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Bottle Up Granular Computing Classification Algorithms

Hongbing Liu and Chang-An Wu

School of Computer and Information Technology, Xinyang Normal University, Xinyang 464000, China
liuhbing@126.com

Abstract

Shape of granule is one of the important issues in granular computing classification problems and related to the classification accuracy, the number of granule, and the join process of two granules. A bottle up granular computing classification algorithm (BUGrC) is developed in the frame work of fuzzy lattices. Firstly, the granules are represented as 4 shapes, namely hyperdiamond granule, hypersphere granule, hypercube granule, and hyperbox granule. Secondly, the granule set is induced by the training set and the bottle up join operator. Thirdly, machine learning benchmark datasets are used to analyze and discuss the BUGrC with different shape granules.

Keywords: *Hyperdiamond granule, hypersphere granule, hypercube granule, hyperbox granule, join operator.*

1. Introduction

Granular computing (GrC) is computing method based on the partition of problem space, and widely used in pattern recognition, information system, etc. Zadeh identified three fundamental concepts of the human cognition process, namely, granulation, organization, and causation [1, 2]. Granulation is a process that decomposes a universe into parts. Conversely, organization is a process that integrates parts into a universe by introducing operation between two granules. Causation involves the association of causes and effects. Information granules based on sets, fuzzy sets or relations, and fuzzy relations is computed in [3]. These studies enable us to map the complexities of the world around us into simple theories.

GrC based algebraic system is a frame computing paradigm that regarded the set of objects as granule, and the join operator and meet operator are the two keys of GrC. The join operator and meet operator are related to the shapes of granule.

The present work uses a fuzzy partial order relation (fuzzy inclusion relation) to form a fuzzy lattice based on a granule set with 4 granular shapes, and bottle up granule computing (BUGrC) paradigm is proposed based on the granule set, which is induced by the training set.

The rest of this paper is presented as follows: Section2 introduces the motivation and related works. BUGrC is described in Section3. Section4 demonstrates the comparative experimental results on two-class and multi-class problems. Section5 summarizes the contribution of our work and presents future work plans.

2. Motivation and Related Work

In this section, we present the motivation of our work, and discuss some related works.

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2.1. Motivation

Granular computing theory are formed based on the granule set and the relation between two granule, the granule set is induced by the training set, and the relation between two granule is determined by the shape of granule. For the training set S, the algebra system $\langle \wp(S), \subseteq \rangle$ is a lattice, where $\wp(S)$ is the power set of S, $\subseteq$ is the inclusion relation between two elements in $\wp(S)$. The power set $\wp(S)$ is regarded as the granule set composed of changeable granule, and $\subseteq$ is the inclusion relation between two granules. The inclusion relation between two granules is different from the traditional inclusion relation which is crisp. Namely, the inclusion relation between two granules is fuzzy. The lattice formed by fuzzy relation is called the fuzzy lattice. The fuzzy inclusion relation is often induced by the join operator and meet operator. The join operator $\vee$ has been used extensively in the fields of classification, neural networks, and machine learning in the context of mathematical morphology [4].

Different granule shapes are demonstrated in different representations, such as the two-point representation of a hyperbox granule and the single-point representation of hyperspherical granule. All samples that belong to a granule are mapped into a hyperbox, represented as a vector. Inconsistency exists between the partial order relation between two granules and that between two vectors. To eliminate this inconsistency, Kaburlasos and colleagues introduced the positive valuation function in linear or nonlinear forms, which satisfies the equality and inequality properties [5-7].

Granular computing classification algorithms with different shape granules are suitable for the different classification problems. Hypersphere granular computing classification algorithm is more suitable the hyperbox granular computing classification algorithm for the same classification algorithm. The steps of improving the testing accuracy of classification involve two aspects: (1) increasing the number of granule induced by the training set, (2) changing the shape of granules.

The objective of this article is to form the bottle up granular computing paradigm with 4 different shapes of granules on the fuzzy lattice, such as hypersphere granules, hypercube granules, hyperdiamond granules, and hyperbox granules.

2.2. Related Work

From 1988, Lin published articles on granular computing and neighborhood systems, mainly focusing on the granular computing model, which included binary relationship, granular structure, granule representation, and applications of granular computing [8–10]. Yao introduced the rough set in granular computing and discussed data mining, rule extraction, and machine learning methods based on granular computing [11-13]. Therefore, the relationship between two granules and the changeable granules based on their relationships are two issues in granular computing. Lattice computing and partial order relation can be used to solve these two issues. Fuzzy lattices have also been used to form classifiers. Kaburlasos and colleagues proposed a fundamentally new and inherently hierarchical approach on neurocomputing, called fuzzy lattice neurocomputing (FLN) [14]. Based on FLN, they designed fuzzy lattice reasoning (FLR) classifiers in which the partially ordered relationship is induced by the positive valuation function. FLR classifiers are applied to estimation of ambient ozone [15], using both lattice theory and granular computing. In granular computing, the fuzzy inclusion relation between two granules is used to form classification algorithms, which can obtain changeable hyperbox granules [16, 17].

3. Bottle Up Granular Computing Classification Algorithms

In this section, we form the bottle up granular computing classification algorithms.

3.1. Representation of Four Kinds Shapes of Granules

In reality, there are different shapes for granules. In the article, four shape granules are called hypersphere granules, hypercube granules, hyperdiamond granules, and hyperbox granules. Hypersphere, hypercube, and hyperdiamond are represented by the center and granularity, such as $G=(c,g_r)$. Hyperbox is represented by the beginning point and the end point, such as $G=[\mathbf{x},\mathbf{y},g_r]$.

(1) Hypersphere granule is represented as a vector including the center and the radii of the hypersphere.

(2) Hypercube granule is represented as a vector including the center and the half side of the hypercube.

(3) Hyperdiamond granule is represented as a vector including the hyperdiamond's center and the half diagonal of hyperdiamond.

(4) Hyperbox granule is represented as a vector including vectors induced the beginning points, the end points, and the distance between the beginning point and the end point.

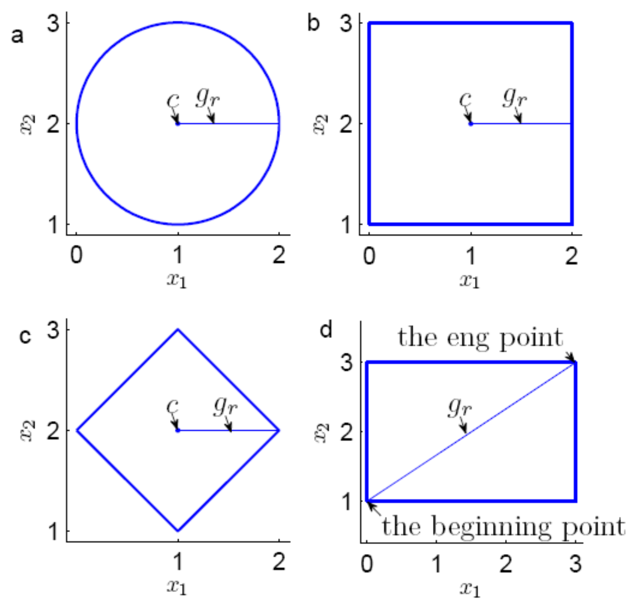

Figure 1. Shapes of granules in 2-dimensional space. (a) hypersphere granule, (b) hypercube granule, (c) hyperdiamond granule, (d) hyperbox granule

In 2-dimensional space, hypersphere granules, hypercube granules, hyperdiamond granules, and hyperbox granules are circle, cube, diamond, and rectangle respectively.

Granularity is the size of granule, we measure the granularity by diameter or radii. In Figure 1, the hypersphere granule (1,2,1) is represented as the circle with center(1,2) and radii 1 in Figure 1(a), and hypercube granule (1,2,1) is represented as the cube with the center (1,2), and the half side is 2 in Figure 1(b), a hyperdiamond granule (1,2,1) is represented as a diamond with the center (1,2), and the half diagonal is 1, and a hyperbox granule (0,1,3,3) is represented as a rectangle with the beginning point(0,1) and the end point (3,3).

3.2. Join Operator for BUGrC

Join operator $\vee$ is the key to design granular computing classification algorithms. Firstly, the atomic granule is represented by the single with granularity 0. Secondly, the join operator is designed to unite the granule with larger granularity compared with the atomic granule. Thirdly, the bottle-up granular computing classification algorithms are proposed by the designed join operator.

Suppose $G_1 \vee G_2 = (C, R)$ is the join hypersphere granule of two hypersphere granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, $C_{12} = C_2 - C_1$ is the vector from C_1 to C_2, where $C_1 = (x_1, x_2, ..., x_N)$, $C_2 = (y_1, y_2, ..., y_N)$, the join hypersphere granule induced by join operator is

$$G_1 \vee G_2 = [C, R] = \left[\frac{1}{2}(P+Q), \frac{1}{2}\|P-Q\|\right] \tag{1a}$$

Where $P = C_1 - r_1 \frac{C_{12}}{\|C_{12}\|}$, $Q = C_2 + r_2 \frac{C_{12}}{\|C_{12}\|}$. The granularity of join hypersphere granule $g_r(G_1 \vee G_2) = R$.

For two hypercube granules $G_1 = (C_1, r_1)$, $G_2 = (C_2, r_2)$, the join hypercube granule is $G_1 \vee G_2 = (C, R)$, where

$$R = \max(\max\{C_1, C_2\} - \min\{C_1, C_2\}) + r_1 + r_2 \quad C = \min\{C_1 - r_1 I, C_2 - r_2 I\} + R \tag{1b}$$

Where $\max\{C_1, C_2\}$ is a vector induced by the minimization of the corresponding components of vector C_1 and C_2, I is the vector with the same dimension as C_1 whose components are both 1. The granularity of join hypercube granule $g_r(G_1 \vee G_2) = R$.

For two hyperdiamond granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the vertex set of G_1 is $S_1 = \{C_1 - r_1 e_i \mid i = 1, 2, ..., N\}$, e_i is the identical vector whose the ith component is 1, the vertex set of G_2 is $S_2 = \{C_2 - r_2 e_i \mid i = 1, 2, ..., N\}$. Suppose $S = S_1 \cup S_2$ is composed of the vectors with length N, $S(:, i)$ is the set including the ith component of each element in set S, the joined hyperdiamond granule is $G_1 \vee G_2 = (C, R)$

$$\begin{aligned} C = \Big(& \frac{1}{2}\big(\max(S(:,1) - \min(S(:,1))\big), \frac{1}{2}\big(\max(S(:,2) - \min(S(:,2))\big), \\ & ..., \frac{1}{2}\big(\max(S(:,N) - \min(S(:,N))\big) \Big) \end{aligned} \tag{1c}$$

$$R = \frac{1}{2}\|S(id1,:) - S(id2,:)\|_1$$

Where $id1 = \arg\max S(:,1)$, $id2 = \arg\min S(:,1)$. The granularity of join hyperdiamond granule $g_r(G_1 \vee G_2) = R$.

For two hyperbox granules $G_1 = (\mathbf{x}_1, \mathbf{y}_1)$ and $G_2 = (\mathbf{x}_2, \mathbf{y}_2)$, the joined hyperbox granule is

$$G_1 \vee G_2 = (\mathbf{x}_1 \wedge \mathbf{x}_2, \mathbf{y}_1 \vee \mathbf{y}_2) \tag{1d}$$

Where $\mathbf{x}_1$ and $\mathbf{x}_2$ are vectors and represented as

$$\mathbf{x}_1 = (x_{11}, x_{12}, ..., x_{1N}), \ \mathbf{x}_2 = (x_{21}, x_{22}, ..., x_{2N})$$

$$\mathbf{y}_1 = (y_{11}, y_{12}, ..., y_{1N}), \ \mathbf{y}_2 = (y_{21}, y_{22}, ..., y_{2N})$$

The operators between vectors are

$$\mathbf{x}_1 \wedge \mathbf{x}_2 = (\min(x_{11}, x_{21}), \min(x_{12}, x_{22}), \ldots, \min(x_{1N}, x_{2N}))$$

$$\mathbf{y}_1 \vee \mathbf{y}_2 = (\max(y_{11}, y_{21}), \max(y_{12}, y_{22}), \ldots, \max(y_{1N}, y_{2N}))$$

The granularity of join hypersphere granule $g_r(G_1 \vee G_2)=\|\mathbf{y}_1 \vee \mathbf{y}_2 - \mathbf{x}_1 \wedge \mathbf{x}_2\|_2$.

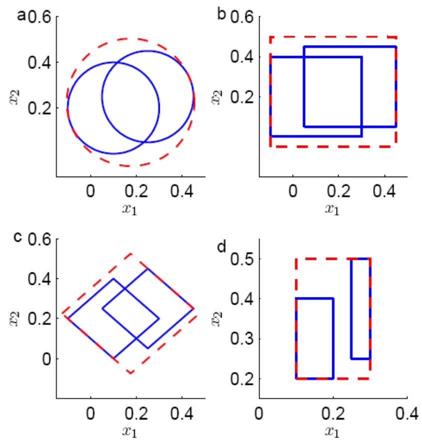

Figure 2. Join operators between granules

We explain the join process in 2-dimensional space. Suppose two G_1=[0.1 0.2 0.2] and G_2=[0.25 0.25 0.2], which can be represented as hypersphere (circle) granule, hypercube (cube) granule, and hyperdiamond (diamond) granule in 2-dimensional space. Two hyperbox (rectangle) granules G_1=[0.1 0.2 0.2 0.4] and G_2=[0.25 0.25 0.2 0.5], they are two hyperboxes (rectangles) in 2-dimensional space. The join granules, which are labelled by the red curves, are listed in Figure 2. In Figure 2 (a), the join circle granule is [0.1750 0.2250 0.2791], in Figure 2 (b), the join cube granule is [0.1750 0.2250 0.275], in Fig.2 (c), the join diamond granule is [0.1750 0.2250 0.3], in Figure 2 (d), the join rectangle granule is [0.1 0.2 0.3 0.5].

3.3 Algebra System Induced by Granule Set and Inclusion Relation

For training set (TS), the algebra system $\langle GS,\mu\rangle$ is formed by granule set (GS) and fuzzy inclusion relation μ. The algebra system $\langle GS,\mu\rangle$ is proved as fuzzy lattice. Because the traditional inclusion relation can't completely reflect the fuzziness, randomness, and uncertainty by which the research objectives are evaluated, Kaburlasos introduced the positive valuation function to form the fuzzy inclusion relation to measure the fuzzy inclusion relation between two granules. The fuzzy inclusion measure is compounded by the positive valuation of granules and their join granule.

$$\mu(G_1, G_2) = \frac{v(G_2)}{v(G_1 \vee G_2)} \tag{2}$$

Where $v(G)$:$GS \rightarrow R$ is the positive valuation function, which is the mapping between granule space and real number, and satisfies the equality property and inequality property: (1) equality property, $G_1 \subseteq G_2$ iff $v(G_1) \leq v(G_2)$, (2) inequality property, $v(G_1)+v(G_2)= v(G_1 \vee G_2)+v(G_1 \wedge G_2)$. We select the increasing function as the positive valuation function, which is compounded by the granularity. The positive valuation function is (3a) or (3b)

$$v(G)=g_r(G)+1 \tag{3a}$$

$$v(G) = \frac{1}{1+e^{-g_r(G)}} \tag{3b}$$

and the fuzzy inclusion measure is

$$\mu(G_1, G_2) = \frac{g_r(G_2) + 1}{g_r(G_1 \vee G_2) + 1} \tag{4}$$

The fuzzy inclusion measure satisfies the following properties.

$$G \in GS, G \neq \varnothing, \mu(G, \varnothing) = 0 \tag{5a}$$

$$\mu(G, G) = 1 \tag{5b}$$

$$G_1 \subseteq G_2 \Rightarrow \mu(G, G_1) \leq \mu(G, G_2) \tag{5c}$$

$$G_1 \wedge G_2 \subset G_1 \Rightarrow \mu(G_1, G_2) < 1 \tag{5d}$$

For real number set R, N-dimensional real number space R^N, granule set (GS) induced by the training set TS in N-dimensional space R^N, the following theorems are achieved.

Theorem 1. Algebra system $\langle R, \leq \rangle$ is lattice, $\leq$ is the less than or equal relation between two real numbers.

Theorem 2. Algebra system $\langle R^N, \leq \rangle$ is lattice, $\leq$ is the less than or equal relation between two vectors, and defined as $x \leq y \Leftrightarrow x_1 \leq y_1, x_2 \leq y_2, \ldots, x_N \leq y_N$.

Theorem 3. Algebra $\langle GS, \subseteq \rangle$ is lattice, $\subseteq$ is the inclusion relation between two granules.

Theorem 4. Algebra $\langle GS, \mu \rangle$ is fuzzy lattice, μ is the fuzzy inclusion relation between two granules.

3.4. Bottle Up Granular Computing Classification Algorithms

For training set TS, the bottle up granular computing classification algorithms are proposed by the following steps. Firstly, the samples are used to form the atomic granule. Secondly, the threshold of granularity is introduced to conditionally join the atomic granules by the aforementioned join operator, and the granule set is composed of all the join granules. Thirdly, if all atomic granules are included in the granules of GS, the join process is terminated, otherwise, the second process is continued. We explain the BUGrC as follows.

Suppose the atomic granules with the same class labels induced by TS are g_1, g_2, g_3, g_4, g_5. The training process of BUGrC can be described as the following structure of tree shown in Figure 3, leafs denote the atomic granules, root denotes GS including its child nodes G_2 and G_3, G_1 is induced by join operation of child nodes g_1 and g_2, G_2 is the join granule of G_1 and g_3, G_3 is the join granule of g_4 and g_5. The whole process of obtaining GS is the bottle up process.

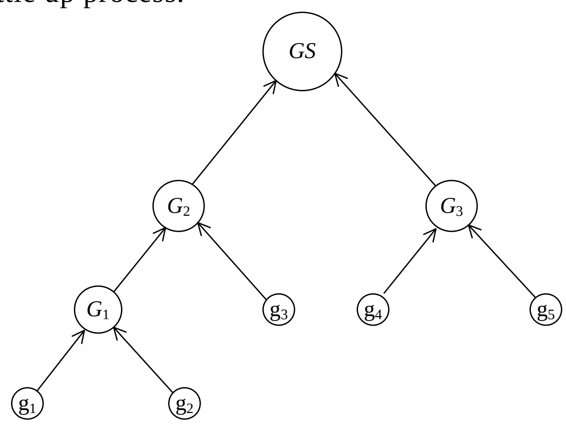

Figure 3. The training process of BUGrC including 5 samples

The training algorithm and testing algorithm are described as algorithm1 and algorithm2.

Algorithm1. BUGrC
Input: Training set TS, threshold ρ of granularity, the class number n
Output: Granule set GS, the class label lab

S1. initialize the granule set $GS=\varnothing$, $lab=\varnothing$
S2. i=1
S3. select the samples with class i, and form set X
  S31. initialize the granule set $GSt=\varnothing$
  S32. j=1
  S33. for the jth sample x_j in X, form the corresponding atomic granule G_j
  S34. k=1
  S35. compute the inclusion measure μ_{jk} induced by formula (3b) between the atomic granule G_j and the kth granule G_k in GSt
  S36. $k=k+1$
  S37. find the maximal inclusion measure μ_{jm}
  S38. if the granularity of the join of G_j and G_m is less than or equal to ρ, the granule G_m is replace by the join, otherwise G_j is the new member of GSt.
  S39. remove x_j until X is empty.
S4. $GS=GS\cup GSt$, $lab=lab\cup\{i\}$
S5. if $i=n$, output GS and class lab, otherwise $i=i+1$

Algorithm2. Testing process
Input: inputs of unknown datum x, granule set GS, the class label lab
Output: class label of x

S1. x is represented as granule g
S2. for i = 1:|GS|
S3. compute the fuzzy inclusion measure μ_i between g and g_i in GS
S4. find the maximal inclusion measure μ_m
S5. find the corresponding class label of the g_m as the label of x

4. Experiments

We evaluated the effectiveness of BUGrC in spaces R^2 and R^N using Intel PIV PC with 2.8 GHz CPU and 2 GB memory, running Microsoft Windows XP Professional and Matlab 7.0. The classification problems and the shapes of granules are shown in spaces R^2 clearly. Data sets in spaces R^N listed in Table 2 are selected from web site (http://sci2s.ugr.es/keel/datasets.php)to verify the performance of BUGrC. The data sets, and their 10-fold cross validation data and 5-fold cross validation data can be found in the Web site. We select 10-fold cross validation data to analyze and discuss BUGrCs with different shape granules from testing accuracy including maximization (max), minimization (min), mean, and standard deviation (std).

4.1. Classification Problems in space R^2

The spiral classification is a difficult problem to be classified, and used to evaluate the performance of classifiers. The data proposed in reference [18] are used to evaluate the performance of GrC. The data set is composed of 312 data including input in R^2 and 3 class labels, the data and the induced granules are showed in R^2, in which we can see the shapes of granules.

The threshold ρ of granularity is form 1 to 0 with step 0.01, the maximal accuracy is the selection indicator of optimization algorithms. Performances of BUGrC with four kinds of shape are listed in Table 1. The training data and their granules were shown in Figure 4 in which the single points are the atomic granules, each point lies in a single granule. From the Table 1, we saw, BUGrCs with hypercube granules and hyperbox granules achieved the optimization performance because of the minimal size of *GS* including 27 granules when ρ=0.19 and 0.11, BUGrC with hyperdiamond granules is poor because of the maximal size of *GS* including 32 granules when ρ=0.22, and BUGrCs with hypersphere granules and hyperdiamond granules touched the best testing accuracy firstly. Granular computing classification algorithms with the minimal size of granule set are our pursuits in the same conditions for the maximal accuracy.

Table 1. Performance of BUGrC with different shape granules

Shapes	ρ	Size	Tr(%)
Hypersphere	0.22	29	100
Hypercube	0.19	27	100
Hyperdiamond	0.22	32	100
Hyperbox	0.11	27	100

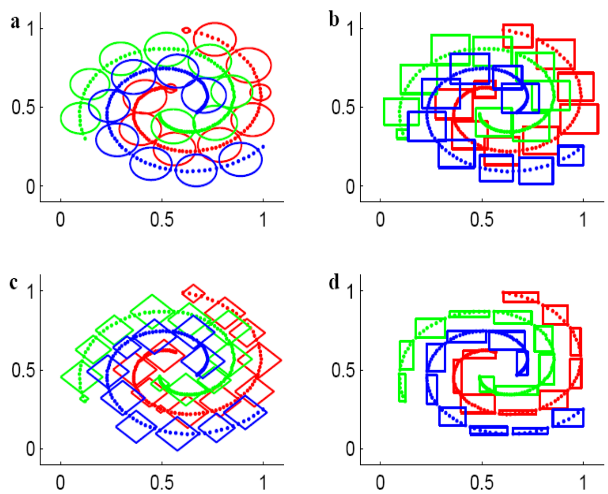

Figure 4. Spiral classification problem and *GS* (a) hypersphere granules, (b) hypercube granules, (c) hyperdiamond granule, (d) hyperbox granule.

4.2. Classification Problems in Space R^N

In order to evaluate the performance of BUGrC in space RN, four data sets listed in Table 2 are selected to perform the algorithms by 10-fold cross validation.

Table 2. Classification problems in R^N

Data sets	Sizes	Attributes	Classes
Balance	625	4	3
Wine	178	13	3
Phoneme	5404	5	2
Segment	2310	19	7

We selected the optimal parameters that maximized the testing accuracy. GrCs with four kinds of shape are performed in the same environment, and the performance is listed in table. From the table 3, we can see, (1) for different classification problems, the BUGrCs with different shape granules achieved the best testing accuracy. For classification problem balance, BUGrCs with hypersphere and hypercube granules achieved the maximal testing accuracies, and for wine problem, BUGrCs with hyperdiamond and hyperbox granules are much better than BUGrCs with hypersphere and hypercube granules. (2) the maximal testing accuracy of BUGrCs less than or equal to KNN algorithm, the maximal testing accuracy of KNN on data set balance is 93.6508%, which equals to BUGrCs, the maximal testing accuracy of BUGrCs on data set phoneme is 92.9760%, which is greater than 92.7911% obtained by KNN. (3) From the aspect of mean of testing accuracies, BUGrCs with hyperbox have the better testing accuracies compared with the other BUGrC.

Table 3. performance of BUGrC on classification problems in space R^N

Data sets	Shapes	Testing accuracies			
		max	min	mean	std
Balance	Hypersphere	**93.6508**	84.1270	87.8386	2.9220
	Hypercube	**93.6508**	79.3651	86.0694	4.6559
	Hyperdiamond	92.0635	79.3651	85.9210	3.8678
	Hyperbox	92.0635	84.1270	**87.9921**	2.7683
Wine	Hypersphere	88.8889	76.4706	80.8824	3.9764
	Hypercube	88.2353	70.5882	78.6601	5.8460
	Hyperdiamond	**94.4444**	82.3529	88.2026	5.5916
	Hyperbox	**100**	88.2353	**94.3464**	3.8175
Phoneme	Hypersphere	**92.9760**	87.9630	90.7467	1.6353
	Hypercube	92.4214	86.8519	89.9880	1.7296
	Hyperdiamond	**92.9760**	88.3333	90.6176	1.4865
	Hyperbox	92.7911	88.5185	90.8208	1.4556
Segment	Hypersphere	**99.5671**	94.8052	**97.4026**	1.4574
	Hypercube	98.7013	92.6407	95.4978	1.7579
	Hyperdiamond	**99.5671**	96.1039	97.5758	0.9882
	Hyperbox	**99.5671**	95.6710	**97.7922**	1.1758

5. Conclusions

The bottle up granular computing classification algorithms with different shape granules are proposed in the article. Firstly, a training datum is represented as an atomic granule. Secondly, the fuzzy inclusion measure between granules is form based on the join operator. Thirdly, the bottle up structure of training process is constructed based on the join operator and the threshold of granularity jointly. Finally, the proposed granular computing classification algorithms are demonstrated by the data set selected from references. BUGrC is affected by the sequence of the training data the same as the other granular computing. For the future work, we will focus on the distance measure between granules and how to form the granule set with mixing granules with different shapes.

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References

[1] L. A. Zadeh, "Fuzzy sets and information granulation", Advances in Fuzzy Set Theory and Applications, North Holland Publishing, (**1979**).

[2] L. A. Zadeh, "Towards a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic", Fuzzy Sets and Systems, vol. 19, (**1997**), pp. 111–127.
[3] L. Wang, X. Liu and W. Pedrycz, "Effective intervals determined by information granules to improve forecasting in fuzzy time series", Expert Systems with Applications, vol. 40, no. 14, (**2013**), pp. 5673-5679.
[4] P. Sussner and E. E. Laureano, "Morphological perceptrons with competitive learning: lattice-theoretical framework and constructive learning algorithm", Information Sciences, vol. 181, no. 10, (**2011**), pp. 1929–1950.
[5] V. G. Kaburlasos and S. E. Papadakis, "A granular extension of the fuzzy-ARTMAP (FAM) neural classifier based on fuzzy lattice reasoning (FLR)", Neurocomputing, vol. 72 nos. 10–12, (**2009**), pp. 2067-2078.
[6] V. G. Kaburlasos, L. Moussiades and A. Vakali, "Fuzzy lattice reasoning (FLR) type neural computation for weighted graph partitioning", Neurocomputing, vol. 72, nos. 10–12, (**2009**), pp. 2121-2133.
[7] S. E. Papadakis and V. G. Kaburlasos, "Piecewise-linear approximation of non-linear models based on probabilistically/possibilistically interpreted intervals' numbers (INs)", Information Sciences, vol. 180, no. 24, (**2010**), pp. 5060-5076.
[8] T. Y. Lin, "Data mining and machine oriented modeling: a granular computing approach", Journal of Applied Intelligence, vol. 13, no. 2, (**2000**), pp. 113–124.
[9] T. Y. Lin, "Granular computing rough set perspective. The Newsletter of the IEEE Computational Intelligence Society, vol. 2, no. 4, (**2005**), pp. 1543-4281.
[10] T. Y. Lin, "Data Mining and Machine Oriented Modeling: A Granular Computing Approach", Journal of Applied Intelligence, vol. 13, no. 2, (**2000**), pp. 113-124.
[11] Y. Y. Yao, N. Zhang, D. Q. Miao and F. F. Xu, "Set-theoretic approaches to granular computing", Fundamenta Informaticae, vol. 115, nos. 2-3, (**2012**), pp. 247-264.
[12] Y. Y. Yao, "Interpreting concept learning in cognitive informatics and granular computing", IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 39, no. 4, (**2009**), pp. 855–866.
[13] Y. Y. Yao, "Information granulation and rough set approximation", International Journal of Intelligent Systems, vol. 16, no. 1, (**2001**), pp. 87–104.
[14] V. G. Kaburlasos and V. Petridis, "Fuzzy lattice neurocomputing (FLN): a novel connectionist scheme for versatile learning and decision making by clustering", International Journal of Computers and their Applications, vol. 4, no. 3, (**1997**), pp. 31-43.
[15] V. G. Kaburlasos, I. N. Athanasiadis and P. A. Mitkas, "Fuzzy lattice reasoning (FLR) classifier and its application for ambient ozone estimation", International Journal of Approximate Reasoning, (**2007**), vol. 45, pp. 152-188.
[16] H. Liu, S. Xiong and Z. Fang, "FL-GrCCA: a granular computing classification algorithm based on fuzzy lattices", Computers and Mathematics with Applications, vol. 61, no. 1, (**2011**), pp. 138–147.
[17] H. Liu, S. Xiong and C. A. Wu, "Hyperspherical granular computing classification algorithm based on fuzzy lattices", Mathematical and Computer Modeling, vol. 57, (**2013**), pp. 661-670.
[18] H. Chang and D. Y. Yeung, "Robust path-based spectral clustering", Pattern Recognition, vol. 41, no. 1, (**2008**), pp. 191-203.

Author

Hongbing Liu, he received his Ph.D. degree from Wuhan University of Technology in 2011. He is an associate professor at Xinyang Normal University. His research interests include intelligent information processing, data mining, rough set and image processing, etc.

