

A Novel Hybrid Algorithm for Constrained Multi-objective Optimization

Zhidan Xu

Institute of Basic Science, Harbin University of Commerce
xuzhidanivy@163.com

Abstract

A new hybrid Optimization Algorithm is proposed to solve Constrained Multi-objective Optimization Problems (CMOPs). The algorithm is named BBO/DE which combines the exploitation ability of Biogeography-based Optimization (BBO) and the exploration ability of Differential Evolution (DE). Meanwhile distance measures and adaptive penalty functions are adopted to handle the constraints so that optimal solutions in the infeasible space can be searched effectively. In addition, the feasible archive is applied to store the non-dominated feasible solutions obtained so far and is updated based on crowding-distance. Experiment results demonstrate that the proposed hybrid algorithm BBO/DE can approximate the true Pareto front and has better distribution.

Keywords: *Biogeography-based Optimization Algorithm, Differential Evolution, adaptive penalty function, constrained multi-objective optimization*

1. Introduction

Evolutionary algorithms (EAs) are random search algorithms and can obtain many optimal solutions in a run so that they are applied extensively to solve optimization problems. However, many real problems are often subject to some constraints, so different constraint-handling methods has been proposed and applied in EAs to put forwards novel constrained multi-objective optimization algorithms. Main constraint-handling methods include ignoring infeasible solutions method [1] which is easy to implement but difficulty in finding feasible solutions especially when feasible regions are small and surrounded by infeasible solutions; Taking constraint violation as optimization objectives method [2] which increases the dimension of the optimization objectives and run time; Feasible solutions preferring over infeasible ones method[3] which ignores the contribution of excellent infeasible solutions, that is infeasible solutions nearby the feasible region, to the Pareto front. However, the performance of constrained multi-objective optimization algorithms (CMOAs) not only relate to constraint-handling methods but also to selection of EAs. Differential Evolution (DE) is a random search algorithm which has mutation operator based on the difference of individuals so that DE has better exploration ability of the population information. Zamuda [4] incorporated the constraint-handling method that feasible solutions preferring over infeasible ones in DE with self-adaptation and local search to solve constrained multi-objective problems (CMOPs), experiment results show the feasibility of the proposed algorithm for CMOPs. Zhang [5] proposed a hybrid of differential evolution and genetic algorithm for CMOPs. The hybrid algorithm adopted two different schemes to generate the offspring population so that it has better performance in the distribution of optimal solutions. With some new (CMOAs) appearing, it is gained attention that infeasible solutions contribute to

the distribution of solutions on Pareto front. Wei [6] proposed an infeasible elitist based particle swarm optimization for CMOPs. The algorithm adopted an infeasible elitist strategy, a new crowding distance function and a new mutation operator with two phases to improve the spread and distribution of solutions. These EAs with different constraint-handling methods show different advantages for (CMOPs), but most algorithms of them need be further improved in the diversity and distribution of the population.

BBO is a novel population-based stochastic search algorithm and had shown excellent exploitation ability of population information for single-objective optimization problems [7-8]. But there are a few reports on applying BBO to solve CMOPs. In view of the complexity of CMOPs, CMOAs with better performance need be proposed to improve the convergence and distribution of optimal solutions set. So considering the exploitation of BBO and the exploration of DE, a novel hybrid algorithm will be put forward to solve CMOPs. In the remainder of the paper, some fundamental conceptions on CMOPs are described in section II. In section III, the original BBO and DE are briefly introduced. The proposed hybrid algorithm BBO/DE is elaborated in section IV. At last, BBO/DE is compared with classic NSGAI on benchmark CMOPs and experiment results are discussed in section V.

2. Basic conceptions

The minimization CMOP is mathematically formulated

$$\begin{aligned} \min \quad & y = f(x) = [f_1(x), f_2(x), \dots, f_m(x)] \\ \text{s.t.} \quad & g_i(x) \leq 0, i = 1, 2, \dots, q \\ & h_j(x) = 0, j = q + 1, q + 2, \dots, n \end{aligned} \quad (1)$$

where $x = (x_1, x_2, \dots, x_n) \in D \subset R^n$ is a decision vector with n decision variables, D is a n dimension decision space, each dimension variable meets $x_i^{\min} \leq x_i \leq x_i^{\max}, i = 1, 2, \dots, n$, x_i^{\max} and x_i^{\min} are its upper limit and lower limit; objective vector $y = (f_1, f_2, \dots, f_m) \in Y \subset R^m$ has m objects and Y is a m dimension objective space; $g_i(x)$ and $h_j(x)$ are inequality constraints and equality constraint respectively. The equality constraints generally should be transformed into inequality form and combined with other inequality constraints as

$$c_j(x) = \begin{cases} \max \{g_i(x), 0\} & i = 1, 2, \dots, q \\ \max \{|h_j(x)| - \delta, 0\} & j = q + 1, q + 2, \dots, m \end{cases} \quad (2)$$

where δ is a tolerance parameter for the equality constraint.

Definition 1(Pareto Domination) Let $x, y \in D$, solution vector x is said to dominate strictly solution y iff

$$\begin{aligned} \forall i \in \{1, 2, \dots, m\} : f_i(x) \leq f_i(y) \\ \exists j \in \{1, 2, \dots, m\} : f_j(x) < f_j(y) . \end{aligned} \quad (3)$$

The domination relation is denoted $x \prec y$.

Definition 2(Pareto optimal) A solution $y \in D$ is called Pareto-optimal with respect to D iff

$$\{x \mid x \prec y, x \in D \text{ and } x \neq y\} = \Phi \quad (4)$$

Definition 3(non-dominated solution) Let $S \subset D$ be a subset of solutions, x is called a non-dominated solution with respect to S iff

$$\{y \mid y \prec x, y \in S\} = \Phi \quad (5)$$

Definition 4(Pareto optimal solution) v is called Pareto-optimal solution if v is non-dominated with respect to all solutions in D .

Definition 5(Pareto front) The image of all non-dominated solutions is called the Pareto front.

3. Biogeography-based optimization and differential evolution

3.1. Biogeography-based optimization

The mathematical model of biogeography main describes the distribution of species. Based on this, Simon proposed Biogeography-based Optimization (BBO) in 2008. In BBO, each individual corresponds with a habitat of species, and each habitat owns certain habitat survival index (HSI) which is similar to the fitness of EAs. If HSI is higher, the habitat has more species number and is considered as an excellent individual. According to species number of each individual, its immigration rate λ_i and emigration rate μ_i can be calculated as following [7]

$$\lambda_i = I \left(1 - \frac{l}{S_{\max}}\right) \quad (6)$$

$$\mu_i = \frac{El}{S_{\max}} \quad (7)$$

where s_{\max} is the maximum species number of all habitats, I and E denote the maximum of immigration rate and emigration rate, l is the species number of the i th individual. If a individual has more species number, it owns smaller immigration rate λ_i and larger emigration rate μ_i . By contraries, the individuals with littler species have larger immigration rate and smaller emigration rate. Based on this, migration operator is implemented. That is Individuals with high emigration rate (excellent individuals) will share their features variables with those with high immigration rate so that more excellent individuals are generated.

According to the species number probability p_i of each individual, mutation probability m_i [7] is defined

$$m_i = P_{\text{mute}} \left(1 - \frac{p_i}{P_{\max}}\right) \quad (8)$$

where P_{mute} is a predefined parameter, the species number probability p_i denotes the probability of each habitat owning certain species, $P_{\max} = \max\{p_i, 1 \leq i \leq S_{\max}\}$. If the species number probability is smaller, individuals have larger mutation probability and have many chances to change excellent individuals. By applying the mutation operator, the diversity of the population can be improved. The detail migration and mutation procedures can refer to [4].

Biogeography-Based Optimization (BBO) generate the next generation population mainly through the above migration and mutation operators. Its evolution process is shown in Table1. Firstly, the initial population is generated randomly. Secondly, calculate the fitness of the individuals of the population and compute the immigration and emigration rate based on the species number of the individual. Thirdly, implement migration operators based on the immigration and emigration rate, and then mutation operators is implemented according to the individual species probability[4]. The algorithm BBO is run repeatedly until it meets the end condition. In BBO, by the

migration operator, the variable information of the excellent individuals can be shared by the individuals with small fitness value, which make excellent characters conserved in the population so that more excellent individuals are generated. Meanwhile, the population can approximate the Pareto front. By the mutation operator, the individuals with the more and fewer individuals have more chance in mutating the individuals with higher fitness value, while some individuals are conserved in the current population to ensure the convergence of the population. After the two operators, the next generation population is produced. The same process is run more times to gain the optimal solutions of problems.

Table 1. Pseudo-code of Biogeography-based optimization algorithm

<i>BBO Algorithm</i>
1 : <i>Generate randomly the initial population $P(t)$, $t = 1$</i>
2 : <i>While $t \leq g_{max}$,</i>
3 : <i>Compute the fitness of individuals in the population $P(t)$</i>
4 : <i>Rank individuals of $P(t)$ by fitness in ascending order</i>
5 : <i>Compute species number, immigration and emigration rate of individuals in $P(t)$</i>
6 : <i>Implement the migration and mutation operator on the population $P(t)$ to gain the next generation population $P(t+1)$</i>
7 : <i>$t = t + 1$</i>
8 : <i>Endwhile</i>

3.2. Differential evolution

Differential Evolution (DE) [9] is also a population-based optimization algorithm. In DE, the mutation, crossover and selection are major operators. The mutation operator is implemented based on the difference of individuals so that DE showed better exploration ability for multi-objective optimization problems (MOPs)[10][11]. By mutation operator, a candidate solution is produced as

$$v_p = x_{r1} + F(x_{r2} - x_{r3}) \quad (9)$$

where x_{r1} , x_{r2} and x_{r3} are three random individuals from the population, F is the scale factor and is often selected in the region (0,2). And then different mutation strategies are designed and proposed to balance the convergence and the diversity of the population [12].

To increase the diversity of the population, the crossover operation is applied to each pair of mutant vector (v_p, x_p) , and then offspring vector u_p is obtained as the follows:

$$u_p(i) = \begin{cases} v_p(i) & rand < CR \\ x_p(i) & otherwise \end{cases} \quad (10)$$

where CR is a predefined constant within the range (0,1) and it determines which dimension variable of u_p are copied from v_p or x_p . After the crossover, the selection operation based on fitness is performed to generate the offspring population.

4. Hybrid algorithm BBO/DE

Considering that infeasible solutions contribute to the distribution of the population, they will be evaluated and evolved with feasible ones. The proposed hybrid algorithm will use objective function values modified [13] to handle the constraints so that optimal solutions in infeasible region can be searched efficiently.

4.1. Objective function values modified

The objective function value modified is consisted of individuals' distance and adaptive penalty. In each dimension objective function, the distance [13]of individual is formulated as

$$d_i(x) = \begin{cases} v(x) & \text{if } r_f = 0 \\ \sqrt{\tilde{f}_i(x)^2 + v(x)^2} & \text{, otherwise} \end{cases} \quad (11)$$

$$v(x) = \frac{1}{m} \sum_{j=1}^m \frac{c_j(x)}{c_{\max}^j} \quad (12)$$

$$\tilde{f}_i(x) = \frac{f(x)_i - f_{\min}^i}{f_{\max}^i - f_{\min}^i} \quad (13)$$

where r_f is the proportion of feasible individuals in current population, f_{\min}^i and f_{\max}^i are the minimum and maximum in the i th dimension objective function, respectively. c_{\max}^j is the maximum of the j th constraint. When there is no feasible individual in the current population, infeasible individuals with smaller constraint violation have small distance; on the other hand, if there is more than one feasible solution in current population, infeasible individuals with smaller objective function value and smaller constraint violation have small distance, and When they have similar objective function values with feasible individuals, the feasible individuals have smaller distance.

To search feasible solutions efficiently, infeasible solutions will be further penalized. The penalties[13]of individual x for its i th objective function as

$$p_i(x) = (1 - r_f)X_i(x) + r_f Y_i(x) \quad (14)$$

$$X_i(x) = \begin{cases} 0 & \text{if } r_f = 0 \\ v(x) & \text{, otherwise} \end{cases} \quad (15)$$

$$Y_i(x) = \begin{cases} 0 & \text{if } x \text{ is feasible} \\ \tilde{f}_i(x) & \text{, otherwise} \end{cases} \quad (16)$$

If the feasibility ratio r_f of the population is small, individuals with higher constraint violation gain larger penalty so that few feasible individuals are generated; on the other hand, when there are many feasible individuals in the current population, infeasible individuals with larger objective function value will be more penalized.

The final modified objective value [13] of individual is

$$F_i(x) = d_i(x) + p_i(x) \quad (17)$$

By using the modified objective value, infeasible solutions with small objective value and constraint violation are excellent individuals and have more chance to produce feasible one and improve the distribution of the population.

4.2. Hybrid Migration Operator

BBO has good exploitation ability but its exploration ability needs to improve. If original migration operator of BBO is directly used to solve MOPs, the population will gather and lose the diversity. Because of this, the mutation operator of DE is incorporated in migration of BBO to propose a novel hybrid migration operator. The operator combines the exploitation ability of BBO and the exploration ability of DE. The hybrid migration operator is shown in Table 2, where F is a random real number in the region $(0, 1)$. During the evolution process, the non-dominated feasible solutions are conserved in the archive. By the hybrid migration operator, individuals with bigger immigration rate will share the character variable from individuals with bigger emigration rate or from individuals mutated of the archive, which make the population have good convergence and diversity.

Table 2. Hybrid migration operator

<i>hybrid migration operator</i>
For $i=1$ to N Randomly select individual p_{s_1}, p_{s_2} from the current population select individual p_s from the archive select individual p_r with probability $\propto \mu_r$ For $j=1$ to D If $\text{rand} < \lambda_i$ then If $\text{rand} < 0.5$ $x_i(j) = p_s(j) + F(p_{s_1}(j) - p_{s_2}(j))$ Else $x_i(j) = p_r(j)$ End if Else $x_i(j) = x_i(j)$ End if End for End for

4.3. Hybrid algorithm BBO/DE

After computing the modified objective, individuals are sorted based on their non-domination relations, and then immigration rate and migration rate are obtained. The hybrid migration operator and original mutation operator are implemented to generate the next generation population. During the process, the non-dominated feasible solutions gained are conserved in the archive. When the size of archive exceeds the prefixed size, individuals with small crowding distance will be deleted until the fixed size.

The process of the proposed algorithm is described as follows:

Step1 : Parameter settings as population size N , maximum generation g_{\max} and the iterative generation $t = 0$.

Step2 : Generate a random initial population $P(t)$, $Q(t)$ and the archive $A(t)$ is empty,

Step3 : Calculate the constraint violation of individuals in $P(t) \cup Q(t)$ according to Eq(2), the non-dominated feasible individuals is selected and conserved in the archive $A(t+1)$. If the

size of $A(t+1)$ exceed the prefixed size, individuals with small crowding distance will be deleted until the fixed size.

Step4 Calculate the modified objective function value of individuals in $P(t) \cup Q(t)$, and implement non-dominated rank sorting and updating operator based on crowding-distance to gain the population $Q(t+1)$, then implement selection operator, hybrid migration operator and mutation operation on $Q(t+1)$ to obtain the offspring $P(t+1)$

Step5 : If $t \geq g_{max}$ is satisfied, export $A(t+1)$ as the output of the algorithm and the algorithm stops; otherwise, $t = t + 1$ and go to step3.

5. Experiment result

The proposed algorithm BBO/DE is tested on some benchmarks to validate the effectiveness for CMOPs. These benchmark problems are all minimization problems and are denoted as OSY [14], TNK [15], and CTP1-CTP4 [16]. OSY and CTP1 have continuous Pareto fronts and Pareto optimal region of OSY is a concatenation of five regions, while every region lies on the intersection of certain constraints, which demand the population to maintain subpopulation at different intersection of constraint boundaries; the remaining ones have disjoint Pareto fronts (TNK, CTP2-CTP4). For the function TNK, because it's Pareto optimal solutions lie on a non-linear constraint surface, and it is difficulty for the optimization algorithms in finding a spread of solutions across the entire Pareto optimal front. Classic algorithm NSGAI [17] is selected to compare with CMBBO to show its performance.

The parameter setting is Population size 100, the maximum number of generations 100 and a real number representation for decision variables. For BBO/DE, $E = I = 1$. For NSGAI [17], crossover probability 0.9, mutation probability $1/n$, n is the number of decision variable, SBX crossover parameter 20, polynomial mutation parameter 20. For above test functions, the optimal Pareto fronts by BBO/DE and NSGAI are shown Figures1-6.

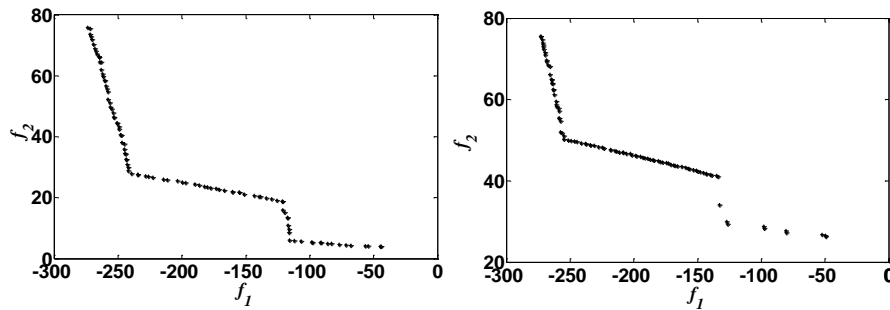


Figure 1. Pareto front gained by CBBMO and NSGAI for OSY

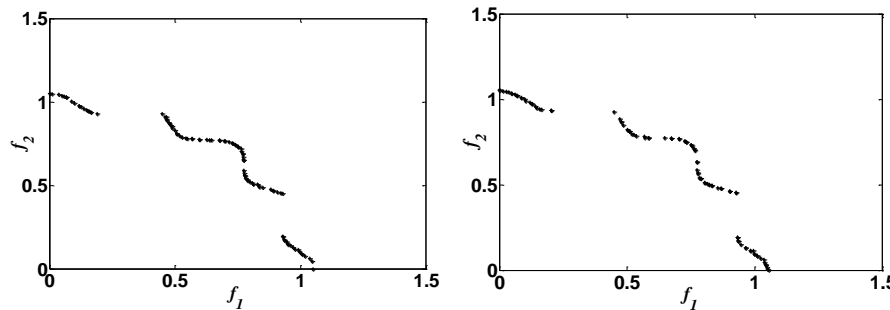


Figure 2. Pareto front gained by CBBMO and NSGAI for TNK

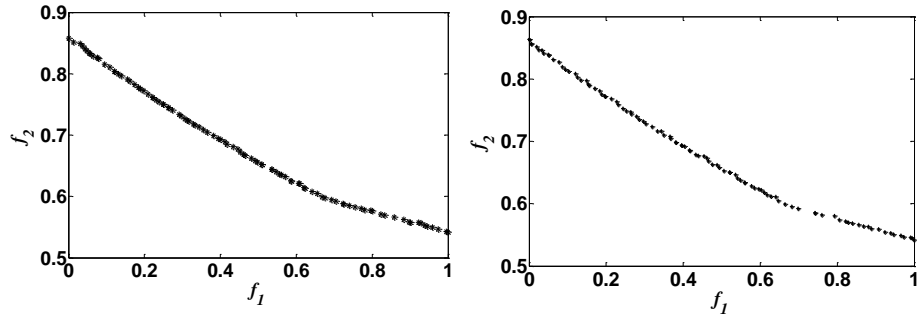


Figure 3. Pareto front gained by CBBMO and NSGAI for CTP1

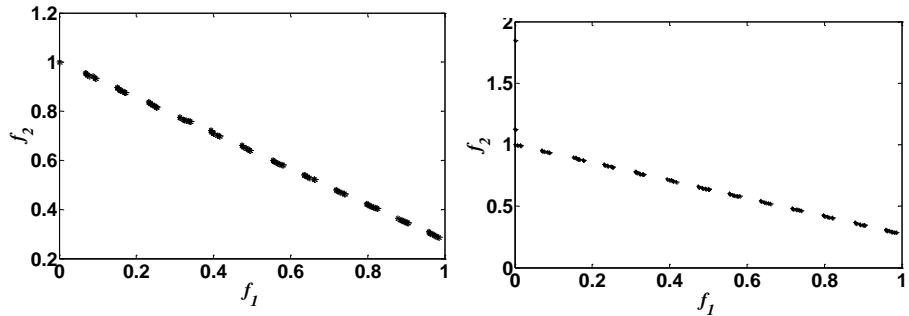


Figure 4. Pareto front gained by CBBMO and NSGAI for CTP2

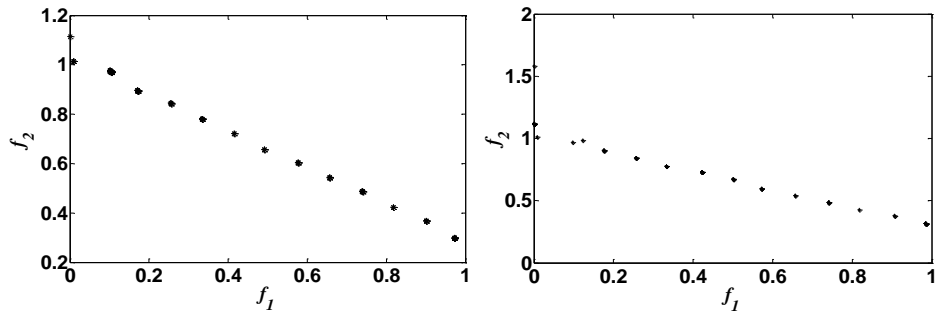


Figure 5. Pareto front gained by CBBMO and NSGAI for CTP3

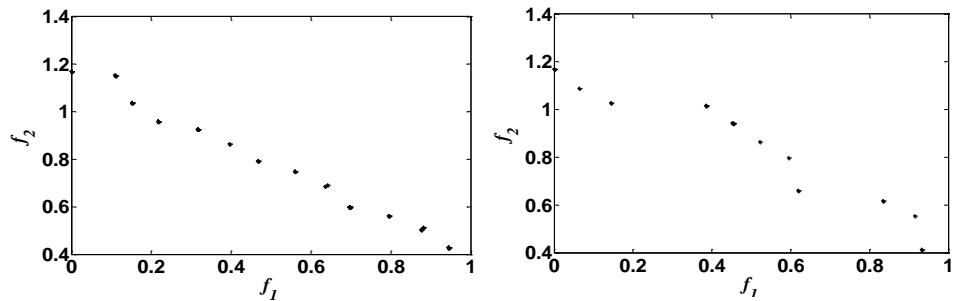


Figure 6. Pareto front gained by CBBMO and NSGAI for CTP4

For OSY in Figure1, it can be seen that the proposed algorithm can converge close to the true Pareto front and evenly distribute on the Pareto front. However, NSGAI can not maintain the diversity of subpopulation. For TNK of Figure 2 and CTP1 of Figure 3, it is shown that the proposed algorithm can search more diverse and wider spread solutions on the true Pareto front. However, NSGAI can not maintain the diversity of

the population during the convergence. For CTP2-CTP3, CBBMO can converge close to the true Pareto front but NSGAI has few solutions far from the Pareto front. For CTP4 in Figure 6, it has many disconnected feasible regions. Pareto solutions set by CBBMO can converge close the Pareto front and evenly distribute on the Pareto front but NSGAI can not find all feasible regions.

6. Conclusion

In this paper, we propose a new hybrid constrained multi-objective optimization algorithm BBO/DE. The algorithm BBO/DE adopts the adaptive penalty function to evaluate infeasible individuals so that infeasible individuals with small object or small constraint violation can contribute to the diversity of population. The migration operator of BBO is combined with the mutation operator of DE to evolve the population so that BBO/DE efficiently combines the exploitation ability of BBO and the exploration ability of DE. Experiment results show that the proposed algorithm BBO/DE is highly competitive in respect of convergence, diversity and even distribution for CMOPs.

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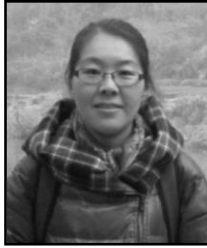
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Author



Zhidan Xu

Female, Born in 1980

Current position: Lecturer, School of Basic Science, Harbin University of Commerce, Harbin, China

Education: Ph.D. in engineering, Harbin Engineering University, China

Main Research Fields: Pattern recognition and intelligent system