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Multi-Objective Optimization Algorithm based on Biogeography with Chaos

Xu Wang[1] and Zhidan Xu[2]

Mathematics and Applied Mathematics, Harbin University of Commerce, Harbin, 150028, China
E-mail: wangxu9696@163.com, xuzhidanivy@163.com

Abstract

Biogeography-based optimization (BBO) has shown excellent exploitation ability of the population information for simple-objective optimization problem. But if BBO is directly applied in multi-objective optimization problems (MOPs), optimal solution set gained by BBO has worse diversity and distribution. To overcome these shortcomings, a chaos migration operator is put forwards to improve the diversity of the population. And then based on the new chaos migration operator, Chaos biogeography multi-objective optimization algorithm (CBBMO) is proposed for MOPs. In CBBMO, the chaos migration operator and original mutation operator of BBO are applied to produce the next generation population. The archive is used to conserve the Pareto optimal solutions. The experiment results show that the proposed algorithm CBBMO is feasible and effective for MOPs.

Keywords: *Biogeography-based optimization; chaos; multi-objective optimization*

1. Introduction

The evolutionary algorithm is a population-based stochastic algorithm by simulating biological evolutionary process and mechanism. In a run, many optimal solutions can be obtained so that it is very suitable to solve multi-objective optimization problems (MOPs). Since the 1970s, the researchers had raised the idea of applying genetic algorithm to solve (MOPs). In recent years, with some new intelligent optimization algorithms appearing, they have been applied extensively for solving (MOPs). These new intelligent optimization algorithms include multi-objective particle swarm optimization algorithm [1-2], multi-objective differential evolutionary algorithm [3], multi-objective simulated annealing algorithm and multi-objective immune algorithm [4] and so on. However, to improve the diversity and distribution of the obtained Pareto solutions set, it is very necessary to study and design new multi-objective optimization algorithm.

As a population-based stochastic algorithm, biogeography-based optimization (BBO) generates the next generation population by simulating the characteristics of the biological species migration. Main operators of BBO are the migration and mutation operators based on the emigration and immigration rate of each individual in the population. Because of sharing of the population information during the migration process, BBO has a better exploitation ability and it presents certain superiority for single-objective optimization (SOPs) [5-10]. However, there are few reports on applying BBO for MOPs. Therefore, in this paper, based on BBO, a new multi-objective optimization algorithm is proposed for MOPs. In the new algorithm, real coding will be adopted and the migration operator with chaos will be put forwards. The chaos migration operator incorporated chaos in the original migration operator

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so that worse individuals can share the characters of excellent individual or those with chaos. The operator can make the population evolve towards Pareto optimal front and keep the diversity of the population. By above operators, the chaos biogeography-based multi-objective optimization algorithm (CBBMO) is proposed to solve MOPs. Experiment results show that the obtained Pareto optimal solution set can approximate the real Pareto front and has good diversity and even distribution.

2. Basic conceptions

Without the loss of generality, we consider the minimization MOPs, which can be denoted as follows:

$$\begin{aligned} \min \quad & y = f(x) = [f_1(x), f_2(x), \cdots f_m(x)] \\ & x = (x_1, x_2, \cdots x_n) \\ & x_i^{\min} \le x \le x_i^{\max}, i = 1, 2 \cdots n \end{aligned} \quad (1)$$

where $x = (x_1, x_2, \cdots, x_n) \in D \subset R^n$ is a decision vector with n decision variables. D is a n dimension decision space. $y = (f_1, f_2, \cdots, f_m) \in Y \subset R^m$ is an objective vector with m objects. Y is a m dimension objective space. Each dimension variable of the decision space is bounded by its upper limits $x_i^{\max}$ and lower limits $x_i^{\min}$.

Definition 1(Pareto Domination) Let $x, y \in D$, and a solution vector x is said to dominate strictly a solution y and is denoted by $x \prec y$ iff

$$\begin{aligned} & 1) \forall i \in \{1, 2, \cdots m\}: f_i(x) \le f_i(y) \\ & 2) \exists j \in \{1, 2, \cdots m\}: f_j(x) \le f_j(y) \end{aligned} \quad (2)$$

Definition 2(Pareto optimal) A solution $y \in D$ is called Pareto-optimal with respect to D iff

$$\{x | x \prec y, x \in D \quad and \quad x \ne y\} = \Phi \quad (3)$$

Definition 3(non-dominated solution) Let $S \subset D$ be a subset of solutions, x is called a non-dominated solution with respect to S iff

$$\{y | y \prec x, y \in S\} = \Phi \quad (4)$$

Definition 4(Pareto optimal solution) v is called Pareto-optimal solution if v is non-dominated with respect to all solutions in D.

Definition 5(Pareto front) The image of all non-dominated solutions is called the Pareto front.

The shape of the Pareto front indicates the nature of the trade-off between the different objective functions. By using the distribution, the diversity and the approximation of solutions to the Pareto front, the performance of the algorithm can be evaluated efficiently.

3. Biogeography-Based optimization (BBO)

Biogeography is a subject that main investigates the time and spatial distribution of the species, namely the distribution of the biocenosis, its composition in the earth surface and their forming reasons. The biogeography was presented by Alfred Wallace and Charles Darwin in the 19th century, and then it was gradually improved and formed an independent subject in the 1960s. Subsequently various mathematic models on species distribution, migration and extinction are constructed. Based on these models, new various optimization algorithms constantly are proposed.

Inspired by the mathematic models of biogeography, Dan Simon proposed Biogeography-Based Optimization in 2008. The mathematic models of biogeography mainly describe how the species appear, extinct and migrate. The habitats suitable for the survival of species have higher habitat suitability index (HIS). The factors related to HIS include the rainfall, the vegetation diversity, the geomorphic features and the temperature of the region. These factors are called suitability index variables (SIVs). The habitats with higher HIS can accommodate more species while the habitats with lower HIS can hold fewer species. In accordance with the species of every habitat, the species conduct corresponding immigration and emigration. Due to the immigration of many species, the species in the habitats with higher HIS will become saturate gradually so that lots of species will emigrate, that is these habitats will have a bigger emigration rate and a smaller immigration rate. On the contrary, because the habitats with lower HIS have fewer species, more species will immigrate into and fewer species will emigrate from the habitats, so these habitats will have a bigger immigration rate and a smaller emigration rate. In other words, the HIS of the habitat is proportional to its species number. The individual with higher fitness will have a bigger emigration rate and a smaller immigration rate, while the individual with lower fitness will have a bigger immigration rate and a smaller emigration rate. The individual with higher fitness will share the SIVs with the individual with lower fitness so that the individuals with lower fitness can accept the characteristic variables from the excellent individuals and improve its fitness as much as possible. Based on migration process, the habitats are as individuals ,the migration and mutation operators are shown as follows.

A. Migration Operators

Sorting by the fitness of the individual, we can obtain the species number of every habitats. Through various migration models [7] in the biogeography, every individual can get different immigration and emigration rates. This paper selects the common linear model. If an individual has k species, its immigration rate λ_k and emigration rate μ_k can be defined as [5]:

$$\lambda_k = I(1 - \frac{k}{S_{\max}}) \tag{5}$$

$$\mu_k = \frac{Ek}{S_{\max}} \tag{6}$$

where $S_{\max}$ denotes the maximum species number of all the habitats , I and E are the maximum immigration and emigration rate. With bigger species number k , the individual has lower immigration rate and higher emigration rate, that is the individual is an excellent individual. The excellent individuals will share their SIVs with worse individuals. The detail migration process is shown in Table1.Where N denotes the size of the evolution population, X_i is the i th individual, $X_{i,k}$ denotes the k th variable of individual X_i , rand denotes a random real number in the range (0,1) . By the migration operator, the individual X_j with higher emigration rate μ_j will share its characters $X_{j,k}$ with the other individuals with higher immigration rate λ_i . These individuals with lower fitness, that is high immigration rate, will accept some variables from excellent individuals with high fitness so that they can be transformed optimal solutions towards the optimal Pareto front.

Table 1. The migration operator

Algorithm 1 The migration operator

```
For i=1 to N do
  select X_i with probability ∝ λ_i
   If rand< λ_i then
    For j=1 to N do
     Select X_j with probability ∝ μ_j
      If rand< μ_j then
        Randomly select a variable X_{j,k} from X_j
        Replace the corresponding variable in X_{i,k} with X_{j,k}
     End if
    End for
   End if
End for
```

B. Mutation Operator

With the migration of the species in different habitats, the species number of these habitats will change constantly. Assuming the probability denote P_s that a habitat with the species number S, the change of the species probability P_s from the time t to $t+\Delta t$ is computed as [5]

$$p_s(t+\Delta t) = (1-\lambda_s-\mu_s)P_s\Delta t + \lambda_{s-1}P_{s-1}\Delta t + \mu_{s+1}P_{s+1}\Delta t \tag{7}$$

Where λ_s and μ_s are the immigration rate and emigration of the habitat with the species number S. The function of probability P_s is a symmetric function about the balance point (namely the corresponding point when $\lambda = \mu$). The individuals with bigger or smaller species number have lower stability probability and have higher chancing in mutating the other individuals. Based on this, the mutation rate m_i [5] is designed as:

$$m_i = P_{mute}\left(1-\frac{p_i}{p_{\max}}\right) \tag{8}$$

This mutation rate is inversely proportional to the species number probability P_s of habitat. If the individual has lower the species number probability P_s, then it has bigger mutation probability, so it has more chance to become a better individual. On the contrary, the individual with higher species number probability has smaller chance to mutate the other individual; therefore, the excellent individual can be reserved in the population. So the mutation can improve the diversity of the population and ensure its convergence. The detail mutation operator is shown in Table 2. In Table 2, a random SIV denotes some variable of a random individual selected. By the operator , the diversity of the population is increased.

Table 2. The mutation operator

Algorithm 2 Mutation operator

```
For i=1 to N do
  calculate m_i according to (8)
  If rand< m_i then
   replace X_{i,k} with a random SIV
  End if
End for
```

C. Algorithm Flow

The Biogeography-Based Optimization (BBO) produces the next generation population mainly through the above migration and mutation operators. Its flowchart can be seen in Table 3. Firstly, generate the initial population randomly. Secondly, calculate the fitness of the individuals in the population and obtain the immigration and emigration rate according to the species number of the individual. Thirdly, implement migration operators based on the immigration and emigration rate to make the population approximate the optimal Pareto front. And then implement mutation operators based on the individual species probability to improve the diversity of the population. The algorithm is run repeatedly until it meets the end condition. In BBO, the variable information of the excellent individuals can be shared by the migration operator, which ensures the convergence of the population. At the same time, the mutation operator can change the worst and best individuals in the current population to generate more excellent individual.

Table 3 .Pseudo-code of Biogeography-based optimization algorithm

BBO Algotithm
1：Generate randomly the initial population $P(t)$, $t = 1$
2：While $t <= g_{max}$,
3： Compute the fitness of individuals in the population $P(t)$
4： Sequence individuals of $P(t)$ in fitness ascending order
5： gain species number, immigration and emigration rate of individuals in $P(t)$
6： Implement migration and mutation operator on $P(t)$ to gain the population $P(t+1)$
7： $t = t + 1$
8：Endwhile

4. Implementation of CBBMO

Based on the excellent performance of BBO for Simple-objective optimization problems, the paper proposes multi-objective optimization algorithm based on BBO which is named as Chaos biogeography multi-objective optimization algorithm (CBBMO). In CBBMO, the real coding is adopted .The same fitness evaluation method as SPEA2 [11] is applied. After evaluation, the migration operator with chaos and mutation operator is applied to generate the next generation population. The non-dominated individuals obtained are conserved in the archive. When the size of the archive exceeds the refined size, archive truncation mechanism of SPEA2 [11] is used to update the archive.

A. Chaos migration operator

The migration operator of BBO is presented for Simple -objective optimization and adopts integer coding so that it is suitable to solve the discrete optimization problems. If it is directly applied to MOPs, the diversity of the solutions decreases gradually with the increasing of the evolutionary generation. Therefore, this paper proposes the chaos migration strategy. Chaos [12] is introduced in the migration operator. The individuals with high migration rate will share their characters variables with chaos to improve the population diversity and the detail operator is shown in Table 4.

Table 4. Chaos migration operator

Algorithm1: Chaos migration operator
For $i = 1$ to N do
select X with probability $\propto \lambda_i$
If rand< λ_i then
For $j = 1$ to N do
select X_j with probability $\propto \mu_i$
If rand< μ_i then
randomly select a variable $X_{j,k}$ from X_j
$X_{i,k} = X_{j,k} e^{(0.1(1-X_{i,k}))}$
End if
End for
End if
End for

B. Process of CBBMO

CBBMO is mainly composed of chaos migration, mutation operator and the updating operation of the archive. The chaos migration operator can share the information of the excellent individuals and enhance the diversity of the population. The adoption of mutation operator can further improve the diversity of the population and generate more excellent individual as much as possible. Apply the archive to conserve the non-dominated individuals in the population. When the archive exceeds the fixed size, it is update by using truncation mechanism of SPEA2 [11] to ensure the even distribution of the population. The main process of CBBMO is shown as Table5. In Table5, the non-dominated individuals gained by evolution are conserved in the archive population $Q(t)$ so that the population can evolve towards the Pareto front. The crowding-distance is applied to update the archive population which makes the population have good even distribution. By the chaos migration and mutation operator, the population can have good diversity. Given all that, CBBMO will have better performance.

Table 5. Chaos Biogeography-Based multi-objective optimization algorithm (CBBMO)

CBBMO Algorithm
Step 1: Generate randomly the initial population $P(0)$, the archive $Q(0)$ is empty set, the number of generation $T = 0$;
Step2: Compute the fitness of individuals in the population $P(t)$ and $Q(t)$ using the method of SPEA2[11]
Step3：Select the non-dominated individuals and conserve in the archive $Q(t+1)$, and then update $Q(t+1)$ by using crowding-distance

Step4：if $t > g_{\max}$, then output $Q(t+1)$ and stop, otherwise go to Step5.

Step5：Compute the fitness of individuals in $Q(t+1)$ and obtain the migration, immigration rate and mutation probability.

Step6：Implement chaos migration and mutation operator on the population $Q(t+1)$ to gain the population $Q'(t+1)$.

Step7：Repair the exceeding boundary of individuals in $Q'(t+1)$ to obtain the next generation population $P(t+1)$, $t = t+1$, go to Step2

5. The simulation experiment

To evaluate the performance of the algorithm CBBMO, benchmark problems ZDT1, ZDT2, ZDT4 and ZDT6 are selected to validate the effectiveness of CBBMO for MOPs; those problems are commonly used to test performance of multi-objective optimization algorithms. These test problems can effectively test if multi-objective optimization algorithm can approximate the true Pareto front and maintain good diversity and distribution.

In experiment, real coding is used. The parameters are set follows: initial mutation probability 0.5; the population size100; the archive size100; the biggest immigration and emigration rate E=I=1and the maximum generations 100.

A. The performance of chaos migration operator

Firstly, to demonstrate the performance of chaos migration, the proposed optimization algorithm CBBMO with chaos migration is run by increasing 10 iterations each time. ZDT4 and ZDT6 are selected to validate the performance of chaos migration and their Pareto fronts are shown in Fig1-2. From bottom to top, iterations times are 10, 20, 30, 40and 50, respectively. In Fig1, it can be seen that the diversity of the population is constantly improved with the iterations times increasing. When the iterations times are 10, 20 and 30, the solutions gained by CBBMO scatter several parts on the whole Pareto front. When the iterations times reach 50, the solutions gained can scatter on the whole Pareto front. For ZDT6, when the iterations times are from 10 to 40, there are some gaps on the Pareto front. Until 50, the solutions gained have good diversity and even distribution. Given all that, the chaos migration can make the population evolve efficiently and have good performance.

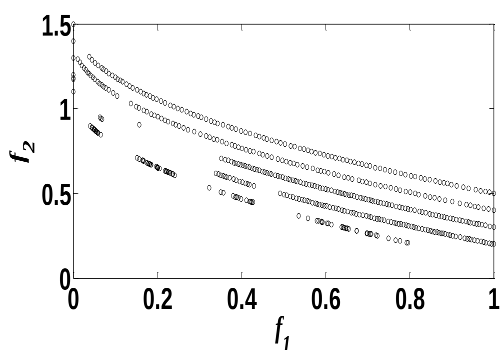

Figure 1. Pareto front of ZDT4 for iterations 10,20,30,40 and 50

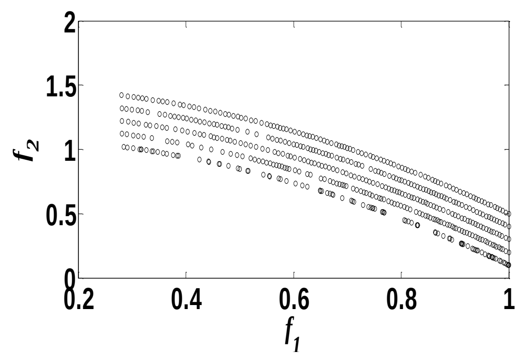

Figure 2. Pareto front of ZDT6 for iterations 10,20,30,40 and 50

B. The performance of CBBMO

To prove the superiority of the proposed CBBMO, it is compared with the multi-objective optimization algorithm with original migration operator of BBO. The optimal Pareto fronts gained by two algorithms are shown in Fig3-6. In Fig3-6, real dot denotes the optimal Pareto fronts obtained by CBBMO, however hollow circle denotes the optimal Pareto fronts obtained by the algorithm with original migration operator of BBO (MOBBO). For ZDT1 and ZDT2, MOBBO only can gain some parts of the Pareto front and has many gaps on it; CBBMO can obtain the whole Pareto front and has good distribution, which shows CBBMO is better than the MOBBO in the diversity and distribution for ZDT1 and ZDT2. Especially for the function ZDT4 with 21^9 local Pareto fronts, CBBMO can also get across all local Pareto fronts to approximate the true Pareto fronts, which indicates the algorithm CBBMO has good convergence. While MOBBO gains some parts of the Pareto front. For ZDT6, CBBMO is better than MOBBO in the diversity and distribution of the population.From these figures, we can see that CBBMO can converge the true optimal Pareto fronts with good diversity and distribution for all test problems. The algorithm with original migration operator of BBO (MOBBO) can also approximate the true Pareto fronts but the diversity and distribution of the population is worse than CBBMO. These results show that multi-objective algorithm with the chaos migration operator can efficiently improve the diversity of the population. Meanwhile CBBMO with the chaos migration is feasible and effective for solving MOPs.

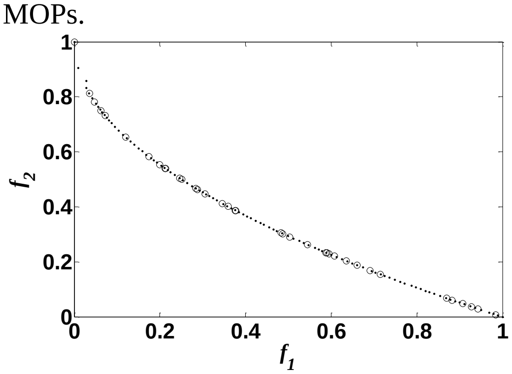

Figure 3. Pareto front of ZDT1

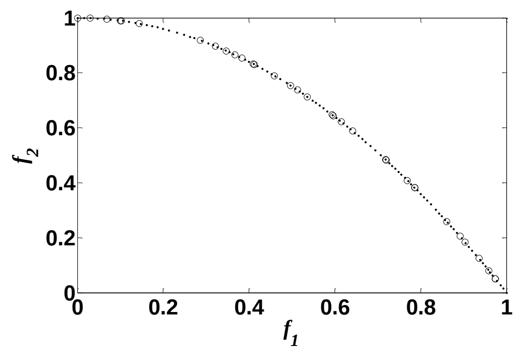

Figure 4. Pareto front of ZDT2

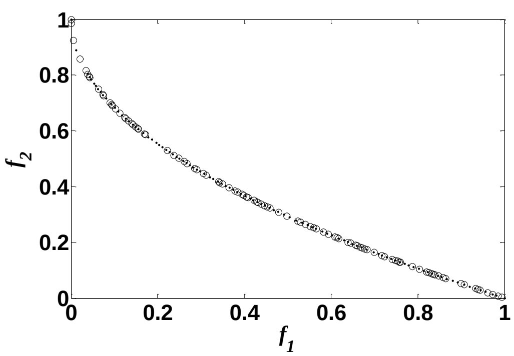

Figure 5. Pareto front of ZDT4

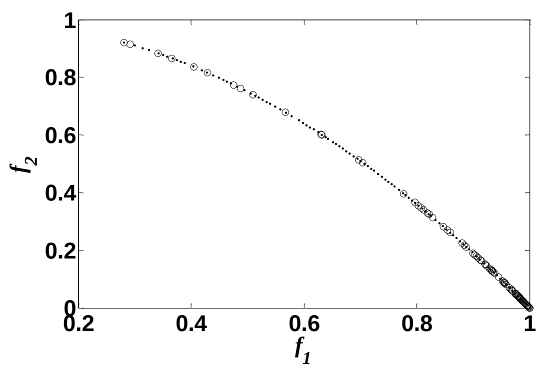

Figure 6. Pareto front of ZDT6

6. Conclusions

Based on the excellent performance of BBO for Simple-objective optimization problem, the paper proposes a new Chaos biogeography multi-objective optimization algorithm CBBMO. The algorithm CBBMO adopts the real coding and the same fitness evaluation method as SPEA2. After evaluation, the chaos migration operator and mutation operator is applied to generate the next generation population. During the evolution, the non-dominated individuals obtained are conserved in the archive. When the size of the archive exceeds the refined size, archive truncation mechanism of SPEA2 is used to update the archive. Benchmark test results show that the proposed algorithm CBBMO is feasible and effective for MOPs.

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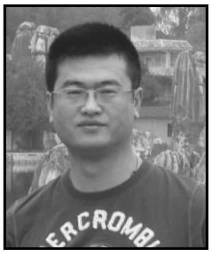
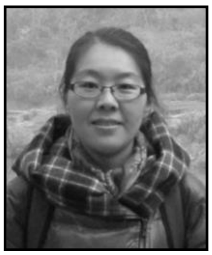
Authors



Xu Wang
Male, Born in 1980
Current position: Lecturer, School of Basic Science, Harbin University of Commerce, Harbin, China
Education: M.S. in Fundamental Mathematics, Civil Aviation University of China, China
Main Research Fields: Iterative Algorithm, Fixed Point Theory and Applications



Zhidan Xu
Female, Born in 1980
Current position: Lecturer, School of Basic Science, Harbin University of Commerce, Harbin, China
Education: Ph.D. in engineering, Harbin Engineering University, China
Main Research Fields: Pattern recognition and intelligent system