be obtained through (9)—(17).

Proof: Selecting $\boldsymbol{\alpha}_r(t)$ is the reference tracking and $\boldsymbol{e}_j^{\mathrm{T}} \alpha_0(k)$ $(\boldsymbol{e}_j^{\mathrm{T}} = [\underbrace{0 \cdots 0}_{j-1}\ 1\ \underbrace{0 \cdots 0}_{m-j}])$ is the controller at time k for the fusion system (1) and (37). For the sake of letting the selected fused states $\boldsymbol{e}_j^{\mathrm{T}} \hat{\boldsymbol{x}}_0(k+i \mid k), i = 1, \cdots, t$ is as far as possible closing to the reference variable $\boldsymbol{\alpha}_r(k+i)$. So we must obtain the control increments $\Delta\boldsymbol{U}(k), \Delta\boldsymbol{U}(k+1), \cdots, \Delta\boldsymbol{U}(k+T_\mu-1)$.

Where T_μ is called the controlled domain, t is defining as the optimal domain.

The expanse function equation is defining [15]:

$$J = (\hat{\boldsymbol{A}}_0 - \boldsymbol{A}_r)^{\mathrm{T}} \boldsymbol{Q}_e (\hat{\boldsymbol{A}}_0 - \boldsymbol{A}_r) + \boldsymbol{\Delta}\bar{\boldsymbol{U}}^{\mathrm{T}} \boldsymbol{\Lambda}_u \boldsymbol{\Delta}\bar{\boldsymbol{U}} \tag{38}$$

From the local system(7), we get

$$\hat{\boldsymbol{\alpha}}(k+i \mid k) = \bar{\boldsymbol{\Phi}}^i(k)\hat{\boldsymbol{\alpha}}(k \mid k) + \bar{\boldsymbol{\Phi}}^{i-1}(k)\boldsymbol{B}(k)\boldsymbol{U}(k) + \cdots + \boldsymbol{B}(k)\boldsymbol{U}(k+i-1) \tag{39}$$

Defining

$$\Delta\bar{\boldsymbol{U}}(k) = \boldsymbol{U}(k) - \boldsymbol{U}(k-1) \tag{40}$$

We have

$$\boldsymbol{U}(k+i-1) = \Delta\bar{\boldsymbol{U}}(k+i-1) + \Delta\bar{\boldsymbol{U}}(k+i-2) + \cdots + \Delta\bar{\boldsymbol{U}}(k) + \boldsymbol{U}(k-1) \tag{41}$$

Putting (41) into (39) obtains

$$\hat{\boldsymbol{\alpha}}(k+i \mid k) = \bar{\boldsymbol{\Phi}}^i(k)\hat{\boldsymbol{\alpha}}(k \mid k) + \sum_{n=0}^{j-1} \left[\sum_{m=0}^{j-n-1} \bar{\boldsymbol{\Phi}}(k)^m \right] \boldsymbol{B}(k)\Delta u(k+n) \tag{42}$$

So that we have

$$\hat{\boldsymbol{A}}_0 = \boldsymbol{e}^{\mathrm{T}}[\bar{\boldsymbol{\Phi}}_x \hat{\boldsymbol{\alpha}}(k \mid k) + \bar{\boldsymbol{\Phi}}_N \Delta\bar{\boldsymbol{U}} + \bar{\boldsymbol{\Phi}}_\mu \bar{\boldsymbol{U}}(k-1)] \tag{43}$$

Putting (43) into (38) yields

$$J = \{\boldsymbol{e}^{\mathrm{T}}[\overline{\boldsymbol{\Phi}}_{x}(k)\hat{\boldsymbol{\alpha}}(k \mid k) + \overline{\boldsymbol{\Phi}}_{N}(k)\Delta\overline{\boldsymbol{U}} + \overline{\boldsymbol{\Phi}}_{\mu}(k)\boldsymbol{U}(k-1)] - \boldsymbol{X}_{r}\}^{\mathrm{T}}\boldsymbol{Q}_{e} \times \{\boldsymbol{e}^{\mathrm{T}}[\overline{\boldsymbol{\Phi}}_{x}(k)\hat{\boldsymbol{\alpha}}(k \mid k) + \overline{\boldsymbol{\Phi}}_{N}(k)\Delta\overline{\boldsymbol{U}} + \overline{\boldsymbol{\Phi}}_{\mu}(k)\boldsymbol{U}(k-1)] - \boldsymbol{X}_{r}\}^{\mathrm{T}} + \boldsymbol{\Delta}\overline{\boldsymbol{U}}^{\mathrm{T}}\boldsymbol{\Lambda}_{u}\boldsymbol{\Delta}\overline{\boldsymbol{U}} \tag{44}$$

And letting $\frac{\partial J}{\partial \boldsymbol{\Delta}\overline{\boldsymbol{U}}} = 0$, we have

$$\overline{\boldsymbol{\Phi}}_{N}^{\mathrm{T}}(k)\boldsymbol{e}\boldsymbol{Q}_{e}\{\boldsymbol{e}^{\mathrm{T}}[\overline{\boldsymbol{\Phi}}_{x}(k)\boldsymbol{\alpha}(k \mid k) + \overline{\boldsymbol{\Phi}}_{\mu}(k)\boldsymbol{U}(k-1)] - \boldsymbol{A}_{r}\} + \overline{\boldsymbol{\Phi}}_{N}^{\mathrm{T}}(k)\boldsymbol{e}\boldsymbol{Q}_{e}\boldsymbol{e}^{\mathrm{T}}\overline{\boldsymbol{\Phi}}_{N}(k)\boldsymbol{\Delta}\overline{\boldsymbol{U}} + \boldsymbol{\Lambda}_{u}\boldsymbol{\Delta}\overline{\boldsymbol{U}} = 0 \tag{45}$$

We have the control increments can be computed via (32). From (32) and (40), (36) is obtained.

This completes the proof.

6. Simulation Example

Consider three-sensor linear stochastic controllable tracking time-varying system (1) and (2), where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} position \\ velocity \end{bmatrix}$ are the states, $\boldsymbol{y}_i(\mathrm{k})$ are the measurements of the *j*th subsystem, $\boldsymbol{d}(t)$ is the unknown stochastic system bias of the *j*th constant subsystem, $\boldsymbol{A}(k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B}(k) = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{\Gamma}(k) = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{D}(k) = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $T = 0.3$ is the sampled period, $H_1 = [1 \quad 0 \quad 0]$, $H_2 = [1 \quad 0 \quad 0]$, $H_3 = [1 \quad 0 \quad 0]$.

$$\boldsymbol{x}(k+1) = \begin{bmatrix} 1 & \mathrm{T} \\ 0 & 1 \end{bmatrix}\boldsymbol{x}(k) + \begin{bmatrix} \frac{\mathrm{T}^2}{2} \\ \mathrm{T} \end{bmatrix}\boldsymbol{U}(k) + \begin{bmatrix} \frac{\mathrm{T}^2}{2} \\ \mathrm{T} \end{bmatrix}\boldsymbol{w}(k) + \begin{bmatrix} \frac{\mathrm{T}^2}{2} \\ \mathrm{T} \end{bmatrix}\boldsymbol{d}(k) \tag{46}$$

$$\boldsymbol{y}_j(k) = \boldsymbol{H}_j\boldsymbol{x}(k) + \boldsymbol{v}_j(k), \quad j = 1, \cdots, L \tag{47}$$

$$\boldsymbol{d}(k+1) = \boldsymbol{d}(k) + \omega_j(k) \tag{48}$$

And $w(t)$ and $\bar{v}_i(t)$ are assumed to be independent Gaussian white noises with zero mean and input noise variances $\sigma_w^2 = 0.01$, and measurement variances $\sigma_{v1}^2 = 0.2$, $\sigma_{v2}^2 = 0.4$, $\sigma_{v3}^2 = 0.9$, $\sigma_{\omega}^2 = 0.1$.

The estimation criterion of the controlled system is the sum of mean square error function (SMSE) of the differences of the state reference track $\boldsymbol{\alpha}_r(t)$ and the controlled state fusion estimator $\boldsymbol{e}_j^{\mathrm{T}}\hat{\boldsymbol{\alpha}}_0(t \mid t)$ weighted by matrices, the diagonal matrices and scalars [16, 17]

$$\mathrm{SMSE}(N) = \sum_{k=0}^{N}\frac{1}{L}\sum_{i=1}^{L}[\boldsymbol{e}_j^{\mathrm{T}}\hat{\boldsymbol{\alpha}}_0^{(i)}(k \mid k) - \boldsymbol{\alpha}_r(k)]^2 \tag{49}$$

where $\boldsymbol{e}_j^{\mathrm{T}}\hat{\boldsymbol{\alpha}}_0^{(i)}(t \mid t)$ is the *j*th Monte Carlo simulation state estimates at time *k*.

In Monte Carlo simulation for 30 times, $\boldsymbol{\alpha}_1(t)$ is selected the controlled state. Setting the control time domain $T_{\mu} = 3$ and the optimize time domain $t = 3$, and the reference track $\boldsymbol{\alpha}_r(t)$ is the 5 units pulse signal, and the error weighted matrix $\boldsymbol{Q}_e = \mathrm{diag}(6, 3, 1) \times 8$, and the controlled weighted matrix $\boldsymbol{\Lambda}_{\mu} = \mathrm{diag}(6, 3, 1) \times 0.1$.

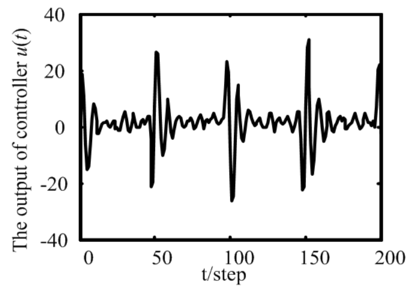

Figure 2. The Output Controller *u(t)* Weighted by Matrices

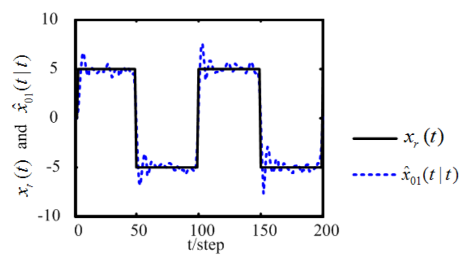

Figure 3. Fusion State Weighted by Matrices $x_{01}(t)$ and State Reference Track $x_r(t)$

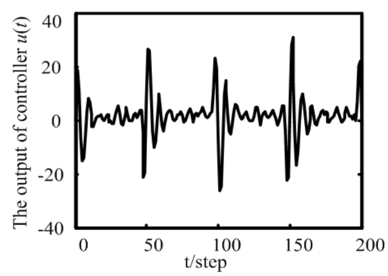

Figure 4. The Output of Controller *u(t)* Weighted by Diagonal

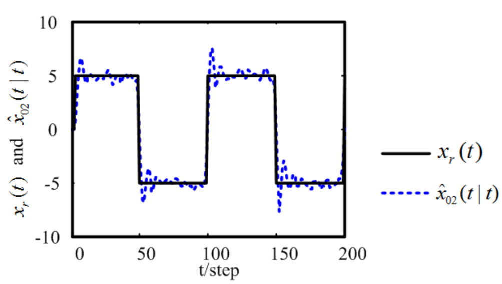

Figure 5. Fusion State Weighted by Diagonal $x_{02}(t)$ and State Reference Track $x_r(t)$

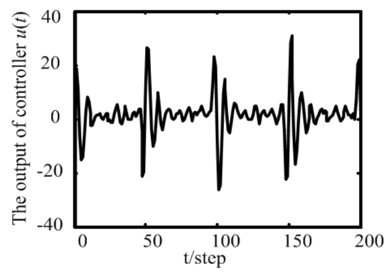

Figure 6. The Output of Controller *u(t)* Weighted by Scalars

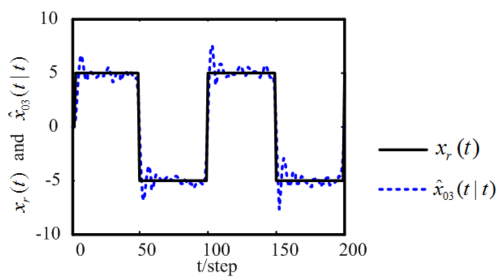

Figure 7. Fusion State Weighted by Scalars $x_{03}(t)$ and State Reference Track $x_r(t)$

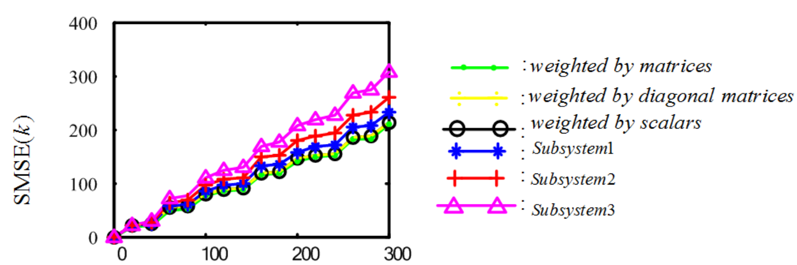

Figure 8. The Curves of the Sum of Mean Squares (SMSE) for Local and Fusion Filters

The simulation results are shown in Figure 2 to Figure 8. Controller output $u(t)$ is shown in Figure 2, Figure 4 and Figure 6. Figure 3, Figure 5 and Figure 7 respectively shows the comparison curves of the state reference track $x_r(t)$ and the fused state estimates $\hat{\alpha}_{0i}(t\mid t)$, $(i=1,2,3)$. From Figure 2 to Figure 7, we can see that the fused estimates $\hat{\alpha}_{0i}(t\mid t)$ follow the tracks of the state reference track $\alpha_r(t)$ as closely as possible, where the straight lines denote the state reference track, and the dashed curves denote the fused state estimates. From the graph, we can draw the conclusion: the algorithm has good convergence, and small overshoot, stable output and so on. The curves of the sum of mean square error (SMSE) for local and fusion filters are shown in Figure 8. We can see that that the accuracy of the fused Kalman filter is higher than each local Kalman filter. Simulation results show no significant difference between the three kinds of distributed fusion algorithm, and the three fusion curves almost coincide. But the scalar weighting fusion predictor has the least amount of calculation, and provides a fast information fusion estimation algorithm.

7. Conclusions

In this paper, a distributed information fusion predictive control algorithm is presented. Firstly, the algorithm combines the distributed fusion Kalman filter with predictive control. Compared with the classic generalized predictive control, this algorithm has a lot of advantages:

(1) The application of the minimization model in this algorithm reduces amount of calculation, and improves the system dynamic performance. So the algorithm can solve the system parameter mismatch problem with the slow changes of model parameters.

(2) Classical generalized predictive control can only deal with linear time invariant system, or linear time-varying system that parameters vary slowly, this is called adaptive generalized predictive control. But Kalman filtering can deal with the time-varying system, so the predictive control system based on Kalman filter can deal with the linear time-varying and time-invariant system together.

(3) This algorithm presented based on Kalman filter. This algorithm does not need to solve the complicated Diophantine equation that is needed in generalized predictive control [18], so it can reduce the computational burden obviously.

(4) Based on the stability of Kalman filtering and the distributed information fusion weighted by matrices, the diagonal matrices and scalars, the algorithm which is presented in this paper has better stability of the system and better ability of anti-jamming.

(5) Comparing with the single sensor case, the accuracy of using this distributed information fusion algorithm is improved greatly.

(6) Although the discrete-time linear time-varying stochastic control system has unknown stochastic system bias, according to the linear minimum variance optimal information fusion criterion based on distributed fusion estimation, a distributed information fusion predictive control algorithm is obtained. The algorithm used Kalman filtering to remove the system bias, and got the optimal control strategy.

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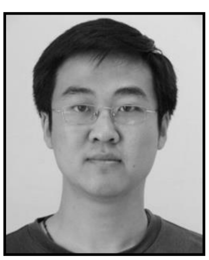
Authors

Ming Zhao is associate professor at Harbin University of Commerce now. She obtained her bachelor's degree and master's degree in Harbin Engineering University. Her major researches are pattern recognition, information fusion, etc.

Yun Li is associate professor at Harbin University of Commerce now. She obtained her bachelor's degree and master's degree in Heilongjiang University. Her major researches are state estimation, information fusion, etc.



Gang Hao is associate professor at Heilongjiang University now. He obtained his bachelor's degree and master's degree in Heilongjiang University, and obtained his Ph.D. in Harbin Engineering University. His major researches are state estimation, information fusion, etc.