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Modified Function Projective Synchronization Using Feedback Error Control

Hongyue Du, Wei Yao, Ge Hu and Jian Wang

School of Automation, Harbin University of Science and Technology, Harbin 150080, PR China

du_hong_yue@163.com

Abstract

Some new simple control methods of modified function projective synchronization in two chaotic systems are investigated in this paper. Based on Lyapunov method, a general feedback error control law is proposed, which contains only feedback error term and easy to implement in practice. Moreover, an adaptive feedback error control scheme is proposed, in which the feedback gain can be automatically adapted to suitable constant. Numerical example is provided to show the effectiveness of proposed method.

Keywords: *Modified function projective synchronization, feedback error control, adaptive control, chaotic system*

1. Introduction

Since the seminal work of Pecora and Carroll [1], in which presented a successful method to synchronize two identical chaotic systems, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields. Up to now, many different synchronization regimes have been studied [2-6]. Recently a new type of synchronization, termed as modified function projective synchronization, have been extensively investigated in [7-13], where the drive and response systems could be synchronized to a desired scaling function matrix. The novelty feature of this synchronization phenomenon is that the scaling functions can be arbitrarily designed to different state variables by means of control, while the unpredictability of the scaling functions in MFPS can additionally enhance the security of communications.

In Ref. [7], the authors gave the MFPS scheme of two coupled Lorenz systems. Ref. [8] investigated adaptive modified function projective synchronization of hyperchaotic systems with unknown parameters. Based on active control scheme, a general method of MFPS with time delay was investigated in Ref. [9]. Ref. [10] investigates the modified function projective synchronization (MFPS) of drive-response dynamical networks using adaptive open-plus-closed-loop control method. More general forms of MFPS have been extensively investigated in Refs. [11-13].

In most previous proposed references [7-13], the designed controllers contain some nonlinear terms of the systems, which is more complicated and hard to implement in practice. Differ from the ones proposed in [7-13], our designed controller contain only feedback error term, which is simple and easy to implement in practice. Furthermore, proposed adaptive method can achieve MFPS even not require any additional information regarding the drive system and the feedback gain of the closed loop control

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part can be automatically adapted to suitable constant. To the best of our knowledge, at present, there are few theoretical results about it.

The remainder of this paper is organized as follows: In Section 2, some preliminaries are briefly outlined. The main theorems for MFPS are given in Section 3. In Section 4, we will choose two groups of examples to show the effectiveness of the proposed methods. Conclusions are finally drawn in Section 5.

2. Preliminaries

The drive system and the response system are defined below

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{1}$$

$$\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{u} \tag{2}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R}$ are the state vectors, $\boldsymbol{f} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ are continuous nonlinear vector functions, $\boldsymbol{u}$ is the vector controller. We define the error vector

$$\boldsymbol{e} = \boldsymbol{\Lambda}(t)\boldsymbol{x} - \boldsymbol{y} \tag{3}$$

where $\boldsymbol{\Lambda}(t)$ is a n-order diagonal matrix, $\Lambda(t) = diag\ (\alpha_1(t), \alpha_2(t), \cdots, \alpha_n(t))$ and $\alpha_i(t) \neq 0$ $(i = 1, 2, \cdots, n)$, is a continuously differentiable function with bounded.

Assumption 1. The derivative of the scaling functions are bounded, that is

$$|\dot{\alpha}_i(t)| \leq a_i^*, \quad i = 1, 2, \cdots, n$$

for all $t \in R^+$, where $a_i^* \in R^+$ is the upper limit of $|\dot{\alpha}_i(t)|$.

Definition 1 (MFPS). For the drive system (1) and the response system (2), it is said that the system (1) and the system (2) are modified function projective synchronization (MFPS), if there exists a scaling function matrix $\boldsymbol{\Lambda}(t)$ such that $\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = 0$.

Our goal is to design a simple controller u such that the controlled response system (2) could be MFPS to the drive system (1), i.e. $\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = 0$.

3. Controller Design

In most previous proposed references [7-13], the designed controllers contain some nonlinear terms of the drive and response systems, which are more complicated and hard to implement in practice. In this section, a general simple scheme is proposed in Theorem1 and an adaptive scheme is proposed in Theorem2, which contain only feedback error term and are easy to implement in practice.

3.1. General Scheme

Theorem 1. Suppose Assumption 1 holds. For a given synchronization scaling function matrix $\boldsymbol{\Lambda}(t)$, if there exists a positive constant p satisfying $p > M_1 + M_2 + M_3$, then the MFPS between the drive system (1) and the response system (2) will occur by the control law as below

$$\boldsymbol{u} = p\,\mathrm{sgn}(\boldsymbol{e}) + q\boldsymbol{e} \tag{4}$$

where $\boldsymbol{e} = \boldsymbol{\Lambda}(t)\boldsymbol{x} - \boldsymbol{y}$, $q = \lambda_{\max}(\frac{\boldsymbol{H} + \boldsymbol{H}^T}{2})$ and $\mathrm{sgn}(\cdot)$ denotes the sign function.

Proof. We define the error vector as

$$\boldsymbol{e} = \boldsymbol{\Lambda}(t)\boldsymbol{x} - \boldsymbol{y} \tag{5}$$

The time derivative of Eq. (3) is

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}(t)\dot{\boldsymbol{x}} - \dot{\boldsymbol{y}} + \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x} \tag{6}$$

Substituting (1), (2) into (6), we have

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{y}) - \boldsymbol{u} + \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x} \tag{7}$$

The vector function $\boldsymbol{f}(\boldsymbol{y})$ is linearized as follows in the neighborhood of the goal value via Taylor expansions

$$\boldsymbol{f}(\boldsymbol{y}) = \boldsymbol{f}(\boldsymbol{z}) + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{z}}(\boldsymbol{y} - \boldsymbol{z}) + \cdots \tag{8}$$

where $\boldsymbol{z} = \boldsymbol{\Lambda}(t)\boldsymbol{x}$. Keeping the first-order terms in Eq. (8) and substituting in Eq. (7), we have

$$\dot{\boldsymbol{e}} = \boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{z}) - \boldsymbol{He} - \boldsymbol{u} + \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x} \tag{9}$$

where $\boldsymbol{H} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{z}}$ is the Jacobian matrix of $\boldsymbol{f}(\boldsymbol{y})$ with respect to $\boldsymbol{z}$.

Construct Lyapunov function

$$V = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{e} \tag{10}$$

With the choice of Eq. (5) the time derivative of V along the trajectories of Eq. (9) is

$$\begin{aligned}
\dot{V} &= \boldsymbol{e}^T\dot{\boldsymbol{e}} \\
&= \boldsymbol{e}^T(\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{z}) - \boldsymbol{He} - \boldsymbol{u} + \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}) \\
&\le \left\|\boldsymbol{e}^T\right\|\left\|\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})\right\| + \left\|\boldsymbol{e}^T\right\|\left\|\boldsymbol{f}(\boldsymbol{z})\right\| + \left\|\boldsymbol{e}^T\right\|\left\|\dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}\right\| + \lambda_{\max}\left(\frac{\boldsymbol{H} + \boldsymbol{H}^T}{2}\right)\boldsymbol{e}^T\boldsymbol{e} - p\left\|\boldsymbol{e}^T\right\| - q\boldsymbol{e}^T\boldsymbol{e}
\end{aligned} \tag{11}$$

where $\lambda_{\max}\left(\frac{\boldsymbol{H} + \boldsymbol{H}^T}{2}\right)$ is the maximum eigenvalue of the matrix $\frac{\boldsymbol{H} + \boldsymbol{H}^T}{2}$.

Because chaos systems and the scaling functions are bounded, $\boldsymbol{x}$, and $\boldsymbol{z}$ are bounded. Furthermore, $\boldsymbol{f}$ is a continuously vector function, there exist the positive constants M_1 and M_2 satisfying $\left\|\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})\right\| \le M_1$, $\left\|\boldsymbol{f}(\boldsymbol{z})\right\| \le M_2$. Because Assumption 1 is held, there exists a positive constant M_3 satisfying $\left\|\dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}\right\| \le M_3$.

$$\dot{V} \le (M_1 + M_2 + M_3 - p)\left\|\boldsymbol{e}^T\right\| \tag{12}$$

Note that the condition of Theorem 1 holds $p > M_1 + M_2 + M_3$, we obtain

$$\dot{V} < 0 \tag{13}$$

According to the Lyapunov stability theorem, $\boldsymbol{e} \to 0$ with $t \to \infty$. The MFPS is achieved under the certain chosen controller $\boldsymbol{u}$ in Eq. (4). This completes the proof.

3.2. Adaptive Scheme

Although the proposed control law in Theorem1 is simple, the feedback gain is hard to achieve. In this subsection, we will further investigate adaptive feedback gain scheme.

Theorem 2. Suppose Assumption 1 holds. For a given synchronization scaling function matrix $\boldsymbol{\Lambda}(t)$, the MFPS between the drive system (1) and the response system (2) will occur by the control law as below

$$\boldsymbol{u} = p\,\mathrm{sgn}(\boldsymbol{e}) \tag{14}$$

$$\dot{p} = k\boldsymbol{e}^T \mathrm{sgn}(\boldsymbol{e}) \tag{15}$$

where $\boldsymbol{e} = \boldsymbol{\Lambda}(t)\boldsymbol{x} - \boldsymbol{y}$, k is a arbitrary positive constant and $\mathrm{sgn}(\cdot)$ denotes the sign function.

Proof. Construct Lyapunov function

$$V = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{e} + \frac{1}{2k}(p - p^*)^2 \tag{16}$$

With the choice of Eq. (14) and Eq. (15) the time derivative of V along the trajectories of Eq.(9) is

$$\begin{aligned}
\dot{V} &= \boldsymbol{e}^T\dot{\boldsymbol{e}} + \frac{1}{k}(p - p^*)\dot{p} \\
&= \boldsymbol{e}^T(\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{z}) - \boldsymbol{He} - \boldsymbol{u} + \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}) + (p - p^*)\boldsymbol{e}^T\mathrm{sgn}(\boldsymbol{e}) \\
&\le \|\boldsymbol{e}^T\|\|\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})\| + \|\boldsymbol{e}^T\|\|\boldsymbol{f}(\boldsymbol{z})\| + \|\boldsymbol{e}^T\|\|\dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}\| + \lambda_{\max}(\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2})\boldsymbol{e}^T\boldsymbol{e} - p^*\boldsymbol{e}^T\mathrm{sgn}(\boldsymbol{e}) \\
&\le (\|\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})\| + \|\boldsymbol{f}(\boldsymbol{z})\| + \|\dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}\| + \lambda_{\max}(\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2})\|\boldsymbol{e}^T\| - p^*)\|\boldsymbol{e}^T\|
\end{aligned} \tag{17}$$

where $\lambda_{\max}(\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2})$ is the maximum eigenvalue of the matrix $\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2}$.

Because chaos systems and the scaling functions are bounded, $\boldsymbol{x}$, and $\boldsymbol{z}$ are bounded. Furthermore, $\boldsymbol{f}$ is a continuously vector function, there exist the positive constants M_1 and M_2 satisfying $\|\boldsymbol{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})\| \le M_1$, $\|\boldsymbol{f}(\boldsymbol{z})\| \le M_2$. Because Assumption 1 is held, there exists a positive constant M_3 satisfying $\|\dot{\boldsymbol{\Lambda}}(t)\boldsymbol{x}\| \le M_3$.

$$\dot{V} \le (M_1 + M_2 + M_3 + \lambda_{\max}(\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2})\|\boldsymbol{e}^T\| - p^*)\|\boldsymbol{e}^T\| \tag{18}$$

Taking $p^* = M_1 + M_2 + M_3 + \lambda_{\max}(\frac{\boldsymbol{H}+\boldsymbol{H}^T}{2})\|\boldsymbol{e}^T\| + 1$, we obtain

$$\dot{V} \le -\|\boldsymbol{e}^T\| \le 0 \tag{19}$$

According to the Lyapunov stability theorem, $\boldsymbol{e}$ is bounded. Since the states of chaotic systems are bounded and Assumption 1 is held, $\dot{\boldsymbol{e}}$ is bounded, i.e. $\dot{\boldsymbol{e}} \in L_\infty$. According to Eq. (19), we have

$$\int_0^\infty \boldsymbol{e}^T\boldsymbol{e}\,dt = \int_0^\infty \|\boldsymbol{e}^T\|^2 dt \le \left[\int_0^\infty \|\boldsymbol{e}^T\| dt\right]^2 \le \left[-\int_0^\infty \dot{V} dt\right]^2 \le \left[V^2(0) - V^2(\infty)\right]^2 < \infty \tag{20}$$

So, $\boldsymbol{e} \in L_2$. According to Barbalat's Lemma, $\boldsymbol{e} \to 0$ with $t \to \infty$. The MFPS is achieved under the certain chosen controller $\boldsymbol{u}$ in Eq. (14). This completes the proof.

4. Illustrative Examples

In this section, we choose chaotic Lü system as an example to show the effectiveness of the proposed method.

We take Lü system as the drive system, which is described by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -x_1 x_3 + c x_2 \\ \dot{x}_3 = x_1 x_2 - b x_3 \end{cases} \tag{21}$$

where x_1, x_2, x_3 are state variables, a, b, c are system parameters. When three real parameters $a = 36, b = 3, c = 20$, the system shows chaotic behavior. Figure 1 depicts the chaotic attractor of Lü system.

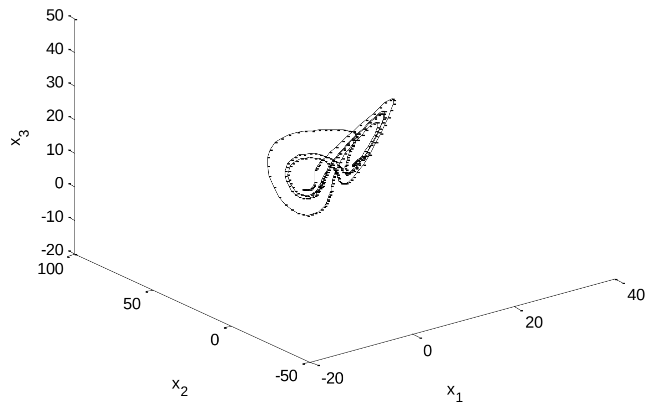

Figure 1. The Chaotic Attractor of Lü System in R^3

The controlled Lü system, as the response system, is described as

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = -y_1 y_3 + c y_2 + u_2 \\ \dot{y}_3 = y_1 y_2 - b y_3 + u_3 \end{cases} \tag{22}$$

where y_1, y_2, y_3 are state variables, u_1, u_2, u_3 are the controllers.

The controller u_1, u_2, u_3 can be designed by Theorem 2 as follows

$$\boldsymbol{u} = \begin{pmatrix} p\,\mathrm{sgn}(e_1) \\ p\,\mathrm{sgn}(e_2) \\ p\,\mathrm{sgn}(e_3) \end{pmatrix} \tag{23}$$

$$\dot{p} = k\left(e_1 \mathrm{sgn}(e_1) + e_2 \mathrm{sgn}(e_2) + e_3 \mathrm{sgn}(e_3)\right) \tag{24}$$

In numerical simulation, we take $p(0)=1, k=4000$. The initial states take $\boldsymbol{x}(0)=[9 \quad 6 \quad 18]^T$, $\boldsymbol{y}(0)=[-6 \quad -8 \quad 9]^T$. The scaling functions take $\alpha_1(t)=0.5\sin(\pi t/6)+1$, $\alpha_2(t)=\sin(\pi t/6)+2$, $\alpha_3(t)=-0.5\sin(\pi t/6)-1$. The simulation result is shown in Figure 2 and Figure 3.

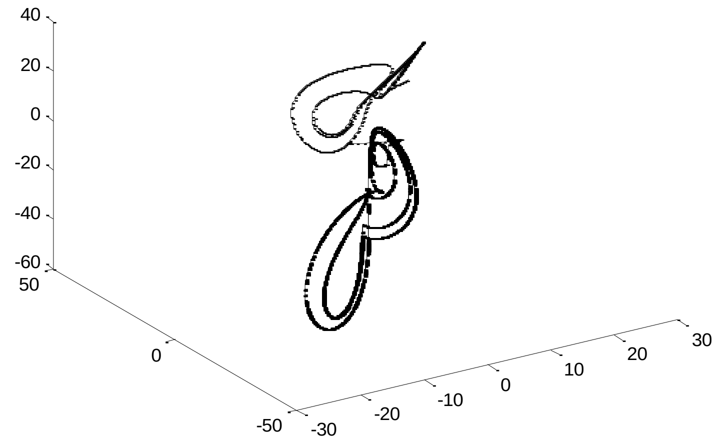

Figure 2. The Synchronized Attractors of Two Chaotic Lü Systems in R^3

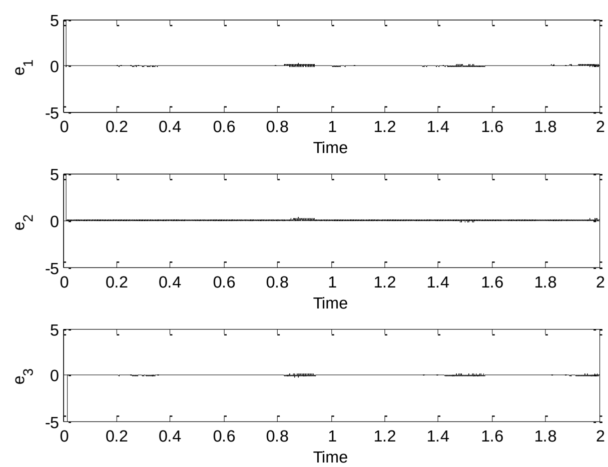

Figure 3. The Synchronization Errors of Two Chaotic Lü Systems

Figure 2 displays the synchronized attractor in R^3. The time evolution of the MFPS errors are depicted in Figure 3, which displays $\boldsymbol{e} \to 0$ with $t \to \infty$. Thus, the required synchronization has been achieved with our designed control law (23).

5. Conclusion

In this paper, some feedback error control schemes of modified function projective synchronization are proposed. The proposed controllers contain only feedback terms and easy to implement in practice. Moreover, proposed adaptive method can achieve

MFPS even not require any additional information regarding the drive system and the feedback gain can be automatically adapted to suitable constant. The proposed schemes can also be used in various synchronization.

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