International Journal of Hybrid Information Technology
Vol.7, No.4 (2014), pp.1-10
http://dx.doi.org/10.14257/ijhit.2014.7.4.01

An Improved Strong Tracking UKF Based on Fading Factor

Jian Feng, Xiao-dong Su, Yu-ru Zhang, Hai-tao Jiang and Jun-ling Li

School of Computer and Information Engineering
Harbin University of Commerce
Harbin, China
dimata@163.com

Abstract

STUKF (Strong tracking UKF) algorithm uses the time-varying fading factor to fade the past data and reduce the impact on current filter value, thus achieves the goal of adjusting the filter gain matrix in real time. But STUKF algorithm needs three UT for each filtering, and compared with the UKF filter, calculating amount of three UT increases seriously, and it is not conducive to application of engineering, therefore this paper presents an improved STUKF algorithm. Compared with the traditional STUKF filter, this new algorithm introduces the formulas of redefined fading factor. By changing the position of the fading factor, it improves the accuracy and robustness of the algorithm and reduces the computational complexity of the algorithm. Finally simulation results show that the new algorithm has higher precision and stronger robustness.

Keywords: *UKF; Strong Ttracking; Kalman Filter; Fading Factor; GPS; D/R*

1. Introduction

UKF [1-2] (Unscented Kalman Filter) algorithm can solve the nonlinear filtering problem of GPS/DR integrated navigation system model [3-4]. However the precision and stability of the filter will be affected badly when the exception of measurement vector happens in integrated navigation system, and it leads to excessive location deviation [5-6]. This paper presents an improved strong tracking UKF algorithm (ISTUKF) based on STUKF algorithm. It solves the problems brought by the introduction of fading factor in the state prediction covariance matrix $P_{k/k-1}$ of the STUKF algorithm directly through fading factor λ_k introduced in measurement prediction covariance matrix and cross-covariance matrix, making algorithm has higher precision and robustness.

2. Unscented Kalman Filter

UKF algorithm is minimum variance estimation based on Unscented Transform (UT), and it is not estimated observation and measurement model, but to estimate the distribution state of random variables. In UKF, the state distributions of random variables are expressed as gaussian distribution variable. When the process error, measurement error and transcendental state submit Gaussian distribution, UKF can be accurate to the third order, otherwise UKF at least can be accurate to the second order for non-gaussian distribution.

ISSN: 1738-9968 IJHIT

2.1. Problem Description

Assuming that system state equation and measurement equation are discrete time nonlinear mathematical model:

$$x_{k+1} = f(x_k, u_k) + w_k \tag{1}$$

$$y_k = h(x_k, u_k) + v_k \tag{2}$$

Where x_k is the system state vector whose dimension is n, and y_k is the measurement vector whose dimension is m respectively. k is the time step, f and h are nonlinear vector discrete function, u_k is deterministic control item, w_k is zero mean system Gaussian white noise sequences with covariance Q_k, and v_k is zero mean measurement Gaussian white noise sequences with covariance R_k. For $k-1 \geq 0$:

$$E[w_k] = 0, E[w_k w_l^T] = Q_k \delta_{kl}$$

$$E[v_k] = 0, E[v_k v_l^T] = R_k \delta_{kl}$$

$$E[w_k v_l^T] = 0, E[x_0 w_k^T] = 0, E[x_0 v_k^T] = 0$$

Initial state is random vector which has the following mean and variance $E[x_0] = m_0, \text{var}[\ x_0] = C_0$.

2.2. UKF Algorithm

Formula (1) and (2) are the system state equation and measurement equation respectively. The process of the UKF algorithm can be summarized as:

a) Initialization

$$\hat{x}_0 = E[x_0] \tag{3}$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \tag{4}$$

b) Calculate 2n+1 sigma points $\tilde{\chi}_{k-1}$ and the corresponding weighted factor $W_i^{(c)}$ and $W_i^{(m)}$

$$\tilde{\chi}_{k-1}^{(0)} = \hat{x}_{k-1} \tag{5}$$

$$\tilde{\chi}_{k-1}^{(i)} = \hat{x}_{k-1} + (\sqrt{(n+\lambda)P_{k-1}})_i, i = 1, \cdots, n \tag{6}$$

$$\tilde{\chi}_{k-1}^{(i)} = \hat{x}_{k-1} - (\sqrt{(n+\lambda)P_{k-1}})_i, i = n+1, \cdots, 2n \tag{7}$$

$$W_0^{(m)} = \lambda/(n+\kappa) \tag{8}$$

$$W_i^{(m)} = 1/[2(n+\kappa)],\ i = 1, \cdots, 2n \tag{9}$$

$$W_0^{(c)} = \lambda/(n+\lambda) + (1-\alpha^2+\beta) \tag{10}$$

$$W_i^{(c)} = 1/[2(n+\lambda)],\ i = 1, \cdots, 2n \tag{11}$$

Where, $\lambda = \alpha^2 (n + \kappa) - n$ is a scaling parameter, in which α is usually set to a small positive value, determining the spread of the sigma points around $\hat{x}_{k-1}$, and κ is the secondary scaling parameter which is set to 0 or 3-n. $\beta = 2$ is optimal for Gaussian distributions.

c) Time updating:

$$\chi_{k/k-1}^{(i)} = f(\tilde{\chi}_{k-1}^{(i)}, u_{k-1}) \qquad (12)$$

$$\hat{x}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(m)} \chi_{k/k-1}^{(i)} \qquad (13)$$

$$P_{k/k-1} = \sum_{i=0}^{2n} W_i^{(c)} [\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}][\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}]^T + Q_{k-1} \qquad 14)$$

$$\zeta_{k/k-1}^{(i)} = h(\chi_{k/k-1}^{(i)}, u_{k-1}), i = 0,1,2,\dots,2n \qquad 15)$$

$$\hat{y}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(m)} \zeta_{k/k-1}^{(i)} \qquad (16)$$

d) Measurement updating:

$$P_{(yy)_{k/k-1}} = \sum_{i=0}^{2n} W_i^{(c)} [\zeta_{k/k-1}^{(i)} - \hat{y}_{k/k-1}][\zeta_{k/k-1}^{(i)} - \hat{y}_{k/k-1}]^T + R_k \qquad 17)$$

$$P_{(xy)_{k/k-1}} = \sum_{i=0}^{2n} W_i^{(c)} [\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}][\zeta_{k/k-1}^{(i)} - \hat{y}_{k/k-1}]^T \qquad (18)$$

$$K_k = P_{(xy)_{k/k-1}} P_{(yy)_{k/k-1}}^{-1} \qquad (19)$$

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k (y_k - \hat{y}_{k/k-1}) \qquad (20)$$

$$P_k = P_{k/k-1} - K_k P_{(yy)_{k/k-1}} K_k^T \qquad (21)$$

When the measured value of the current moment is get, we can update the state vector and covariance matrix through the above steps.

3. Improved Strong Tracking UKF Algorithm

3.1. Strong Tracking Filter (STF)

If a filter has the following good qualities: stronger robustness of model uncertainty, strong traceability about mutation status, even when the system is stable, keep traceability for slowly changing status and mutation status, moderate computational complexity, it is referred to as strong tracking filter (STF) [7].

Strong tracking filter of nonlinear systems which are constituted by Formula (1) and (2) has the following structure:

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k v_k = \hat{x}_{k/k-1} + K_k (y_k - \hat{y}_{k/k-1}) \qquad (22)$$

The orthogonality principle is a sufficient condition for making the filter to be a strong tracking filter, namely choosing an appropriate time-varying gain matrix K_k which makes following formula is established:

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = \min \tag{23}$$

$$E[v_{k+j} v_k^T] = 0, k = 1,2,\cdots, j = 1,2,\cdots, \tag{24}$$

Where, Formula (24) demands that residual value sequence of the different time must keep the orthogonal everywhere.

3.2. STUKF Algorithm

STUKF algorithm [8] is a kind of strong tracking UKF algorithm, in order to overcome the flaws of the UKF, it uses the time-varying fading factor to fade the past data and reduce the impact on current filter value. In the UKF algorithm, state prediction covariance matrix $P_{k/k-1}$, measurement prediction covariance matrix $P_{(yy)_{k/k-1}}$ and the cross-covariance matrix $P_{(xy)_{k/k-1}}$ between state and measurement can be adjusted in real-time through introduction of time-varying fading factor to adjust the real-time filter gain matrix. Formula (14), (17) and (18) are modified as shown below:

$$P_{k/k-1} = \lambda_k \sum_{i=0}^{2n} W_i^{(c)} [\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}][\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}]^T + Q_{k-1} \tag{25}$$

$$P_{(yy)_{k/k-1}} = \sum_{i=0}^{2n} W_i^{(c)} [\zeta'^{(i)}_{k/k-1} - \hat{y}'_{k/k-1}][\zeta'^{(i)}_{k/k-1} - \hat{y}'_{k/k-1}]^T + R_k \tag{26}$$

$$P_{(xy)_{k/k-1}} = \sum_{i=0}^{2n} W_i^{(c)} [\chi'^{(i)}_{k/k-1} - \hat{x}'_{k/k-1}][\zeta'^{(i)}_{k/k-1} - \hat{y}'_{k/k-1}]^T \tag{27}$$

Where, $\lambda_k \geq 1$ is the time-varying fading factor.

With the modified formula (25), (26) and (27) instead of (14), (17) and (18) respectively, STUKF algorithm can be get with the same steps as UKF algorithm. This strong tracking filter is based on the STF and introduces fading factor in state prediction variance matrix. STUKF algorithm needs three UT for each filtering: firstly, calculating the state prediction $\hat{x}_{k/k-1}$ and state prediction covariance matrix $P_{k/k-1}$; secondly, calculating the residual value γ_k ,the measurement prediction covariance matrix $P_{(yy)_{k/k-1}}$ and cross-covariance matrix $P_{(xy)_{k/k-1}}$ without fading factor, then calculating the fading factor λ_k and the new state prediction covariance matrix $P_{k/k-1}$ with fading factor; thirdly, calculating $P_{(yy)_{k/k-1}}$ and $P_{(xy)_{k/k-1}}$ with fading factor, then calculating gain matrix K_k , state estimation $\hat{x}_k$ and estimation covariance matrix P_k. Compared with the UKF filter, calculating amount of three UT increases seriously, and it is not conducive to application of engineering.

3.3. ISTUKF Algorithm

ISTUKF algorithm is an improved STUKF algorithm. In view of the problems brought by the introduction of fading factors in the state prediction covariance matrix $P_{k/k-1}$ of the STUKF algorithm directly, fading factor λ_k is introduced in measurement prediction covariance matrix and cross-covariance matrix. New algorithm is the same as traditional UKF, however without introducing fading factor in measurement prediction covariance matrix and cross-covariance matrix.

a) According to $\hat{x}_{k/k-1}$, $P_{k/k-1}$ and the sampling strategy, we need to do second UT and calculate sigma sampling point set $\xi_j (j = 0,1,\cdots,2n)$ after calculating $P_{k/k-1}$.

b) Calculate the residual value γ_k, fading factor λ_k, measurement prediction covariance matrix $P_{(yy)_{k/k-1}}$ and cross-covariance matrix $P_{(xy)_{k/k-1}}$ with the fading factor λ_k.

$$\eta_i = h(\xi_i), i = 0,1,2,\ldots,2n \tag{28}$$

$$\hat{y}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(m)} \eta_i \tag{29}$$

$$\gamma_k = y_k - \hat{y}_{k/k-1} \tag{30}$$

$$P_{(yy)_{k/k-1}} = \lambda_k \sum_{i=0}^{2n} W_i^{(c)} [\eta_i - \hat{y}_{k/k-1}][\eta_i - \hat{y}_{k/k-1}]^T + R_k \tag{31}$$

$$P_{(xy)_{k/k-1}} = \lambda_k \sum_{i=0}^{2n} W_i^{(c)} [\chi_{k/k-1}^{(i)} - \hat{x}_{k/k-1}][\eta_i - \hat{y}_{k/k-1}]^T \tag{32}$$

$$\lambda_k \sum_{i=0}^{2n} W_i^{(c)} [\eta_i - \hat{y}_{k/k-1}][\eta_i - \hat{y}_{k/k-1}]^T = V_k^{\gamma} - R_k \tag{33}$$

Where $V_k^{\gamma} = E[\gamma_k \gamma_k^T]$

$$N_k = V_k^{\gamma} - R_k \tag{34}$$

$$N_k = \lambda_k M_k \tag{35}$$

Through the formula (35), subprime expression of fading factor is:

$$\lambda_k = \begin{cases} \lambda', & \lambda' > 1 \\ 1, & \lambda' \le 1 \end{cases} \tag{36}$$

$$\lambda' = {tr[N_k]} \big/ {tr[M_k]}$$

Covariance matrix of residual value is estimated by the following formula:

$$V_k^{\gamma} = \begin{cases} \gamma_1 \gamma_1^T \\ \varepsilon V_{k-1}^{\gamma} + \gamma_k \gamma_k^T \big/ 1 + \varepsilon \end{cases} \tag{37}$$

Where, $0 < \varepsilon \le 1$ is forgetting factor.

Compared with STUKF algorithm, the position of fading factor and formula are different in this new algorithm.

4. GPS/DR Integrated Navigation System Model

Global Positioning System (GPS) and Dead Reckoning System (D/R) have their respective characteristic, which can be combined for the GPS/DR integrated navigation system [9]. Because the error of sensor and external interference makes the reckoning error increase [10], the data of DR must be filtered. However in DR navigation system, the measurement equation is nonlinear. So the nonlinear filtering algorithm is used to estimate the state of DR and improves the navigation accuracy.

In order to analyze ISTUKF algorithm, this paper designs a simulation experiment about GPS/DR integrated navigation system model by the Matlab simulation. GPS/DR system adopts the northeast coordinate system in which x axis points to the east and y axis points north. In the GPS/DR integrated navigation system, we define position, velocity and acceleration as system state, therefore, state equation of the system can be summarized as follows:

$$x_k = \begin{bmatrix} x_{e(k)} \\ v_{e(k)} \\ a_{e(k)} \\ x_{n(k)} \\ v_{n(k)} \\ a_{n(k)} \end{bmatrix} = \Phi_{k/k-1} x_{k-1} + \begin{bmatrix} 0 \\ 0 \\ w_{ae} \\ 0 \\ 0 \\ w_{an} \end{bmatrix} \tag{38}$$

Where, $x_{e(k)}, v_{e(k)}$ and $a_{e(k)}$ are the east position, velocity and acceleration respectively, however, $x_{n(k)}, v_{n(k)}$ and $a_{n(k)}$ are the north ones. w_{ae} and w_{an} are zero mean Gaussian noises of east and north accelerations, and $\Phi_{k/k-1}$ is the state transition matrix as shown below:

$$\Phi_{k/k-1} = diag\{\Phi_e, \Phi_n\} \tag{39}$$

Where, $\Phi_e = \Phi_n = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$, T is the sampling period.

Take the east location e_{obs} and north one n_{obs} of GPS receiver outputs, the angular rate ω of gyroscope output and the distance S of the odometer's output during one sampling period as the measurement value. Measurement equation of integrated navigation system can be summarized as follows:

$$y_k = \begin{bmatrix} e_{obs(k)} \\ n_{obs(k)} \\ \omega_k \\ s_k \end{bmatrix} = \begin{bmatrix} x_{e(k)} \\ x_{n(k)} \\ \dfrac{v_{n(k)} a_{e(k)} - v_{e(k)} a_{n(k)}}{(v_{e(k)})^2 + (v_{n(k)})^2} \\ T\sqrt{(v_{e(k)})^2 + (v_{n(k)})^2} \end{bmatrix} + \begin{bmatrix} w_{xe} \\ w_{xn} \\ w_{\omega} \\ w_s \end{bmatrix} \tag{40}$$

Where, w_{xe} and w_{xn} are the measurement noises of east and north locations of the GPS receiver, w_{ω} is output of the gyroscope, w_s is the measurement noises of the odometer.

5. Simulation Experiment

According to the above definition of GPS/DR integrated navigation system model, the conditions of simulation experiment are set as shown below: Assuming the vehicle starts from the origin (0, 0) in an almost straight line trajectory with a speed of $20\ m/s$ and a course of 45^{o} for 300s, the speed of east and north which can be calculated through above assumptions is $10\sqrt{2} m/s$, and outliers appear in GPS receiver's measurement values at time 20s. Furthermore, the sampling period T=1s, initial system state vector $x_0 = [0, 10, 0, 0, 10, 0]$, initial state prediction covariance matrix $P_0 = diag\ [1,1,1,1,1,1]$, independent noises of system and measurement can be set as follows: $\sigma_{ae}^2 = 0.0009$, $\sigma_{an}^2 = 0.0009$, $\sigma_{xe}^2 = 15.5^2$, $\sigma_{xn}^2 = 16.5^2$, $\sigma_{\omega}^2 = 0.00002$, $\sigma_{s}^2 = 0.5$. STUKF [10] algorithm and ISTUKF algorithm were used respectively in simulation of GPS/DR integrated navigation system, and results are shown in Figure 1-3 as below.

From Figure 1-3, it can be seen that when the outlier appears in the measurement data, the filtering precision and stability of the STUKF will be influenced badly. However ISTUKF algorithm filters out very commendably the disturbance brought by the outliers, and it is superior to STUKF algorithm in the filtering precision, robustness and speed.

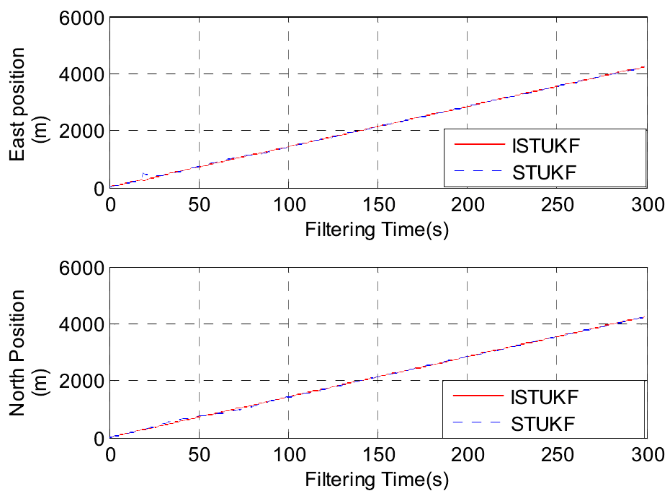

Figure 1. The Contrasts About the East Position and North Position of ISTUKF and STUKF

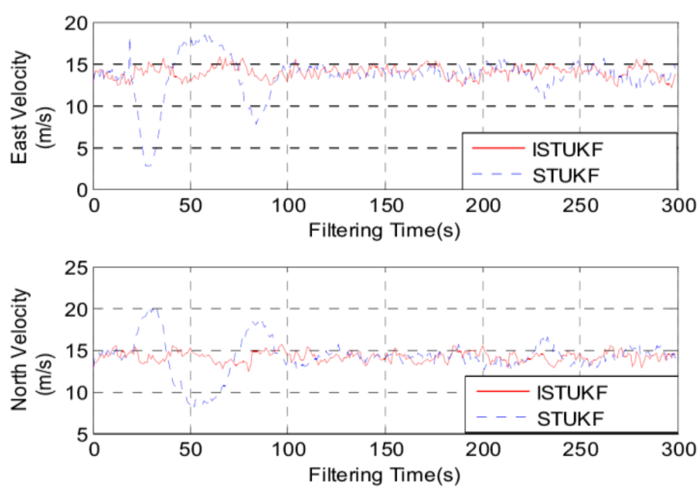

Figure 2. The Contrasts about the East Velocity and North Velocity of ISTUKF and STUKF

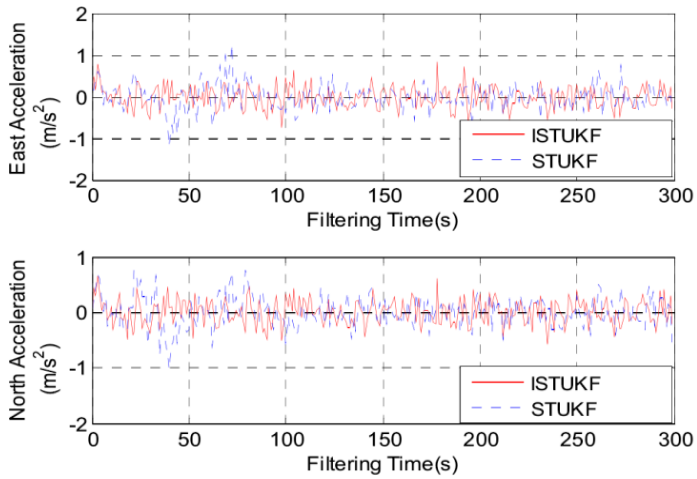

Figure 3. The Contrasts about the East Acceleration and North Acceleration of ISTUKF and STUKF

6. Conclusion

On the base of UKF and STUKF algorithm, this paper presents an ISTUKF algorithm that solves the problem of filtering precision and poor stability of UKF affected by the uncertainty factors in GPS/DR integrated navigation system. Simulation results show that the filtering precision and robustness of ISTUKF algorithm is better than the STUKF algorithm.

Acknowledgements

The author would like to thank the anonymous referees for their valuable suggestions. This work was supported by the Natural Science Foundation of Heilongjiang Province of China (Grant No. F201210).

References

[1] A. Giannitrapani, N. Ceccarelli and F. Scortecci, "Comparison of EKF and UKF for Spacecraft Localization Via Angle Measurements", IEEE Transactions on Aerospace and Electronic Systems, vol.47, no.1, **(2011)**, pp. 75-84.

[2] L. Wenling and J. Yingmin. "Consensus-based Distributed Multiple-model UKF for Jump Markov Nonlinear Systems", IEEE Transactions on Automatic Control, vol. 57, no.1, **(2012)**, pp. 227-233.

[3] J. Dah-Jing, L. Chun-Nan. "Unscented Kalman Filter with Nonlinear Dynamic Process Modeling for GPS Navigation" .GPS Solutions, vol. 9, no. 12, **(2008)**, pp. 249-260.

[4] Z.W. Wu, M.L. Yao and HG. Ma. "Low-Cost Antenna AttitudeEstimation by Fusing Inertial Sensing and Two-Antenna GPS for Vehicle-Mounted Satcom-on-the-Move", IEEE Transactions on Vehicular Technology, vol. 62, no. 3, **(2013),** pp.1084-1096.

[5] FAA Cheein, "Covariance-Based Measurement Selection Criterion for Gaussian-Based Algorithms", ENTROPY, vol. 15, no. 1, **(2013)**, pp. 287-310.

[6] FD Rincon, M Esposito and PHH de Araujo, "Calorimetric Estimation Employing the Unscented Kalman Filter for a Batch Emulsion Polymerization Reactor". Macromolecular Reaction Engineering, vol.7, no. 1, **(2013)**, pp.24-35.

[7] QI LiFeng, T. JianFeng and F. XinXi. "A Modified Unscented Kalman Non-linear Filter". Journal of Projectiles, Rockets, Missiles and Guidance, vol. 31, no. 2, **(2011)**, pp.179-181.

[8] W. Xiaoxu, Z. Lin, X. Quanxi. "Strong Tracking Filter Based on Unscented Transformation", Control and Decision, vol. 25, no.7, **(2010)**, pp.1063-1068.

[9] D. An, J. A. Rios and D. Liccardo. "A UKF Based GPS/DR Positioning System for General Aviation", Proceedings of the 18th International Technical Meeting of the Satellite Division of The Institute of Navigation, ION GNSS **(2005)**, pp. 989-998.
[10] HT Zhang, YJ Zhao, "The Performance Comparison and Analysis of Extended Kalman Filters for GPS/DR Navigation", OPTIK, vol. 122, no.9, **(2013)**, pp. 777-781.

Authors

Jian FENG, lecturer of School of Computer and Information Engineering, Research Interests: information detection and estimation, artificial intelligence.

Xiao-dong SU, Prof. Su Xiaodong, master instructor, vice president of School of Computer and Information Engineering in charge of teaching, Research Interests: information detection and estimation, wireless communication theory and technology, management information systems.

